VALTION TEKNILLINEN TUTKIMUSLAITOS
STATENS TEKNISKA FORSKNINGSANSTALT
THE STATE INSTITUTE FOR TECHNICAL RESEARCH, FINLAND

JULKAISU 153 PUBLICATION

AN APPROXIMATE SOLUTION OF A QUASI-LINEAR DIFFUSION PROBLEM

TABLES AND NOMOGRAMS FOR CONCENTRATION IN SLABS IN A SPECIAL CASE

S.S.H. KASI
S.E. PIHLAJAVAARA

HELSINKI 1969

VALTION TEKNILLINEN TUTKIMUSLAITOS

STATENS TEKNISKA FORSKNINGSANSTALT

(Lönnrotsgatan 37, Helsingfors. Puh. 64 89 31)

Anstalten är underställd handels- och industriministeriet. Det konstituerades 16. 1. 1942 (författningssamling n:o 44).

På Statens tekniska forskningsanstalt ankommer att bedriva teknisk forskningsverksamhet i vetenskapligt och allmännyttigt syfte mm. på byggnadstekniska området, att utföra materialprovningsuppdrag jämte andra forskningsuppdrag samt att bistå tekniska högskolan i undervisnings- och forskningsarbete.

Publikationerna erhålles genom Akademiska Bokhandeln, Centralgatan 2, Helsingfors.

STATENS BYGGEFORSKNINGSINSTITUT

(Borgergade 20, København K. Tlf. Minerva 56 30)

SBI er en selvstaendig institution, der ledes af en bestyrelse udpeget af boligministeren.

SBI er oprettet ved lov nr. 123 af 19. marts 1947.

SBI har til opgave "– at følge, fremme og samordne teknisk, økonomisk og anden undersøgelses- og forsøgsvirksomhed, som kan bidrage til en forbedring og billiggørelse af byggeriet, samt at udøve oplysningsvirksomhed angående byggeforskningens resultater".

Publikationerna erhålles genom bokhandeln eller Teknisk Forlag, Vester Farimagsgade 31, København V.

NORGES BYGGFORSKNINGSINSTITUTT

(Forskningsveien 36, Postadresse: Postboks 322 Blindern, Oslo 3, Tlf. 69 58 80)

NBI er et selvstendig institut under Norges Teknisk-Naturvitenskaplige Forskningsråd, som oppnevner styret. NBI blev oprettet 1953 og avlöste det midlertidige Kontoret for byggforskning fra 1949.

NBI skal fremme byggforskningen ved å klargjöre oppgavene og få dem löst, ved å virke for frivillig koordinering av tiltak og sörge for att försöksresultater blir gjort kjent.

NBI skal samarbeide med myndigheter, organisasjoner o. a. og bistå offentlige og private oppdragsgivere.

Publikationerna erhålles hos Norges Byggforskningsinstitutt, Forskningsveien 1, Blindern, Oslo.

STATENS RÅD FÖR BYGGNADSFORSKNING

(Linnégatan 64, Stockholm Ö. Tel. 08/63 56 20)

BFR - Byggforskningsrådet sorterar under Socialdepartementet.

BFR skall främja forskning och rationalisering inom byggnadsfacket och verka för att verksamheten inriktas på särskilt viktiga uppgifter, lämna medelsbidrag för forsknings- och försöksverksamhet inom byggnadsområdet samt draga försorg om att resultaten av denna verksamhet blir på lämpligt sätt offentliggjorda.

BFR sprider under samlingsnamnet Byggforskningen forsknings- och försöksresultat i form av handböcker, handlingar, rapporter, småskrifter, särtryck av tidskriftsartiklar m. m.

STATENS INSTITUT FÖR BYGGNADSFORSKNING

(Linnégatan 64, Stockholm Ö. Tel. 08/63 56 20)

sorterar under Socialdepartmentet,

bedriver sådan forsknings- och försöksverksamhet, som är ägnad att främja en rationell utveckling av planering, produktion och förvaltning inom byggnadsområdet (företrädesvis problem, som ej är föremål för uppmärksamhet från andra forskningsinstitutioners eller enskilda forskares sida).

Publikationerna erhålles genom AB Svensk Byggtjänst, Kungsgatan 32, Stockholm C. Postgiro 540 33. Tfn. 08/24 28 60.

23

UDC 532.685:517.94
666.972.015.22

VALTION TEKNILLINEN TUTKIMUSLAITOS
STATENS TEKNISKA FORSKNINGSANSTALT
THE STATE INSTITUTE FOR TECHNICAL RESEARCH, FINLAND

JULKAISU 153 PUBLICATION

AN APPROXIMATE SOLUTION OF A QUASI-LINEAR DIFFUSION PROBLEM

TABLES AND NOMOGRAMS FOR CONCENTRATION IN SLABS IN A SPECIAL CASE

S.S.H. KASI
S.E. PIHLAJAVAARA

HELSINKI 1969

VTT OFFSETPAINO 1969
388/4

Preface

For a number of years, one of the present authors, S.E. Pihlajavaara, has been engaged in research work on the drying of concrete [1]. The diffusion theory constitutes the basis of the mathematical treatment of the problem involved. The diffusion-type of equation, with diffusivity dependent upon concentration, has aroused particular interest, and some years ago nomograms were published on the drying of a concrete slab with the moisture conductivity (diffusivity) linearly dependent upon moisture [2]. The task of finding solutions for the cases in which the diffusivity is more generally dependent upon concentration was entrusted to Mr S. Kasi, M.Sc. (Eng.), Assistant of Physics at the Technical University of Helsinki.

For this work, he has prepared a program system for electronic computer ELLIOTT 503. There are only few restrictions as for the dependence of the diffusivity on the concentration. This dependence occurs, explicitly, only in the first part of the program. This part of the program has been programmed for two special cases. Finally, for another case, which has been of immediate practical interest [1], the results have been presented in the form of tables and nomograms. The manuscript has been written by Mr. Kasi.

The text of this paper has been read and criticised by Dr. Pentti Laasonen, Professor of Mathematics, and Dr. Pekka Jauho, Professor of Technical Physics, both of the Technical University of Helsinki.

The authors have been fortunate enough to receive financial support from the State Commission for Technology. Acknowledgement is also due to Professor Arvo Nykänen, director of this laboratory.

June, 1969

STATE INSTITUTE FOR TECHNICAL RESEARCH
Laboratory of Concrete Technology

S.E. Pihlajavaara

Contents

1. Introduction

The macroscopic, deterministic treatment of the random movement of a large number of mass entities, particles or quanta is governed by the diffusion equation, which in a one-dimensional case, assuming an isotropic medium and no sources, takes the form:

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial C}{\partial x}\right) = \frac{\partial C}{\partial h}, \qquad C = C(x,h). \tag{1}$$

C is a quantity expressing concentration (= density, temperature, etc.), h is time, and k is a so-called diffusion coefficient or diffusivity, (or conductivity). The greatest generality of the diffusivity is

$$k = k\,(x,h;C), \tag{2}$$

which means that the diffusivity in medium can vary spatially and temporally, and can be dependent upon concentration. Often, it can be so arranged that C is deleted from (2), and (1) then becomes a linear equation. However, many cases exist in which this idealisation of diffusion is not valid; consequently, attention is directed towards the quasi-linear diffusion problem, in which the concentration dependence of diffusivity, but only that, is assumed to be:

$$k = k\,(C). \tag{3}$$

When Formulae (1) and (3) are combined, and the region of variables x and h, an initial condition, and two boundary conditions are defined, set by the physical interests concerned, then the problem can be written in this manner:

$$\left.\begin{aligned} &C = C(x,h):\\ &\frac{\partial}{\partial x}\left(k\,(C)\,\frac{\partial C}{\partial x}\right) = \frac{\partial C}{\partial h}, \quad 0 < x < 1, \quad h > 0; \quad 0 < k(C) < \infty;\\ &C(x,0) = C_0, \quad \frac{\partial C}{\partial x}\,(0,h) = 0, \quad C(1,h) = C_e. \end{aligned}\right\} \tag{4}$$

C_0 is a uniform initial value of concentration, C_e is its constant value on the other boundary of the x-interval, and the concentration function of diffusivity, $k(C)$, is expressed above to be non-zero, positive, and bounded. The continuity of $k(C)$ is preferable. Discontinuity can in some degree be approved, although with a loss of accuracy.

The variables in (4) will be made dimensionless, defining a relative concentration u, a dimensionless diffusion coefficient $D(u)$, a new spatial coordinate z, and instead of time, the so-called »Fourier number», in conformity with

$$u = \frac{C - C_e}{C_0 - C_e}\,, \tag{5}$$

$$k = k(C) = k_e\,D(u)\,, \tag{6}$$

$$z = x/l\,, \tag{7}$$

$$Fo = k_e h/l^2\,, \tag{8}$$

respectively. If $k(C)$ is continuous, then $D(u)$ is also. k_e is a constant, with a dimension of diffusivity. Problem (4) becomes

$$\left.\begin{array}{l} u = u(x, Fo): \\[2mm] \dfrac{\partial}{\partial z}\left(D(u)\,\dfrac{\partial u}{\partial z}\right) = \dfrac{\partial u}{\partial Fo}\,, \quad 0 < z < 1, \quad Fo > 0; \;\; 0 < D(u) < \infty\,; \\[2mm] u(z,0) = 1, \quad u(1,Fo) = 0, \quad \dfrac{\partial u}{\partial z}\,(0,Fo) = 0. \end{array}\right\} \tag{9}$$

A program for the solution of (9) has been prepared in this work.

The solution of problem (9) is also that of the problem with the corresponding equation in the interval $-1 < z < 1$, when the last boundary condition in (9) is replaced by $u(-1,Fo) = 0$. (Both geometries are illustrated in Fig. 1).

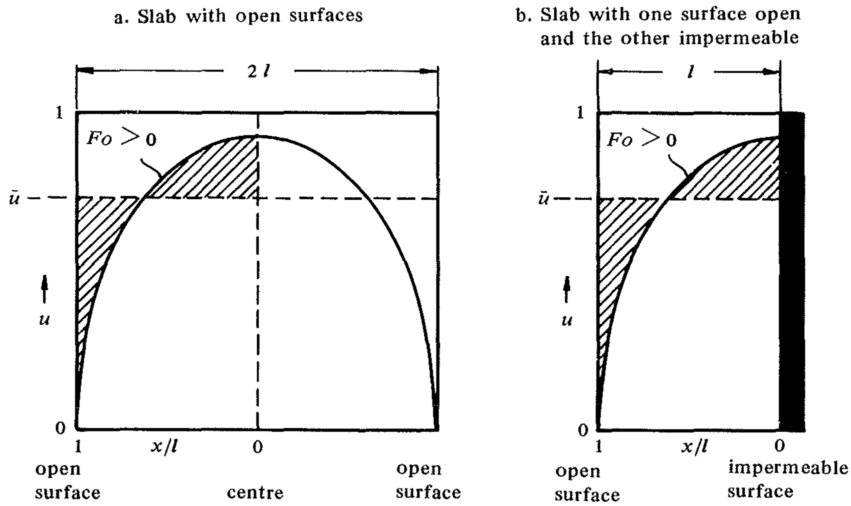


Figure 1. Showing the meaning of the symbols and giving examples of distribution of concentration u and corresponding average concentration $\bar{u}$ in an infinite slab or wall with finite thickness (2 l or l), at a fixed time or Fourier number $Fo = k_e h/l^2$.

The program prepared gives the following results:

distribution of concentration $u = u(z, Fo)$

average concentration $\bar{u} = \bar{u}(Fo)$

flux on the open surface $\phi = \phi(Fo)$,

(Cf. Fig. 1).

This problem has arisen from the investigations conducted by PIHLAJAVAARA [1] on the drying of concrete. The problem in (4) and (9) then corresponds to the drying of a large (considered infinite) concrete slab of constant thickness, with uniform initial moisture distribution, when on both surfaces of the slab there is the same constant moisture content, and l is one half of the thickness of the slab (Fig. 1a), or the constant moisture content exists only on one surface, the other surface is impermeable, and l is the thickness of the slab (Fig. 1b). Pihlajavaara has found experimentally the functional dependence $k(C)$ between moisture conductivity and the moisture content of some concretes [1, pp. 41 ··· 45]. $D(u)$ could have been constructed on the basis of this relation and (5), then containing as parameters the constants C_0 and C_e. However, he points out [1] that in practice the formula

$$D(u) = 1 + bu^n \tag{10}$$

is applicable also where b and n are constants determined by experimental data, including also the values of C_0 and C_e. The solution of (9) with (10) is the special case in which thorough calculations were made by application of the program.

2. Solution of the problem and its programming

The procedure involved in solving the problem (9) will be improved, and the set of soluble problems enlarged, but the present form of the Algol program for the solution is reported here [Appendices I ··· IV], along with an elucidation of its course.

This program has two main parts. The first of these [Appendix I and IV], includes all those stages in which the relations (5) and (6) are required; the second part of the program [Appendix II], however, remains unchanged with alteration in function $D(u)$ or its presentation.

For the whole program, an auxiliary quantity is used

$$s = \frac{\int_0^u D\,\mathrm{d}u'}{\mathrm{denom}}\,, \qquad \mathrm{denom} = \int_0^1 D\,\mathrm{d}u'\,, \tag{11}$$

which thus has the range [0,1]. s is a smoother function of z than u. The direction of the z-axis is now turned: $z' = 1 - z$. With s used as the function, the problem is then

$$\left.\begin{aligned} &s = s(z', Fo):\\ &D\,\frac{\partial s}{\partial z'^2} = \frac{\partial s}{\partial Fo}\,, \quad 0 < z' < 1, \quad Fo > 0; \quad D = D(u(s));\\ &s(z',0) = 1, \quad s(0,Fo) = 0, \quad \frac{\partial s}{\partial z'}(1,Fo) = 0. \end{aligned}\right\} \tag{12}$$

2.1. *Initial calculation*

The most useful method for the solution of a parabolic problem such as (12) is the replacement of its differential equation by a sufficiently precise difference equation, which has a stable and quickly achievable solution. It is observable from (9) and (12) that at the point ($z' = 0$, $Fo = 0$), u and s have a step-wise change from 0 to 1. This fact entails that the direct use of any difference scheme is complicated and time-consuming. We shall, consequently, adopt another method applicable when Fo is near 0; the step-wise change can thus be avoided.

This method has been illustrated by Crank in [3, pp. 207 ··· 210]. According to him, we can, for as long as the auxiliary quantity s at the middle point of the slab, $z' = 1$, is not appreciably reduced, treat the problem as one of semi-infinite diffusion:

$$\left.\begin{aligned}
&s = s(z',Fo):\\
&D\frac{\partial^2 s}{\partial z'^2} = \frac{\partial s}{\partial Fo}, \quad z' > 0, \quad Fo > 0;\\
&s(z',0) = s(\infty,Fo) = 1, s(0,Fo) = 0.
\end{aligned}\right\} \tag{13}$$

In the problem with boundary conditions of this kind, a change can be made in the variables first introduced by Boltzmann; this reduces the partial differential equation of problem (13) to an ordinary differential equation [3, pp. 148 ··· 154]. The new variable is

$$y = z'/(2\sqrt{Fo}), \tag{14}$$

and problem (13) then changes to

$$\left.\begin{aligned}
&s = s(y):\\
&-2y\frac{ds}{dy} = D\frac{d^2 s}{dy^2}, \quad y > 0;\\
&s(0) = 0, \quad s(\infty) = 1.
\end{aligned}\right\} \tag{15}$$

The integral form of (15) is derived direct by quadrature:

$$s = \frac{\int_0^y \exp\left(-\int_0^{y'} \frac{2y''}{D(s)}\,dy''\right) dy'}{\int_0^\infty \exp\left(-\int_0^{y'} \frac{2y''}{D(s)}\,dy''\right) dy'}. \tag{16}$$

The solution of (16), $s(y)$ is arrived at iteratively, and from this is further calculated the function

$$s(z', \mathrm{Focr}) = s(z'/2\sqrt{\mathrm{Focr}}), \tag{17}$$

where Focr is the most extreme value of Fo at which (13) is still considered appropriate. The function $u(z', \mathrm{Focr})$ is then calculated by a special procedure, located in the first part of the program. The distributions of concentration which express the solution of (9) when $Fo \leqslant \mathrm{Focr}$ can now be calculated, (u of (9) replaced by U), by means of

$$U(z,Fo) = \begin{cases} u\left(\sqrt{\dfrac{\mathrm{Focr}}{Fo}}\,(1-z), \mathrm{Focr}\right), & \text{when } \sqrt{\dfrac{\mathrm{Focr}}{Fo}}\,(1-z) \leqslant 1\\[2ex] 1, & \text{when } \sqrt{\dfrac{\mathrm{Focr}}{Fo}}\,(1-z) > 1. \end{cases} \tag{18}$$

These distributions for stated values of Fo are calculated in the program, and listed in a table, and as curves (as presented here in nomograms). In a nomogram, and in a table, z increases from right to left, i.e., from 0 to 1. In the first three columns remaining in the table there are listed: 1) the value of the Fourier number, and 2) the average concentration

$$\bar{u}(Fo) = 1 - (1 - \bar{u}(\text{Focr}))\sqrt{\frac{Fo}{\text{Focr}}}\,, \tag{19}$$

where

$$\bar{u}(\text{Focr}) = \int_0^1 u(z, \text{Focr})\,\mathrm{d}z,$$

and 3) the flux on the open surface,

$$\phi(Fo) = \left(-D\,\frac{\partial u}{\partial z}\,(Fo)\right)_{z=1} = -\,\frac{\mathrm{d}\bar{u}}{\mathrm{d}Fo}\,(Fo) = \frac{1 - \bar{u}(\text{Focr})}{2\sqrt{Fo\,\text{Focr}}}\,. \tag{20}$$

2.2. *Net-analysis*

In this concluding section of the solution of problem (13), the use of a difference equation is valid. An appropriate difference scheme, a modified Crank-Nicolson equation, presented by J. DOUGLAS, Jr. [4, pp. 503 ··· 509] has been introduced. It is assumed that the area of the coordinates ($0 \leqslant z' \leqslant 1$, $Fo \geqslant \text{Focr}$) is divided by the lines parallel to the coordinate-axes. The range of z' is divided into 80 ($= K$) equal intervals of length $\Delta z' = 0.0125$, and $z_i = i\Delta z'$. The Fourier number, starting at Focr, grows in the solution line by line with the step of ΔFo calculated from the stability conditions, and from two conditions adapted with a view to the purposeful progress of the program:

$$\frac{\Delta Fo}{\Delta z'} = \min\left(\frac{2}{\sqrt{a^{**}D^{*}}}\,,\ \ \frac{2}{\sqrt{(aD)^{*}}}\,,\ \ Fo/7,\ \ .025\right),$$

where

$$\left.\begin{aligned} a^{**} &= \max_{1 \leqslant i \leqslant 79}\left(-\frac{\mathrm{d}D}{\mathrm{d}s}\,\frac{\partial^2 s}{\partial z'^2}\right)_i \\ D^{*} &= \max_{0 \leqslant i \leqslant 80} D_i, \\ (aD)^{*} &= \max_{1 \leqslant i \leqslant 79}\left(D\,\frac{\mathrm{d}D}{\mathrm{d}s}\,\frac{\partial^2 s}{\partial z'^2}\right)_i. \end{aligned}\right\} \tag{21}$$

In the calculation, the derivatives are replaced by corresponding differences. The modified structure of the Crank-Nicolson equation enables its solution by the method of direct elimination, [5, pp. 103 ··· 105].

At some round values of Fo, there occurs the listing of a distribution of $u(z, Fo)$ as a curve, and also as numbers after the values of Fo, $\bar{u}(Fo)$ and $\phi(Fo)$ on the same line in the table. Let Fo' be the round value of the Fourier number at which the last listing occurred. A computer is prepared for the following listing as soon as

$$/u(z', Fo') - u(z', Fo)/ > 0.06, \quad \text{for any of } z_i = 0.1,\ 0.2, \ldots, 1,$$

and then, as soon as Fo is the closest to some of the round values of the Fourier number, the new listing is done. Let this round value be denoted by Fo''. Then the function

$s(z', Fo'')$ is first calculated as an array from the known arrays of $s = s(z', Fo)$ and $D = D(z', Fo)$, in accordance with

$$\left.\begin{aligned} &s(0, Fo'') = 0, \\ &s(z_i, Fo'') = s_i + \frac{\Delta F D_i}{(\Delta z')^2} (s_{i-1} - 2s_i + s_{i+1}), \\ &i = 1, 2, \ldots, 80, \end{aligned}\right\} \tag{22}$$

where

$$\Delta F = Fo'' - Fo. \tag{23}$$

The values of $u(z', Fo'')$ are calculated by the procedure given in the first part of the program. (The modification of the Crank-Nicolson equation is also based upon the formula of (22)).

$$\bar{u}(Fo'') = \int_0^1 u(z, Fo'') \, \mathrm{d}z, \tag{24}$$

and $\phi(Fo'')$ is derived by

$$\phi(Fo'') = -\frac{\mathrm{d}\bar{u}}{\mathrm{d}Fo} \tag{25}$$

which is situated as the third number on the line in the table, and also by a Lagrange interpolation for

$$\phi(Fo'') = \left[D \frac{\partial u}{\partial z'} (Fo'') \right]_{z'=0} = (\text{denom}) \left[\frac{\partial s}{\partial z'} (Fo'') \right]_{z'=0}, \tag{26}$$

(Cf. (20)). The value of (26) for the flux is on its own line in the table, just below the value of (25). The value of (26) is generally a better approximation for the flux than (25). The curve of $u(z, Fo'')$ has also been drawn with the aid of a Lagrange interpolation.

2.3. *Checking precision and stability*

The opportunity available for checking the accuracy of the results of this program is provided by the equivalence of the negative derivative of the average concentration to the flux on the open surface, (Cf. (20), (25), and (26)).

In the Boltzmann-transformed part of the program, check made once is valid for the whole part, since it is based upon the integration of this equivalence. The flux on the surface is (Cf. (11), (13), (14)),

$$\phi(Fo) = \left(D \frac{\partial u}{\partial z'} \right)_{z'=0} = \frac{1}{2\sqrt{Fo}} \left(D \frac{\mathrm{d}u}{\mathrm{d}y} \right)_{y=0} = \frac{(\text{denom})}{2\sqrt{Fo}} \left(\frac{\mathrm{d}s}{\mathrm{d}y} \right)_{y=0}.$$

The integral of this from 0 to Fo is the total amount of diffusion substance extracted from the (semi-infinite) slab. This becomes

$$(\text{denom}) \sqrt{Fo} \left(\frac{\mathrm{d}s}{\mathrm{d}y} \right)_{y=0}.$$

This must be equal to the integral of the negative derivative of the average concentration expressed by

$$-\int_0^{Fo} \frac{d\bar{u}}{dFo}(Fo)\ dFo = 1 - \bar{u}(Fo) = \int_0^{\infty}(1-u)\,dz' = 2\sqrt{Fo}\ \int_0^{\infty}(1-u)\,dy.$$

The new equality divided by $\sqrt{Fo}$ becomes

$$2\int_0^{\infty}(1-u)\,dy - (\text{denom})\left(\frac{ds}{dy}\right)_{y=0} = 0. \tag{27}$$

The value of the left hand side of (27) is calculated in the program and printed in the table, after all the lines presenting the results of the Boltzmann-transformed solution. Its closeness to 0 characterises the accuracy of this part of the calculation.

The check on stability during the net-analysis part of the program has the same basis. There, the flux $\phi(Fo)$ has been calculated in two ways, expressed by formulae (25) and (26), of which (25) is approximated by

$$-\frac{\bar{u}(Fo'') - \bar{u}(Fo)}{\Delta F}$$

where the denominator has the form of (23). This approximation gives the true value of the derivative concerned at some value of Fo either smaller or larger than Fo''. The flux of (26), derived approximatively by interpolation, however, belongs to Fo''. For stability, these values for (25) and (26) should be almost equal, and the sign of their difference should vary at random. When such behaviour is apparent, the absence of at least severe stability disturbances should be guaranteed, as in study of the qualities of the program, it was found that instability first becomes perceptible at the values near the ends of the range of z.

2.4. *Different forms of the first part of the program*

A more thorough calculation was made when its first part was formed in correspondence with (10), [Appendix I], (Cf. Chapter 3).

Another first part of the program produced, [Appendix IV], is of more general nature. In this, function $D(u)$ is given to the computer as an one-dimensional array with $M + 1$ elements. M is the number of equal intervals into which the range of u, [0,1], is divided. The elements of the array of $D(u)$ are the values of D at the division, and end points of the u-range. The whole program, without drawings, was calculated with the elements of D being determined in accordance with $D(u) = 1 + 40\,u^5$. Table 21 presents the results of this calculation; they agree well with the values given in Table 15.

We are not yet certain that the main program will function perfectly in all special cases.

2.5. *Function from averages*

After every step of *Fo*, with the relevant calculations, the values of *Fo* and $\bar{u}$ are punched into a paper tape, punch (2). Thus, at the end of the calculation, this paper tape includes a presentation of function $\bar{u}(Fo)$ from Fo = Focr to the value of *Fo*, at which the calculation is finished.

For drawing the curve $\bar{u}(Fo)$, we have an extra program. [Appendix III], in which it is drawn in a rectangular coordinate system, with the coordinate of *Fo* logarithmic. At small Fourier numbers the curve is drawn in accordance with (19), and elsewhere Lagrange interpolation is applied. The drawing of different curves on the same figure can be performed arbitrarily many times.

3. Solution in a special case

3.1. *Problem and program*

The preliminary aim of construction of our program is solution of the problem indicated by formulae (9) and (10). In (10), for parameters b and n, there are selected almost all the combinations of

$$\begin{cases} b = 0, 2, 5, 10, 20, 40, 80 \\ n = 1, 2, 5, \infty. \end{cases}$$

$$D(b=0) = D(n=\infty) = 1. \tag{28}$$

The computer employed is an Elliott 503. The paper tapes of *the main program* are input in the order:

Tape (1) – – CALCOMP PROCEDURES 1, SHORTENED
Tape (2) – – START OF DIFFUSION CALC. WHEN $D = 1 + b\,u^n$
Tape (3) – – UNCHANGEABLE PART OF DIFFUSION CALC.

(Data) – – The values of b and n, in this order.

The duration of one calculation increases with b, and is about 5 ··· 15 minutes. The *extra program* for drawing function $\bar{u}(Fo)$ includes the following paper tapes:

Tape (4) – – CALCOMP, ROTATED, SHORTENED
Tape (5) – – CURVE OF AVERAGES

(Data) – – Paper tapes from punch (2) in the running of the main program. Another data tape will be read in after the release of DWAIT.

3.2. *Results*

The results obtained from the main program with the different values of b and n are presented in tables and Nomograms 1 ··· 18. Curves of averages have been drawn in Nomograms 19 ··· 27, both when n is constant, and separately when b is constant.

The curve of averages obtained, when $b = 0$, belongs to all the bundles of curves, since, according to (28), it is the solution towards which the solution with constant b converges when n moves to infinity, and towards which the solutions with constant n converge when b moves to zero. The curve of averages with any values of b and n is everywhere situated below (or on) the curve of $b = 0$ (or $n = \infty$).

3.3. *Precision of results*

The only possibilities for direct estimation of the errors in results are indicated in (2.3). However, the problem in which $b = 0$ has the exact solution, and in all cases the program in a little modified structure was also suitable for calculation with other divisions of z, i.e., with different values of K, and with other Fourier number steps.

The exact solution when $b = 0$ is obtained, e.g., by CRANK in [3]. The corresponding result of our program is presented in Table 1 and in Nomogram 1. On comparison of the values given in the table with the exact results, the precision is determined: $\Delta \bar{u} \leqslant 0.0004$, $\Delta u \leqslant 0.0012$ everywhere, and $\Delta \phi/\phi \leqslant 0.1$ %.

The precision achieved during the Boltzmann-transformed part of the program is especially good. When $b = 0$, the error of the printed values in this part of the program is less than $2\ 10^{-5}$. The same order of error also occurs when b is small; errors gradually increase with increase in b, although the class of about 10^{-4} is not exceeded.

The net-analysis part of the program was tested with more precision in the cases of ($b = 40$, $n = 5$), and (80, 1). The step of the Fourier number then obeyed (Cf. (21)),

$$\Delta Fo = \frac{f \Delta z}{\sqrt{a^{**} D^{*}}} \quad ,$$

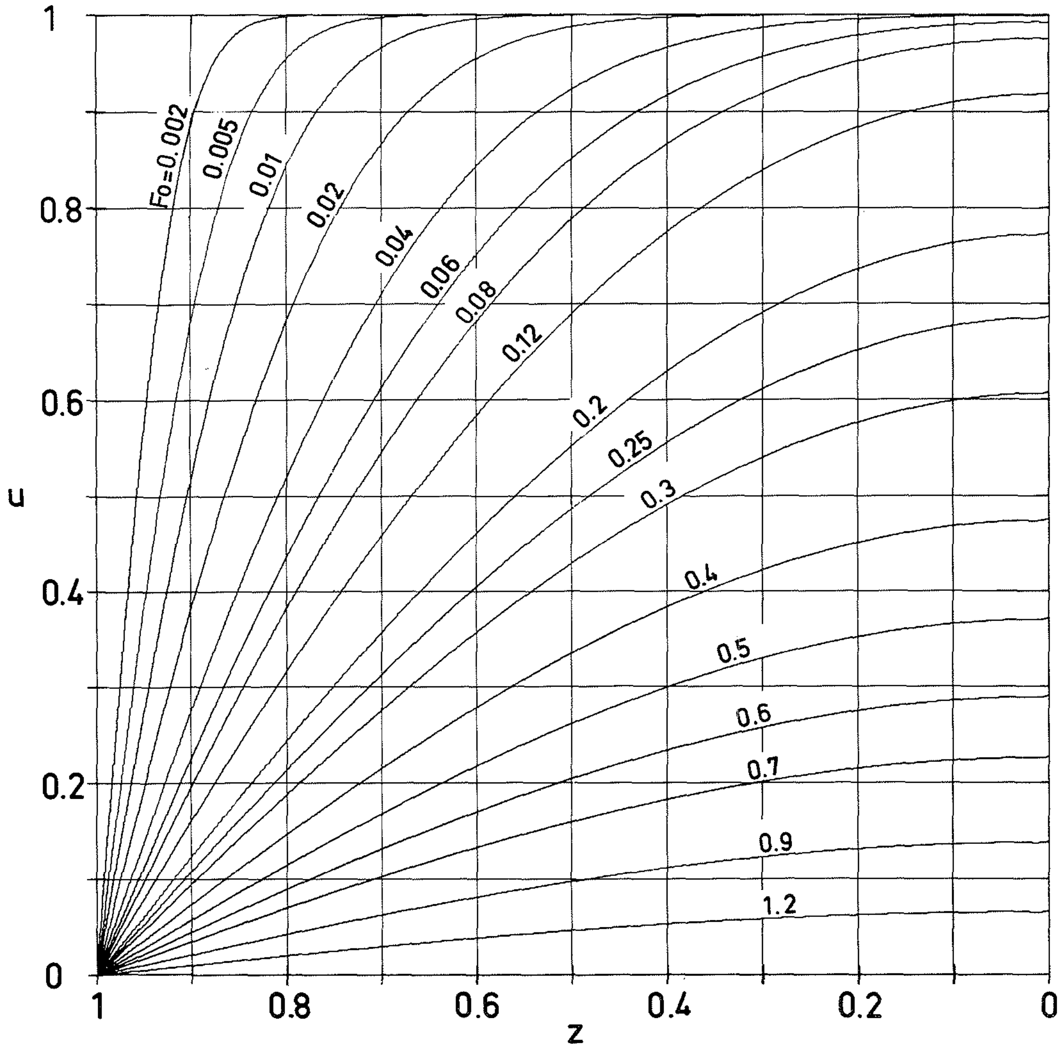
where $\Delta z = 1/K$. Parameters K and f have been varied. In both cases, (Tables 15 and 19, and 16 and 20), it has been found that the deviations: $\Delta \bar{u} \leqslant 0.0003$, $\Delta u \leqslant 0.0006$, when $z < 0.99$, occur in the results.

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 1
b = 0

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00200	.94954	12.62	.1256	.3074	.5708	.7643	.8862	.9519	.9823	.9984	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00500	.92021	7.979	.0797	.1974	.3829	.5467	.6827	.7887	.8664	.9545	.9973	.9999	1.000	1.000	1.000	1.000	1.000	1.000
.01000	.88717	5.642	.0564	.1403	.2763	.4041	.5205	.6232	.7112	.8427	.9661	.9953	.9996	1.000	1.000	1.000	1.000	1.000
.02000	.84043	3.989	.0399	.0995	.1974	.2923	.3829	.4680	.5467	.6827	.8664	.9545	.9876	.9973	.9995	.9999	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000008																		
.04000	.77423	2.904 / 2.817	.0282	.0703	.1401	.2088	.2759	.3410	.4036	.5200	.7107	.8426	.9230	.9663	.9868	.9954	.9985	.9997
.06000	.72345	2.375 / 2.299	.0230	.0574	.1145	.1711	.2267	.2813	.3344	.4356	.6129	.7513	.8509	.9168	.9567	.9788	.9893	.9918
.08000	.68070	1.945 / 1.991	.0199	.0497	.0993	.1485	.1971	.2449	.2919	.3824	.5461	.6822	.7883	.8659	.9189	.9520	.9699	.9748
.12000	.60896	1.590 / 1.625	.0162	.0406	.0811	.1214	.1614	.2010	.2400	.3162	.4586	.5842	.6900	.7749	.8390	.8834	.9094	.9187
.20000	.49556	1.290 / 1.242	.0124	.0310	.0620	.0929	.1236	.1541	.1843	.2438	.3572	.4610	.5526	.6299	.6915	.7362	.7632	.7739
.25000	.43748	1.124 / 1.086	.0109	.0271	.0542	.0812	.1081	.1348	.1613	.2134	.3132	.4050	.4866	.5560	.6116	.6522	.6769	.6868
.30000	.38651	.9877 / .9556	.0096	.0239	.0477	.0715	.0952	.1187	.1420	.1879	.2760	.3572	.4295	.4912	.5408	.5770	.5991	.6081
.40000	.30191	.7697 / .7451	.0075	.0186	.0372	.0558	.0742	.0925	.1107	.1466	.2153	.2788	.3354	.3837	.4225	.4510	.4683	.4757
.50000	.23587	.6012 / .5820	.0058	.0145	.0291	.0430	.0580	.0723	.0865	.1145	.1682	.2178	.2620	.2997	.3301	.3524	.3659	.3719
.60000	.18428	.4698 / .4546	.0045	.0114	.0227	.0340	.0453	.0565	.0676	.0895	.1314	.1702	.2047	.2342	.2579	.2753	.2859	.2908
.70000	.14398	.3444 / .3552	.0036	.0089	.0177	.0266	.0354	.0441	.0528	.0699	.1027	.1329	.1599	.1830	.2015	.2151	.2234	.2274
.90000	.08788	.2242 / .2168	.0022	.0054	.0108	.0162	.0216	.0269	.0322	.0427	.0627	.0811	.0976	.1117	.1230	.1313	.1363	.1393
1.2000	.04191	.1071 / .1034	.0010	.0026	.0052	.0077	.0103	.0128	.0154	.0203	.0299	.0387	.0466	.0533	.0587	.0626	.0650	.0669

NOMOGRAM 1



$\underline{u = u(z, Fo)}$

$z = x/l$

$Fo = k_e h / l^2$

b = 0

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

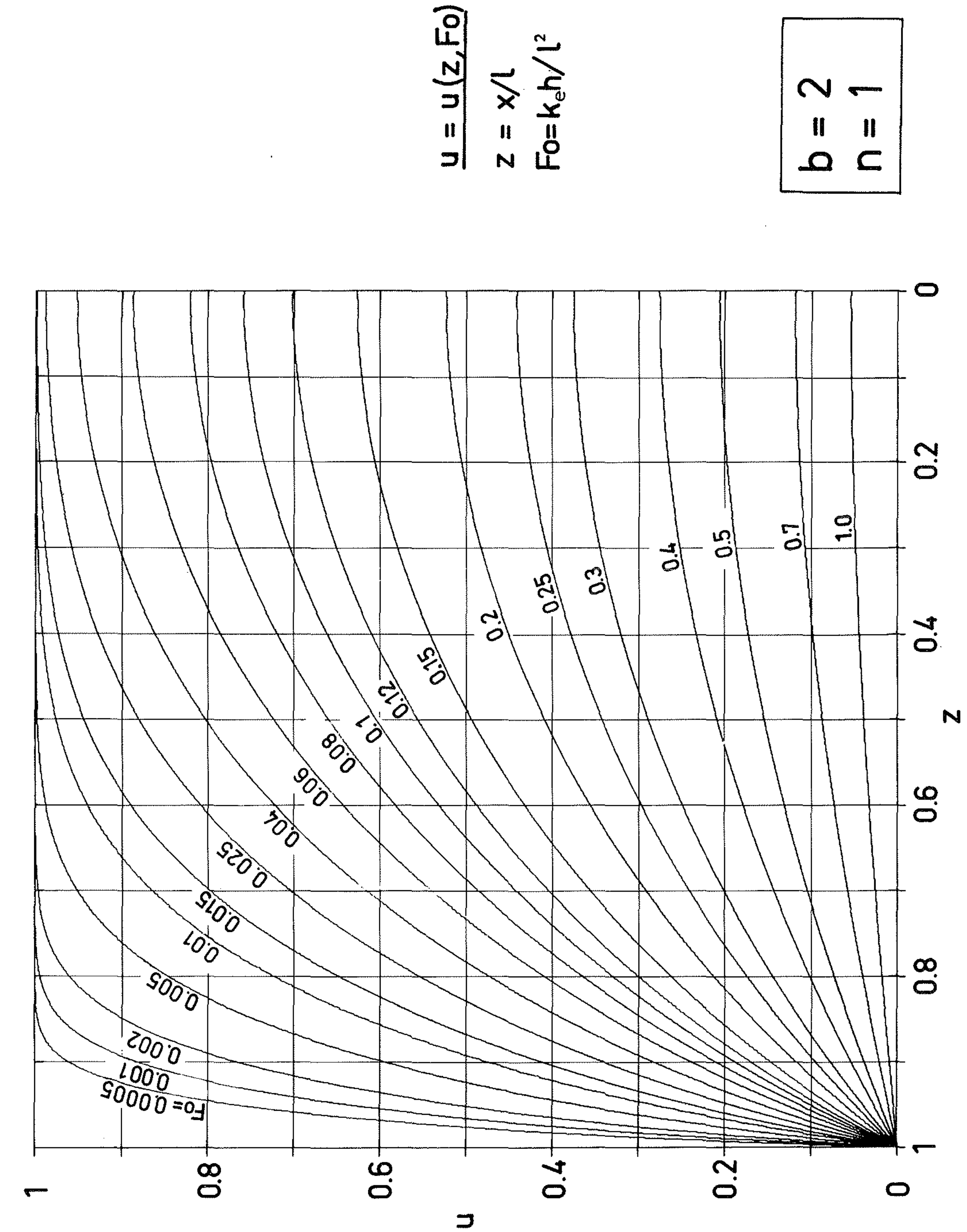
TABLE 2
$b = 2$
$n = 1$

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) z = .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00050	.96700	33.00	.2588	.5115	.7640	.8968	.9606	.9872	.9965	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.95333	23.33	.1941	.4016	.6354	.7852	.8800	.9372	.9695	.9944	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00200	.93400	16.50	.1437	.3090	.5115	.6573	.7640	.8416	.8968	.9606	.9965	.9999	1.000	1.000	1.000	1.000	1.000	1.000
.00500	.89565	10.44	.0951	.2132	.3698	.4937	.5944	.6769	.7448	.8454	.9514	.9881	.9978	.9997	1.000	1.000	1.000	1.000
.01000	.85242	7.379	.0690	.1585	.2830	.3863	.4743	.5501	.6159	.7230	.8641	.9397	.9763	.9919	.9976	.9994	.9999	1.000

DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000043

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	.990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.01500	.81924	6.088 6.026	.0569	.1326	.2402	.3316	.4110	.4809	.5428	.6473	.7966	.8893	.9439	.9738	.9888	.9956	.9983	.9989
.02500	.76665	4.654 4.665	.0445	.1053	.1940	.2713	.3398	.4014	.4572	.5542	.7033	.8079	.8800	.9280	.9585	.9766	.9860	.9889
.04000	.70463	3.828 3.670	.0355	.0848	.1582	.2235	.2825	.3363	.3857	.4736	.6150	.7219	.8023	.8615	.9037	.9317	.9476	.9535
.06000	.63831	2.892 3.013	.0292	.0700	.1320	.1880	.2392	.2864	.3302	.4092	.5399	.6426	.7231	.7851	.8312	.8629	.8815	.8867
.08000	.58293	2.514 2.584	.0251	.0606	.1151	.1648	.2106	.2530	.2927	.3648	.4857	.5822	.6592	.7194	.7647	.7962	.8148	.8203
.10000	.53470	2.202 2.270	.0221	.0535	.1022	.1469	.1884	.2271	.2633	.3295	.4415	.5318	.6043	.6614	.7046	.7348	.7527	.7579
.12000	.49206	1.942 2.021	.0197	.0478	.0916	.1322	.1700	.2054	.2387	.2997	.4037	.4880	.5560	.6098	.6506	.6792	.6961	.7008
.15000	.43676	1.731 1.694	.0167	.0409	.0787	.1140	.1472	.1784	.2079	.2623	.3555	.4317	.4935	.5425	.5798	.6060	.6216	.6272
.20000	.36170	1.338 1.319	.0131	.0321	.0622	.0906	.1176	.1432	.1675	.2126	.2910	.3558	.4087	.4508	.4830	.5056	.5191	.5239
.25000	.30264	1.028 1.003	.0104	.0256	.0500	.0732	.0953	.1165	.1367	.1745	.2409	.2963	.3418	.3783	.4062	.4259	.4376	.4409
.30000	.25544	.8472 .8480	.0084	.0208	.0407	.0598	.0782	.0959	.1128	.1448	.2013	.2489	.2884	.3201	.3444	.3617	.3719	.3754
.40000	.18530	.5986 .5519	.0056	.0142	.0279	.0412	.0541	.0666	.0788	.1018	.1434	.1790	.2089	.2331	.2518	.2651	.2731	.2768
.50000	.13714	.4139 .4182	.0041	.0100	.0197	.0293	.0386	.0477	.0565	.0735	.1045	.1315	.1543	.1730	.1875	.1979	.2041	.2056
.70000	.07780	.2206 .2340	.0022	.0053	.0105	.0157	.0208	.0258	.0307	.0402	.0579	.0737	.0873	.0986	.1074	.1138	.1176	.1181
1.0000	.03510	.0935 .1095	.0010	.0022	.0045	.0068	.0090	.0112	.0133	.0176	.0256	.0328	.0392	.0445	.0488	.0519	.0537	.0537

NOMOGRAM 2



u = u(z,Fo)
z = x/l
Fo=k_eh/l²
b = 2
n = 1
Fo= 0.0005
0.001
0.002
0.005
0.01
0.015
0.025
0.04
0.06
0.08
0.1
0.12
0.15
0.2
0.25
0.3
0.4
0.5
0.7
1.0
u
1
0.8
0.6
0.4
0.2
0
z
0
0.2
0.4
0.6
0.8
1

TABLE 3

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

$b = 2$
$n = 2$

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) ← z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00050	.97033	29.67	.2778	.5630	.8078	.9197	.9699	.9903	.9974	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.95804	20.98	.2026	.4433	.6890	.8264	.9061	.9517	.9768	.9958	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00200	.94066	14.84	.1457	.3367	.5636	.7100	.8078	.8744	.9197	.9699	.9974	.9999	1.000	1.000	1.000	1.000	1.000	1.000
.00500	.90617	9.383	.0931	.2247	.4071	.5447	.6488	.7286	.7907	.8776	.9628	.9910	.9983	.9998	1.000	1.000	1.000	1.000
.01000	.86730	6.635	.0661	.1622	.3062	.4260	.5238	.6039	.6701	.7711	.8931	.9537	.9820	.9938	.9982	.9995	.9999	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000014																		
.01500	.83748	5.435 5.417	.0540	.1334	.2561	.3630	.4540	.5310	.5965	.7005	.8363	.9137	.9570	.9801	.9915	.9967	.9987	.9993
.02500	.79018	4.152 4.195	.0419	.1039	.2025	.2924	.3725	.4431	.5052	.6082	.7531	.8459	.9061	.9445	.9684	.9823	.9894	.9917
.04000	.73449	3.422 3.313	.0331	.0823	.1620	.2368	.3058	.3687	.4256	.5234	.6695	.7704	.8414	.8912	.9254	.9476	.9601	.9646
.06000	.67481	2.801 2.700	.0270	.0672	.1330	.1960	.2553	.3107	.3619	.4525	.5943	.6970	.7724	.8276	.8671	.8935	.9088	.9131
.09000	.60240	2.177 2.153	.0218	.0544	.1080	.1600	.2099	.2573	.3020	.3831	.5150	.6145	.6895	.7455	.7863	.8140	.8302	.8355
.12000	.54228	1.853 1.844	.0184	.0460	.0915	.1360	.1790	.2204	.2598	.3326	.4542	.5484	.6205	.6751	.7150	.7424	.7583	.7638
.15000	.49091	1.603 1.592	.0159	.0397	.0791	.1178	.1555	.1919	.2270	.2924	.4043	.4927	.5614	.6139	.6525	.6790	.6945	.6994
.20000	.41949	1.310 1.280	.0128	.0320	.0637	.0951	.1258	.1558	.1849	.2401	.3371	.4161	.4789	.5274	.5635	.5884	.6030	.6073
.25000	.36150	1.058 1.054	.0105	.0263	.0525	.0784	.1040	.1290	.1535	.2003	.2844	.3547	.4116	.4562	.4897	.5130	.5267	.5311
.40000	.23814	.6389 .6373	.0064	.0159	.0318	.0476	.0632	.0787	.0940	.1236	.1789	.2275	.2687	.3022	.3280	.3463	.3571	.3601
.50000	.18306	.4642 .4750	.0047	.0119	.0237	.0355	.0472	.0588	.0703	.0927	.1350	.1728	.2055	.2326	.2537	.2688	.2779	.2805
.60000	.14159	.3529 .3604	.0035	.0090	.0180	.0269	.0358	.0447	.0534	.0706	.1031	.1326	.1584	.1800	.1970	.2093	.2167	.2187
.80000	.08558	.2095 .2137	.0021	.0053	.0107	.0160	.0213	.0265	.0317	.0420	.0615	.0795	.0953	.1088	.1195	.1273	.1321	.1333
1.2000	.03173	.0771 .0784	.0008	.0020	.0039	.0059	.0078	.0097	.0117	.0154	.0226	.0293	.0353	.0403	.0444	.0474	.0492	.0495

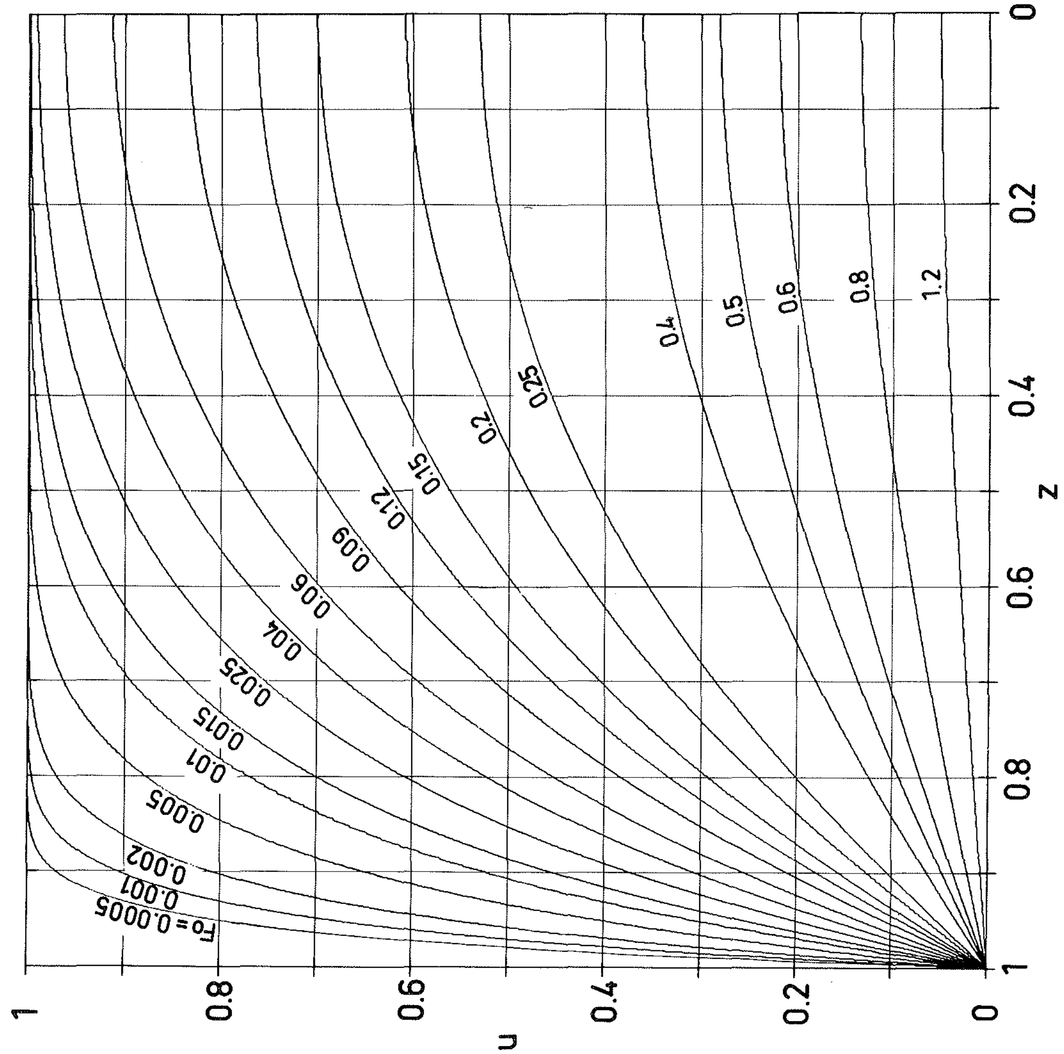
NOMOGRAM 3

$\underline{u = u(z, Fo)}$

$z = x/l$

$Fo = k_e h/l^2$

$b = 2$
$n = 2$

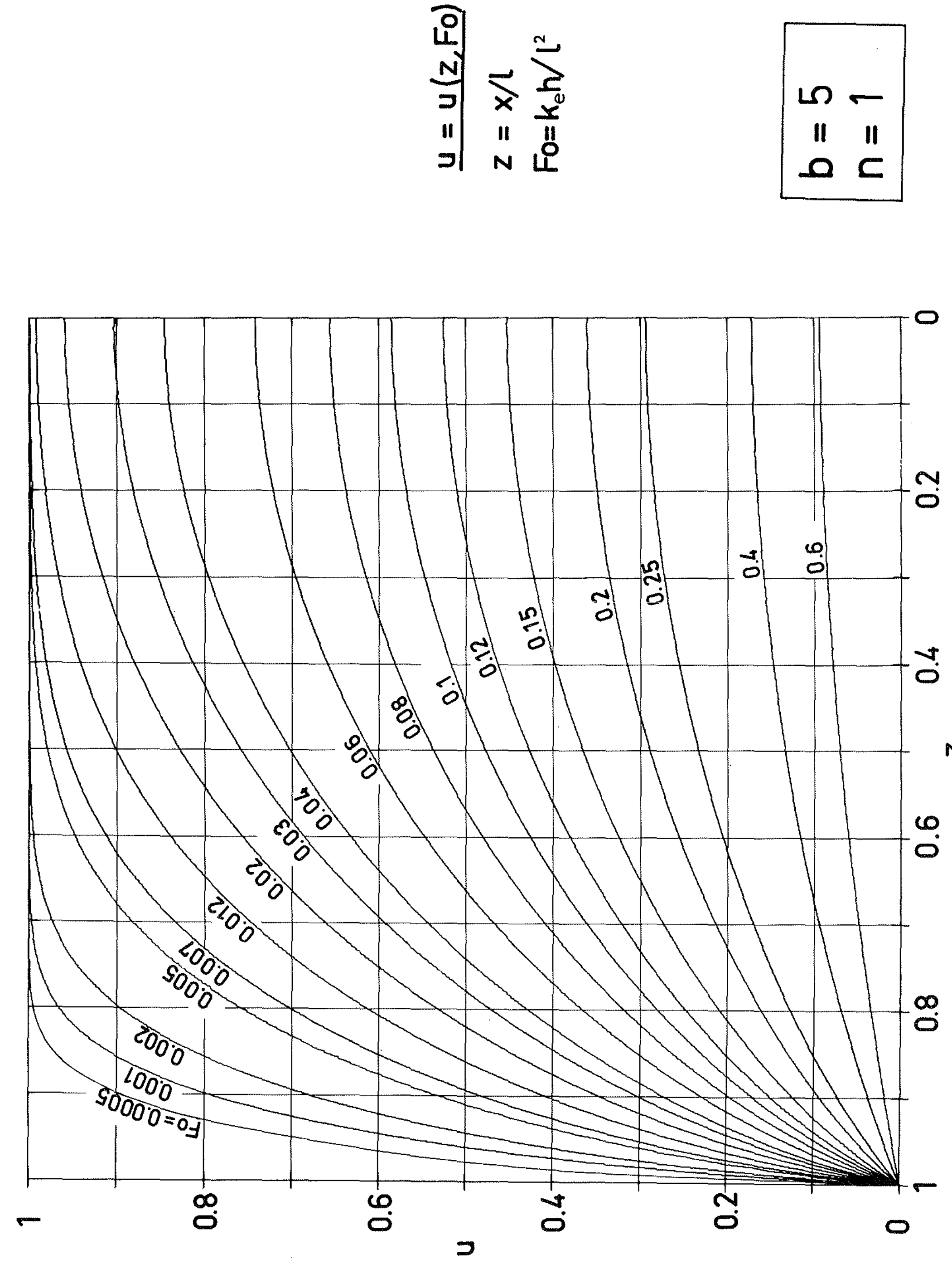


Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 4
$b = 5$
$n = 1$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) ← z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00050	.95810	41.90	.2539	.4640	.6804	.8136	.8964	.9459	.9738	.9952	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.94074	29.63	.1973	.3735	.5674	.7000	.7949	.8629	.9110	.9661	.9970	.9999	1.000	1.000	1.000	1.000	1.000	1.000
.00200	.91519	20.95	.1515	.2965	.4640	.5861	.6804	.7547	.8136	.8964	.9738	.9952	.9994	.9999	1.000	1.000	1.000	1.000
.00500	.86749	13.25	.1049	.2143	.3473	.4494	.5328	.6030	.6628	.7586	.8826	.9481	.9796	.9930	.9979	.9995	.9999	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000134																		
.00700	.84313	10.57 11.17	.0906	.1890	.3101	.4045	.4827	.5494	.6072	.7024	.8340	.9128	.9575	.9811	.9924	.9972	.9990	.9997
.01200	.79463	8.328 8.530	.0720	.1538	.2573	.3397	.4091	.4694	.5227	.6131	.7477	.8395	.9015	.9420	.9673	.9821	.9896	.9922
.02000	.73479	6.876 6.590	.0576	.1256	.2140	.2856	.3468	.4006	.4488	.5322	.6621	.7577	.8286	.8804	.9170	.9412	.9550	.9600
.03000	.67522	5.189 5.417	.0481	.1062	.1837	.2473	.3022	.3508	.3947	.4714	.5937	.6870	.7589	.8137	.8541	.8818	.8980	.9025
.04000	.62552	4.588 4.642	.0419	.0937	.1637	.2218	.2721	.3170	.3576	.4291	.5441	.6330	.7025	.7562	.7962	.8240	.8403	.8455
.06000	.54351	3.550 3.654	.0335	.0762	.1353	.1851	.2286	.2677	.3032	.3660	.4681	.5478	.6106	.6596	.6963	.7219	.7371	.7415
.08000	.47801	3.010 2.969	.0277	.0637	.1145	.1579	.1962	.2307	.2621	.3180	.4093	.4809	.5375	.5817	.6150	.6382	.6519	.6560
.10000	.42403	2.359 2.424	.0231	.0540	.0981	.1362	.1701	.2008	.2289	.2790	.3612	.4259	.4772	.5173	.5475	.5687	.5811	.5861
.12000	.37908	1.992 2.086	.0197	.0462	.0849	.1186	.1489	.1764	.2016	.2468	.3213	.3802	.4270	.4636	.4912	.5105	.5219	.5249
.15000	.32412	1.592 1.646	.0157	.0372	.0693	.0977	.1234	.1470	.1687	.2078	.2727	.3243	.3654	.3977	.4220	.4391	.4491	.4520
.20000	.25550	1.122 1.169	.0112	.0270	.0510	.0728	.0928	.1113	.1285	.1598	.2124	.2546	.2884	.3150	.3352	.3493	.3577	.3599
.25000	.20597	.8395 .8436	.0083	.0203	.0387	.0558	.0716	.0865	.1004	.1259	.1692	.2043	.2327	.2551	.2721	.2840	.2911	.2937
.40000	.11736	.4179 .3850	.0040	.0099	.0192	.0282	.0368	.0449	.0528	.0675	.0932	.1148	.1325	.1468	.1577	.1653	.1699	.1720
.60000	.06199	.1853 .1886	.0019	.0046	.0091	.0134	.0177	.0218	.0259	.0335	.0475	.0596	.0698	.0781	.0846	.0892	.0919	.0928

NOMOGRAM 4


u = u(z,Fo)
z = x/l
Fo=k_eh/l²
b = 5
n = 1
Fo=0.0005
0.001
0.002
0.005
0.007
0.012
0.02
0.03
0.04
0.06
0.08
0.1
0.12
0.15
0.2
0.25
0.4
0.6
u
0
0.2
0.4
0.6
0.8
1
z
0
0.2
0.4
0.6
0.8
1

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 5
b = 5
n = 2

Fo	ū	$\frac{d\bar{u}}{dFo}$ ∅	u(z,Fo) ← z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00050	.96494	35.06	.3004	.5474	.7535	.8629	.9258	.9619	.9817	.9967	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.95041	24.79	.2266	.4474	.6510	.7704	.8482	.9008	.9366	.9762	.9979	.9999	1.000	1.000	1.000	1.000	1.000	1.000
.00200	.92987	17.53	.1669	.3547	.5474	.6687	.7535	.8159	.8629	.9258	.9817	.9967	.9996	1.000	1.000	1.000	1.000	1.000
.00500	.88912	11.09	.1086	.2489	.4166	.5319	.6176	.6844	.7382	.8191	.9156	.9634	.9858	.9951	.9986	.9996	.9999	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000030																		
.00900	.85117	8.548	.0817	.1933	.3396	.4471	.5298	.5960	.6507	.7361	.8480	.9142	.9536	.9763	.9886	.9949	.9977	.9987
.01500	.80793	8.254 6.528	.0636	.1534	.2796	.3781	.4563	.5205	.5744	.6606	.7794	.8562	.9075	.9418	.9641	.9778	.9852	.9877
.02500	.75209	6.397 4.985	.0494	.1207	.2265	.3141	.3865	.4472	.4991	.5838	.7048	.7874	.8462	.8883	.9179	.9376	.9488	.9526
.04000	.68649	4.956 3.986	.0389	.0958	.1835	.2598	.3252	.3814	.4303	.5116	.6304	.7138	.7746	.8194	.8516	.8735	.8861	.8906
.06000	.61714	3.893 3.059	.0310	.0769	.1493	.2146	.2725	.3236	.3688	.4450	.5586	.6393	.6988	.7429	.7749	.7966	.8093	.8137
.08000	.56044	3.105 2.536	.0258	.0642	.1257	.1826	.2341	.2805	.3222	.3936	.5016	.5791	.6365	.6791	.7101	.7312	.7434	.7478
.12000	.47163	2.585 1.927	.0191	.0476	.0940	.1382	.1796	.2180	.2533	.3156	.4131	.4845	.5379	.5777	.6067	.6264	.6379	.6416
.20000	.35077	1.915 1.182	.0120	.0299	.0595	.0884	.1163	.1431	.1686	.2156	.2941	.3547	.4012	.4364	.4622	.4799	.4902	.4938
.25000	.29753	1.196 .9313	.0095	.0236	.0471	.0701	.0926	.1145	.1355	.1751	.2432	.2975	.3401	.3727	.3968	.4133	.4230	.4260
.40000	.19095	.9498 .5263	.0053	.0133	.0266	.0397	.0527	.0656	.0781	.1024	.1469	.1850	.2166	.2418	.2609	.2742	.2821	.2849
.50000	.14546	.5298 .3902	.0039	.0097	.0193	.0289	.0384	.0479	.0571	.0752	.1090	.1387	.1640	.1846	.2006	.2118	.2186	.2209
.70000	.08659	.3856 .2252	.0022	.0055	.0110	.0165	.0219	.0273	.0326	.0431	.0630	.0810	.0968	.1101	.1205	.1281	.1326	.1344
1.0000	.04083	.2132 .1036 .0953	.0010	.0025	.0051	.0076	.0101	.0126	.0151	.0199	.0293	.0378	.0454	.0519	.0571	.0608	.0631	.0641

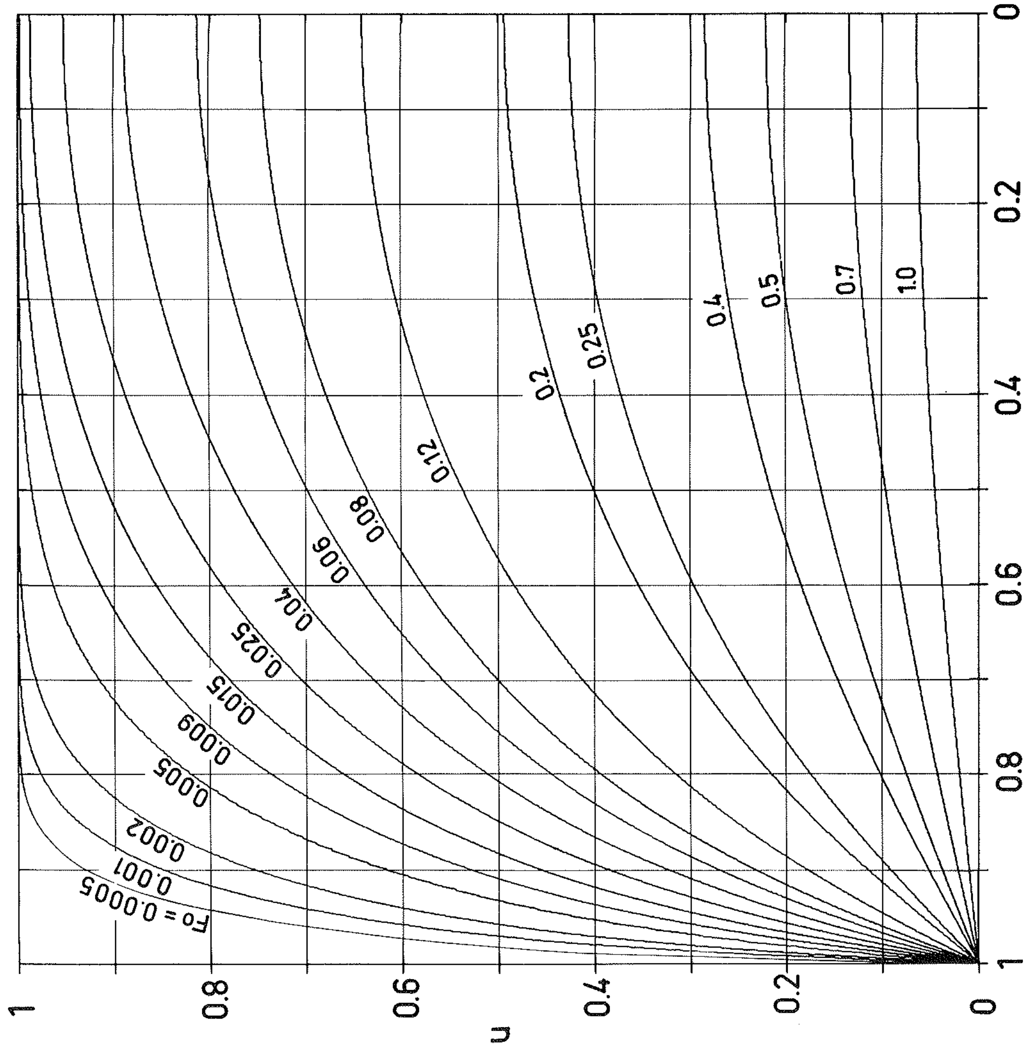
NOMOGRAM 5

$u = u(z, Fo)$

$z = x/l$

$Fo = k_e h / l^2$

$b = 5$
$n = 2$



Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 6
$b = 5$
$n = 5$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) ← z .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00050	.97097	29.03	.2851	.6151	.8323	.9150	.9561	.9780	.9896	.9981	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.95895	20.53	.2035	.4781	.7365	.8462	.9048	.9402	.9627	.9864	.9988	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00200	.94194	14.51	.1445	.3521	.6151	.7546	.8323	.8815	.9150	.9561	.9896	.9981	.9998	1.000	1.000	1.000	1.000	1.000
.00500	.90621	9.179	.0917	.2270	.4351	.5947	.7002	.7701	.8194	.8838	.9497	.9769	.9919	.9973	.9992	.9998	1.000	1.000
DERIVATIVE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000121																		
.00800	.88386	7.468 7.254	.0725	.1802	.3520	.5012	.6150	.6960	.7545	.8322	.9149	.9561	.9780	.9896	.9954	.9981	.9992	.9994
.01200	.85775	6.121 5.922	.0592	.1474	.2906	.4229	.5351	.6227	.6890	.7792	.8773	.9286	.9584	.9763	.9869	.9929	.9959	.9970
.02000	.81537	4.449 4.587	.0458	.1144	.2269	.3350	.4349	.5222	.5946	.7001	.8192	.8836	.9232	.9489	.9656	.9762	.9820	.9841
.03000	.77519	3.706 3.747	.0375	.0935	.1860	.2763	.3627	.4427	.5140	.6270	.7634	.8386	.8855	.9165	.9373	.9508	.9583	.9608
.05000	.70990	2.906 2.889	.0289	.0721	.1438	.2145	.2835	.3500	.4128	.5234	.6764	.7656	.8214	.8583	.8832	.8992	.9083	.9112
.07000	.65729	2.422 2.410	.0241	.0602	.1201	.1794	.2377	.2946	.3495	.4507	.6061	.7040	.7660	.8070	.8344	.8521	.8620	.8653
.10000	.59234	1.959 1.957	.0196	.0489	.0976	.1459	.1937	.2406	.2864	.3736	.5206	.6243	.6933	.7392	.7699	.7896	.8007	.8043
.15000	.50684	1.542 1.500	.0150	.0375	.0749	.1120	.1489	.1853	.2211	.2905	.4159	.5166	.5907	.6426	.6779	.7006	.7134	.7173
.20000	.43948	1.234 1.216	.0122	.0304	.0607	.0909	.1208	.1505	.1798	.2369	.3432	.4342	.5067	.5607	.5988	.6239	.6381	.6426
.25000	.38393	1.007 1.017	.0102	.0254	.0508	.0761	.1012	.1261	.1507	.1990	.2899	.3704	.4376	.4903	.5291	.5553	.5704	.5753
.40000	.26147	.6505 .6551	.0066	.0164	.0327	.0490	.0652	.0813	.0973	.1287	.1888	.2437	.2922	.3328	.3649	.3879	.4018	.4064
.50000	.20381	.5021 .5054	.0051	.0126	.0252	.0378	.0503	.0628	.0751	.0994	.1459	.1888	.2268	.2592	.2850	.3038	.3152	.3191
.60000	.15912	.3907 .3932	.0039	.0098	.0196	.0294	.0392	.0488	.0584	.0773	.1136	.1471	.1768	.2022	.2226	.2376	.2466	.2497
.80000	.09710	.2381 .2396	.0024	.0060	.0120	.0179	.0239	.0298	.0356	.0471	.0693	.0897	.1079	.1234	.1359	.1451	.1506	.1526
1.0000	.05927	.1453 .1462	.0015	.0037	.0073	.0109	.0146	.0182	.0217	.0288	.0423	.0547	.0658	.0753	.0830	.0885	.0920	.0931

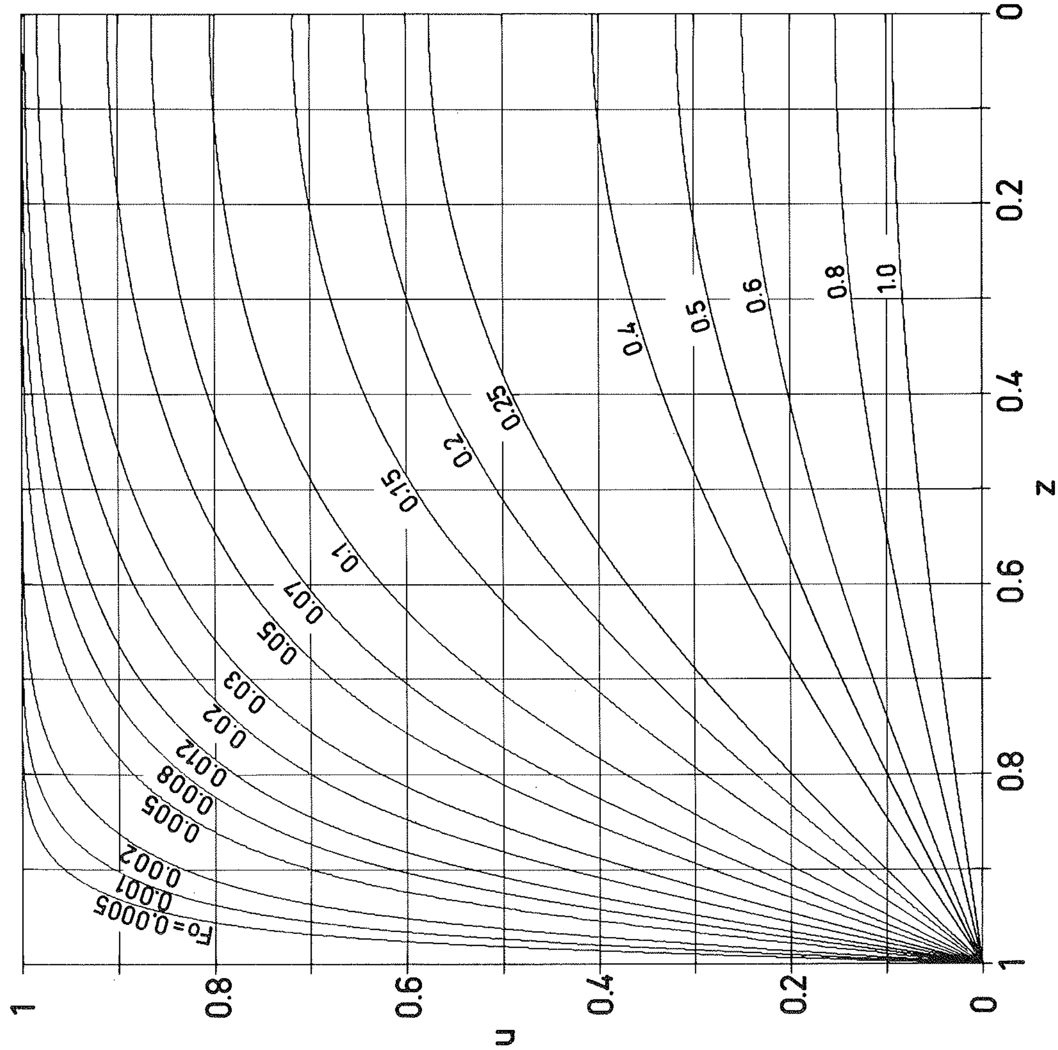
NOMOGRAM 6

$u = u(z, Fo)$

$z = x/l$

$Fo = k_e h / l^2$

$b = 5$
$n = 5$



Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 7
$b = 10$
$n = 1$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00020	.96617	84.58	.3204	.5388	.7498	.8697	.9371	.9724	.9891	.9988	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.94650	53.50	.2409	.4192	.6075	.7333	.8215	.8836	.9265	.9738	.9981	.9999	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.92435	37.83	.1921	.3426	.5078	.6253	.7148	.7843	.8387	.9136	.9801	.9968	.9997	1.000	1.000	1.000	1.000	1.000
.00200	.89301	26.75	.1517	.2774	.4192	.5240	.6075	.6761	.7333	.8215	.9265	.9738	.9922	.9981	.9996	.9999	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000318																		
.00400	.84866	18.83	.1170	.2225	.3425	.4334	.5077	.5708	.6253	.7147	.8387	.9136	.9569	.9802	.9917	.9968	.9988	.9994
		18.92																
.00600	.81466	15.43	.1009	.1948	.3030	.3859	.4545	.5133	.5648	.6514	.7780	.8625	.9182	.9535	.9749	.9869	.9929	.9946
		15.45																
.00900	.77302	12.62	.0867	.1699	.2673	.3425	.4053	.4596	.5077	.5897	.7146	.8038	.8677	.9129	.9436	.9632	.9741	.9775
		12.61																
.01500	.70698	9.632	.0711	.1422	.2269	.2931	.3487	.3974	.4407	.5158	.6337	.7224	.7901	.8411	.8784	.9039	.9187	.9237
		9.741																
.02500	.62237	7.429	.0573	.1170	.1897	.2470	.2955	.3381	.3763	.4429	.5491	.6308	.6946	.7439	.7808	.8063	.8214	.8263
		7.429																
.04000	.52681	5.459	.0446	.0934	.1542	.2027	.2439	.2803	.3130	.3701	.4618	.5328	.5885	.6318	.6642	.6868	.7002	.7045
		5.490																
.05000	.47655	4.637	.0385	.0817	.1364	.1803	.2178	.2509	.2807	.3328	.4167	.4817	.5327	.5725	.6023	.6230	.6353	.6392
		4.612																
.07000	.39744	3.510	.0294	.0640	.1090	.1456	.1772	.2051	.2303	.2746	.3459	.4013	.4450	.4789	.5044	.5222	.5327	.5370
		3.364																
.09000	.33836	2.658	.0232	.0514	.0891	.1202	.1472	.1712	.1929	.2312	.2931	.3413	.3793	.4089	.4312	.4467	.4559	.4584
		2.597																
.12000	.27313	1.766	.0168	.0382	.0679	.0929	.1147	.1343	.1521	.1837	.2349	.2751	.3068	.3315	.3501	.3631	.3708	.3727
		1.837																
.15000	.22606	1.352	.0127	.0295	.0533	.0737	.0918	.1081	.1230	.1496	.1931	.2272	.2543	.2754	.2914	.3025	.3091	.3112
		1.353																
.20000	.17123	.9158	.0085	.0201	.0372	.0523	.0659	.0784	.0898	.1104	.1445	.1715	.1930	.2099	.2226	.2315	.2367	.2390
		.8760																
.30000	.10775	.4589	.0044	.0108	.0205	.0295	.0378	.0456	.0529	.0662	.0887	.1070	.1217	.1333	.1421	.1483	.1520	.1534
		.4505																
.50000	.05141	.1748	.0017	.0042	.0082	.0121	.0158	.0193	.0227	.0292	.0405	.0501	.0580	.0644	.0693	.0728	.0748	.0758
		.1659																

← z

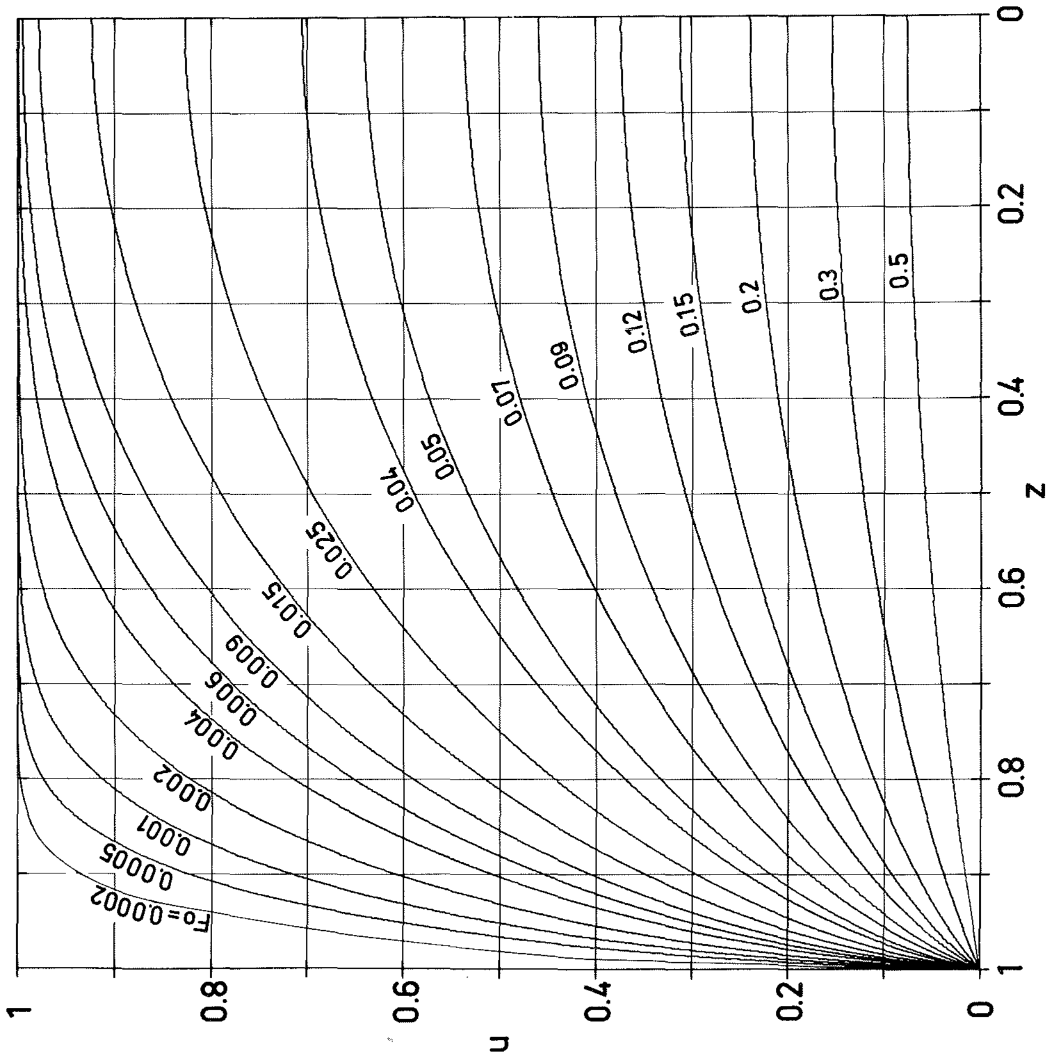
NOMOGRAM 7

$u = u(z, Fo)$

$z = x/l$

$Fo = k_e h/l^2$

$b = 10$
$n = 1$

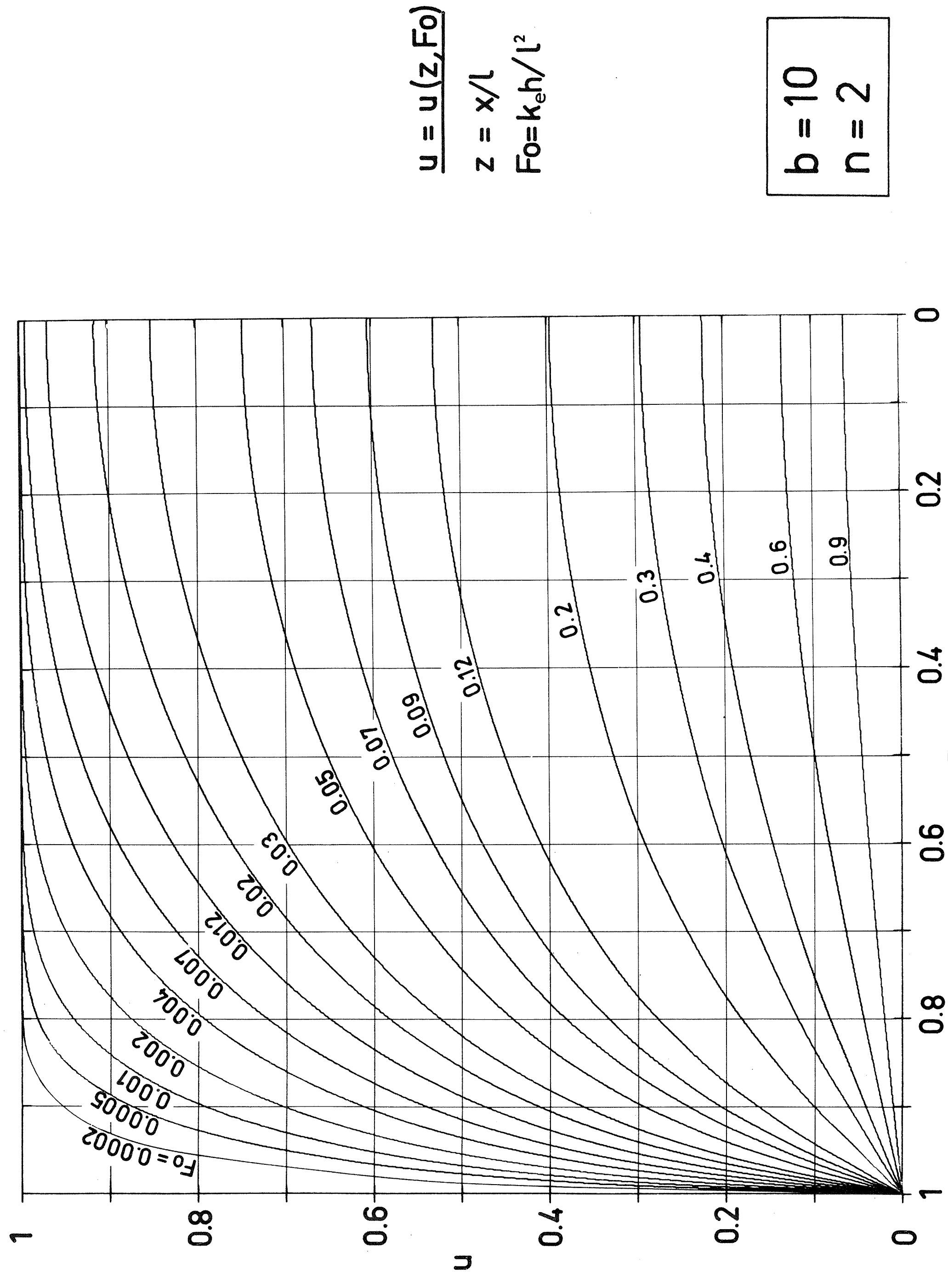


Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 8
$b = 10$
$n = 2$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ ∅	u(z,Fo) z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00020	.97317	67.06	.4152	.6427	.8206	.9101	.9575	.9815	.9928	.9992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.95758	42.42	.3149	.5255	.7041	.8078	.8750	.9200	.9501	.9825	.9987	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.94002	29.99	.2474	.4413	.6138	.7195	.7931	.8472	.8876	.9411	.9867	.9979	.9998	1.000	1.000	1.000	1.000	1.000
.00200	.91517	21.21	.1688	.3623	.5255	.6290	.7041	.7619	.8078	.8750	.9501	.9825	.9948	.9987	.9997	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000239																		
.00400	.88004	14.85 15.02	.1410	.2900	.4413	.5402	.6137	.6718	.7194	.7931	.8876	.9411	.9710	.9867	.9945	.9979	.9992	.9996
.00700	.84131	11.40 11.35	.1094	.2372	.3771	.4712	.5422	.5991	.6464	.7215	.8240	.8893	.9318	.9593	.9764	.9866	.9919	.9934
.01200	.79222	8.733 8.665	.0848	.1920	.3191	.4077	.4756	.5305	.5766	.6510	.7564	.8279	.8784	.9143	.9393	.9558	.9651	.9680
.02000	.73174	6.505 6.693	.0660	.1543	.2674	.3499	.4140	.4665	.5108	.5830	.6872	.7600	.8132	.8525	.8807	.8998	.9109	.9142
.03000	.67216	5.402 5.365	.0532	.1271	.2273	.3037	.3642	.4140	.4564	.5256	.6262	.6971	.7493	.7881	.8163	.8355	.8467	.8504
.05000	.58131	3.843 3.872	.0386	.0940	.1749	.2408	.2949	.3403	.3792	.4433	.5367	.6028	.6516	.6878	.7142	.7321	.7426	.7462
.07000	.51325	2.922 2.987	.0298	.0734	.1398	.1968	.2452	.2867	.3226	.3823	.4700	.5321	.5779	.6120	.6367	.6536	.6634	.6669
.09000	.45961	2.346 2.413	.0240	.0594	.1148	.1642	.2075	.2453	.2785	.3343	.4171	.4760	.5195	.5519	.5753	.5913	.6006	.6034
.12000	.39678	1.815 1.832	.0183	.0454	.0889	.1291	.1656	.1984	.2279	.2784	.3549	.4099	.4506	.4809	.5029	.5179	.5266	.5294
.20000	.28612	1.058 1.054	.0105	.0263	.0521	.0771	.1010	.1236	.1449	.1832	.2452	.2918	.3271	.3535	.3728	.3860	.3937	.3962
.30000	.20403	.6333 .6386	.0064	.0160	.0319	.0475	.0628	.0777	.0921	.1191	.1660	.2036	.2332	.2559	.2726	.2842	.2910	.2933
.40000	.15127	.4212 .4261	.0043	.0108	.0216	.0323	.0428	.0532	.0633	.0828	.1180	.1477	.1720	.1912	.2056	.2157	.2216	.2238
.60000	.08786	.2338 .2351	.0023	.0057	.0115	.0172	.0228	.0284	.0339	.0447	.0651	.0832	.0988	.1116	.1216	.1287	.1330	.1343
.90000	.04096	.1039 .1075	.0010	.0026	.0051	.0077	.0102	.0127	.0152	.0201	.0295	.0381	.0456	.0521	.0572	.0609	.0632	.0638

NOMOGRAM 8



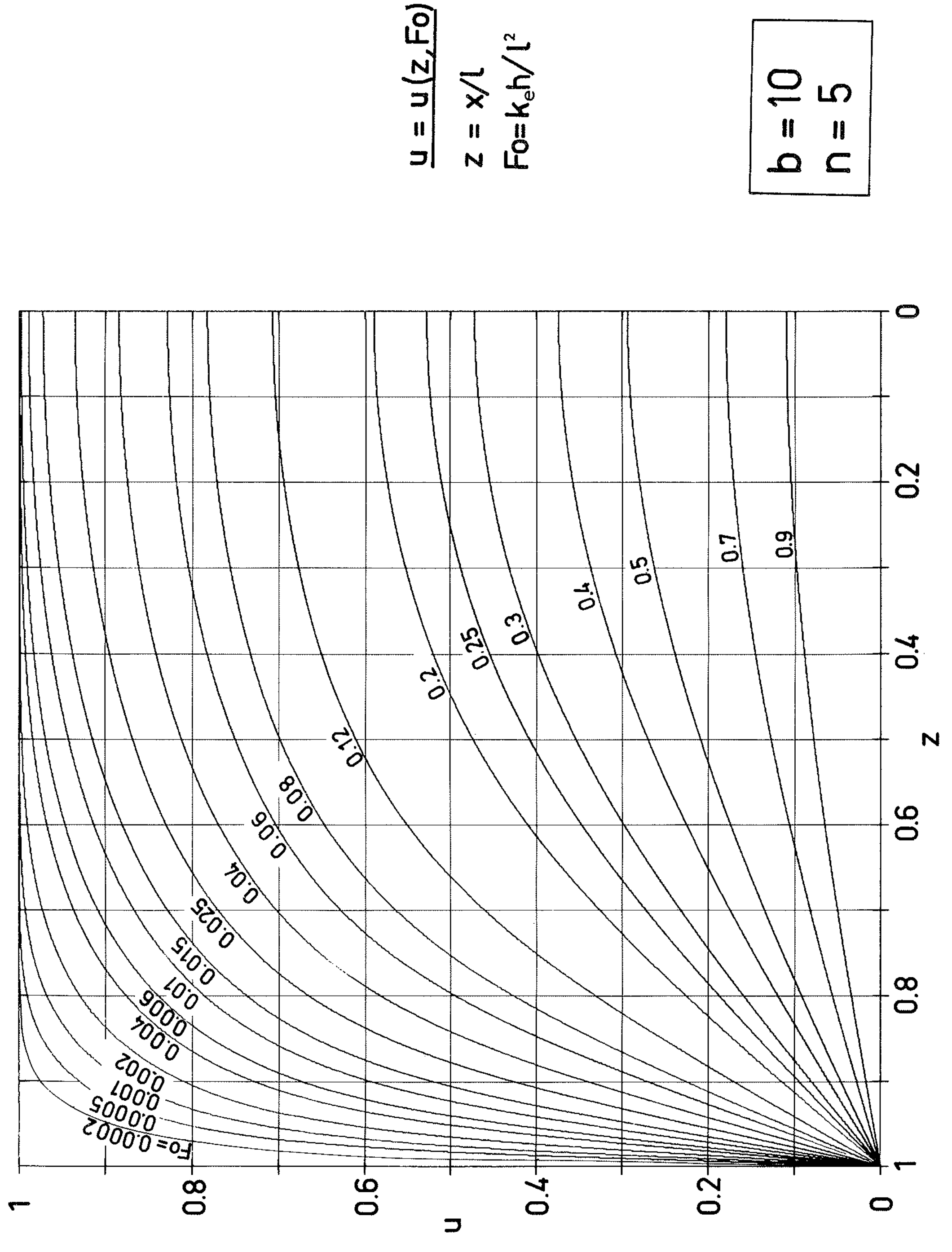
u = u(z,Fo)
z = x/l
Fo=k_eh/l^2
b = 10
n = 2
Fo = 0.0002
0.0005
0.001
0.002
0.004
0.007
0.012
0.02
0.03
0.05
0.07
0.09
0.12
0.2
0.3
0.4
0.6
0.9
u
1
0.8
0.6
0.4
0.2
0
z
0
0.2
0.4
0.6
0.8
1

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 9
$b = 10$
$n = 5$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ / Ø	u(z,Fo) z = .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00020	.97974	50.65	.4690	.7570	.8948	.9502	.9771	.9902	.9962	.9996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.96796	32.04	.3135	.6296	.8101	.8863	.9293	.9560	.9730	.9907	.9993	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00100	.95469	22.65	.2244	.5099	.7291	.8223	.8763	.9120	.9369	.9680	.9930	.9989	.9999	1.000	1.000	1.000	1.000	1.000
.00200	.93593	16.02	.1595	.3849	.6296	.7441	.8101	.8543	.8863	.9293	.9730	.9907	.9973	.9993	.9999	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = .000127																		
.00400	.90938	11.04 11.32	.1130	.2788	.5100	.6480	.7290	.7830	.8222	.8763	.9369	.9680	.9845	.9930	.9971	.9989	.9996	.9998
.00600	.88904	9.051 9.248	.0923	.2291	.4359	.5822	.6735	.7344	.7784	.8392	.9089	.9472	.9698	.9833	.9911	.9954	.9975	.9981
.01000	.85677	7.265 7.167	.0716	.1782	.3481	.4908	.5932	.6640	.7154	.7858	.8671	.9136	.9432	.9627	.9755	.9835	.9879	.9894
.01500	.82452	5.678 5.847	.0584	.1457	.2874	.4167	.5216	.5997	.6576	.7371	.8283	.8809	.9152	.9385	.9544	.9648	.9706	.9727
.02500	.77359	4.521 4.519	.0452	.1127	.2238	.3304	.4270	.5083	.5731	.6651	.7702	.8301	.8691	.8958	.9142	.9263	.9332	.9354
.04000	.71412	3.554 3.518	.0352	.0878	.1749	.2600	.3412	.4159	.4813	.5823	.7023	.7696	.8126	.8419	.8619	.8750	.8825	.8850
.06000	.65182	2.844 2.774	.0277	.0693	.1381	.2059	.2720	.3353	.3943	.4950	.6275	.7032	.7508	.7827	.8043	.8184	.8264	.8288
.08000	.60138	2.343 2.306	.0231	.0576	.1149	.1716	.2272	.2813	.3331	.4268	.5638	.6464	.6985	.7331	.7563	.7713	.7798	.7825
.12000	.52174	1.739 1.735	.0173	.0433	.0865	.1294	.1717	.2133	.2539	.3310	.4604	.5508	.6106	.6504	.6770	.6941	.7037	.7068
.20000	.40881	1.144 1.163	.0116	.0291	.0581	.0869	.1155	.1438	.1717	.2261	.3260	.4096	.4739	.5205	.5528	.5738	.5856	.5894
.25000	.35604	.9503 .9605	.0096	.0240	.0480	.0718	.0955	.1190	.1422	.1876	.2727	.3470	.4079	.4547	.4884	.5109	.5238	.5280
.30000	.31186	.8028 .8130	.0081	.0203	.0406	.0608	.0809	.1008	.1205	.1592	.2324	.2980	.3536	.3981	.4315	.4544	.4677	.4721
.40000	.24146	.6259 .6071	.0061	.0152	.0304	.0455	.0605	.0754	.0902	.1193	.1749	.2257	.2703	.3075	.3367	.3575	.3700	.3743
.50000	.18810	.4795 .4666	.0047	.0117	.0233	.0350	.0465	.0580	.0694	.0918	.1348	.1744	.2095	.2392	.2630	.2802	.2906	.2942
.70000	.11468	.2902 .2826	.0028	.0071	.0141	.0212	.0282	.0352	.0421	.0557	.0818	.1059	.1274	.1457	.1605	.1713	.1779	.1802
.90000	.06999	.1770 .1722	.0017	.0043	.0086	.0129	.0172	.0215	.0257	.0340	.0499	.0646	.0777	.0889	.0980	.1046	.1086	.1100

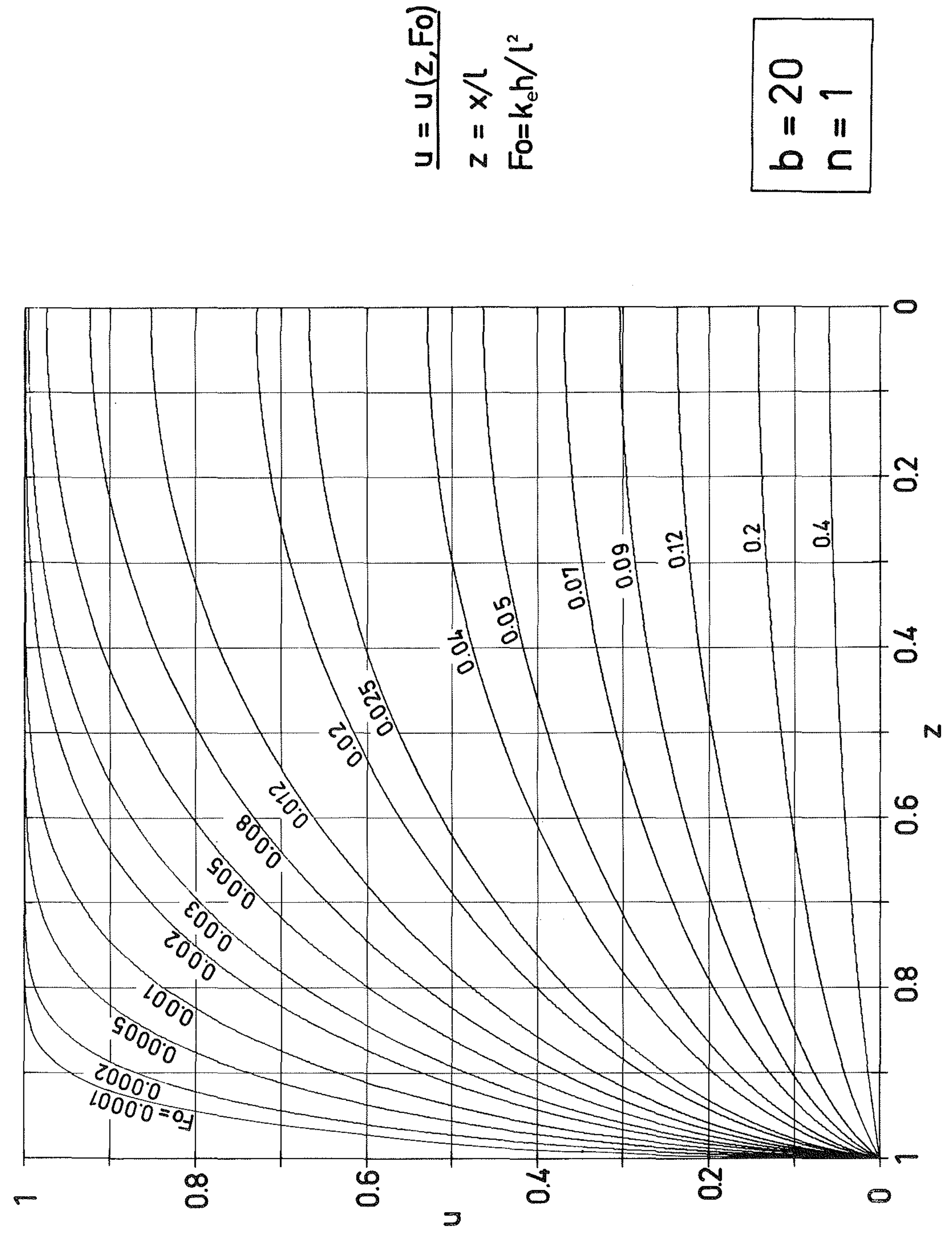
NOMOGRAM 9


u = u(z,Fo)
z = x/l
Fo=k_eh/l²
b = 10
n = 5
Fo= 0.0002
0.0005
0.001
0.002
0.004
0.006
0.01
0.015
0.025
0.04
0.06
0.08
0.12
0.2
0.25
0.3
0.4
0.5
0.7
0.9
u
z
0
0.2
0.4
0.6
0.8
1

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 10
$b = 20$
$n = 1$

Fo	ū	dū/dFo Ø	u(z,Fo) z = .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00010	.96816	159.2	.3489	.5629	.7667	.8810	.9440	.9762	.9910	.9991	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.95497	112.6	.2877	.4728	.6625	.7843	.8657	.9196	.9540	.9872	.9995	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.92880	71.20	.2209	.3709	.5327	.6469	.7331	.7998	.8516	.9221	.9831	.9975	.9998	1.000	1.000	1.000	1.000	1.000
.00100	.89931	50.34	.1796	.3063	.4462	.5485	.6296	.6959	.7509	.8353	.9341	.9774	.9936	.9985	.9997	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000548																		
.00200	.85745	34.44 35.52	.1416	.2514	.3706	.4598	.5324	.5937	.6465	.7328	.8514	.9220	.9621	.9831	.9932	.9975	.9991	.9998
.00300	.82546	28.21 29.10	.1243	.2234	.3316	.4133	.4805	.5379	.5880	.6718	.7936	.8740	.9263	.9591	.9785	.9892	.9943	.9956
.00500	.77468	23.28 22.53	.1059	.1919	.2874	.3601	.4204	.4724	.5185	.5970	.7168	.8028	.8651	.9095	.9402	.9600	.9711	.9742
.00800	.71519	17.71 17.77	.0907	.1664	.2512	.3162	.3704	.4175	.4595	.5319	.6454	.7306	.7956	.8447	.8806	.9051	.9194	.9241
.01200	.65144	14.75 14.32	.0784	.1457	.2217	.2802	.3291	.3718	.4099	.4760	.5810	.6613	.7238	.7719	.8078	.8326	.8472	.8526
.02000	.55363	10.88 10.47	.0629	.1190	.1833	.2330	.2747	.3111	.3437	.4004	.4909	.5606	.6153	.6576	.6894	.7115	.7245	.7279
.02500	.50576	8.551 8.801	.0556	.1066	.1654	.2109	.2492	.2826	.3125	.3646	.4478	.5120	.5623	.6014	.6307	.6511	.6631	.6677
.04000	.39929	5.721 5.729	.0403	.0796	.1261	.1623	.1929	.2197	.2437	.2855	.3524	.4041	.4446	.4761	.4997	.5161	.5259	.5290
.05000	.34850	4.339 4.475	.0333	.0670	.1076	.1394	.1662	.1898	.2110	.2479	.3069	.3526	.3884	.4163	.4371	.4517	.4603	.4637
.07000	.27566	2.964 2.965	.0238	.0495	.0814	.1067	.1282	.1472	.1642	.1940	.2418	.2787	.3078	.3304	.3473	.3591	.3661	.3684
.09000	.22571	2.168 2.093	.0177	.0380	.0638	.0846	.1024	.1181	.1323	.1571	.1971	.2281	.2524	.2714	.2856	.2955	.3014	.3039
.12000	.17489	1.414 1.362	.0121	.0269	.0465	.0626	.0765	.0889	.1001	.1198	.1517	.1765	.1960	.2113	.2227	.2307	.2354	.2375
.20000	.10315	.5829 .5878	.0056	.0130	.0237	.0329	.0412	.0486	.0555	.0677	.0878	.1036	.1161	.1259	.1333	.1385	.1415	.1424
.40000	.04142	.1597 .1590	.0016	.0038	.0074	.0107	.0138	.0168	.0195	.0247	.0336	.0409	.0468	.0515	.0551	.0576	.0591	.0595

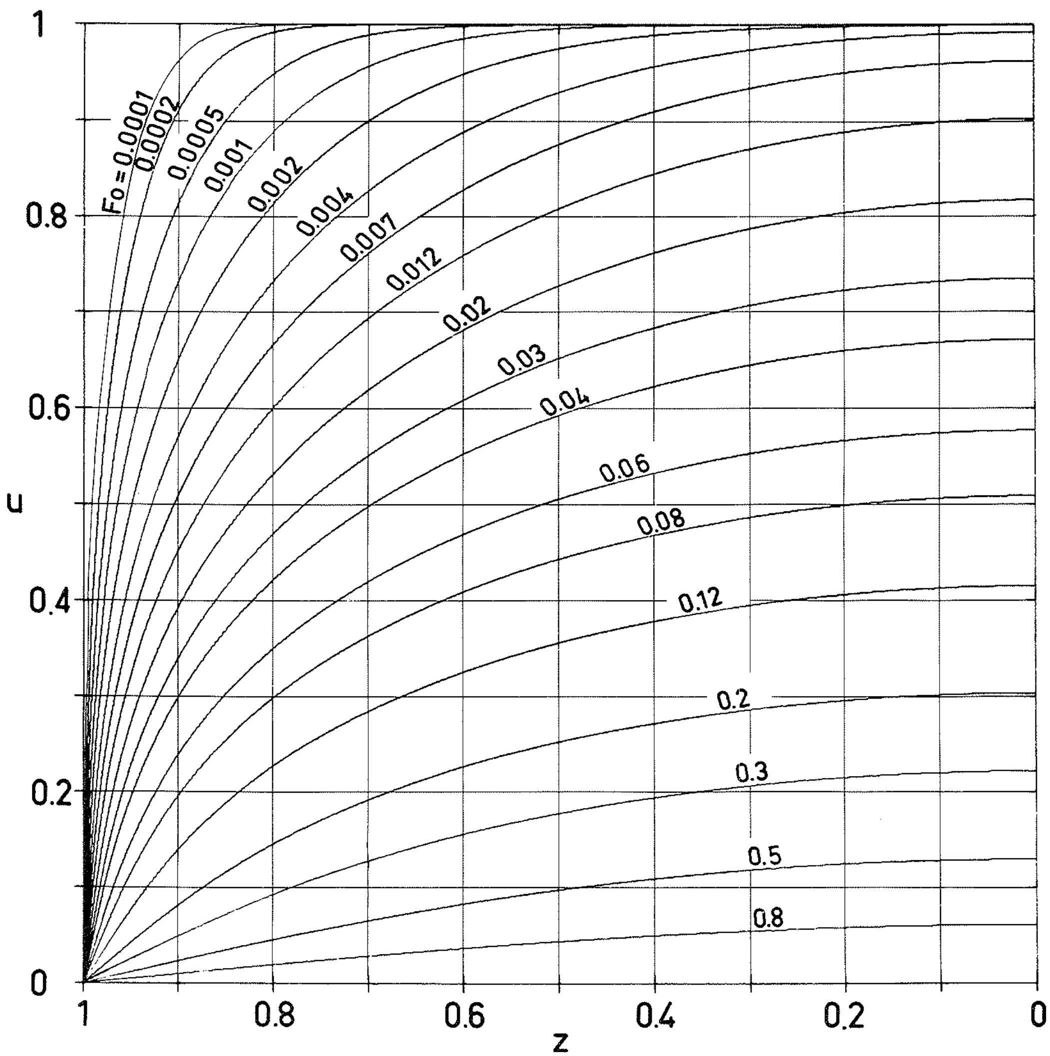

u = u(z,Fo)
z = x/l
Fo=k_e h/l²
b = 20
n = 1
u
z
0
0.2
0.4
0.6
0.8
1
Fo=0.0001
0.0002
0.0005
0.001
0.002
0.003
0.005
0.008
0.012
0.02
0.025
0.04
0.05
0.07
0.09
0.12
0.2
0.4

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 11
$b = 20$
$n = 2$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) ← z															
			.990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00010	.97581	121.0	.4723	.6767	.8388	.9204	.9632	.9845	.9941	.9994	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.96578	85.54	.4031	.5962	.7589	.8517	.9098	.9468	.9698	.9917	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.94590	54.10	.3192	.4956	.6505	.7464	.8137	.8630	.9000	.9485	.9890	.9984	.9998	1.000	1.000	1.000	1.000	1.000
.00100	.92349	38.25	.2614	.4249	.5710	.6643	.7325	.7851	.8270	.8885	.9566	.9853	.9958	.9990	.9998	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.001136																		
.00200	.89170	25.97	.2062	.3588	.4953	.5839	.6502	.7028	.7461	.8134	.8998	.9484	.9752	.9890	.9956	.9984	.9995	.9999
.00400	.84695	27.03 18.67	.1617	.2979	.4248	.5080	.5709	.6217	.6642	.7323	.8269	.8884	.9293	.9564	.9738	.9844	.9900	.9916
.00700	.79760	19.18 14.39	.1299	.2524	.3712	.4499	.5097	.5582	.5993	.6659	.7617	.8279	.8754	.9097	.9339	.9499	.9591	.9621
.01200	.73514	14.48 11.00	.1028	.2108	.3214	.3954	.4518	.4978	.5368	.6006	.6937	.7596	.8083	.8444	.8706	.8885	.8988	.9022
.02000	.65972	10.98 8.245	.0788	.1707	.2717	.3405	.3931	.4361	.4725	.5321	.6194	.6814	.7276	.7620	.7871	.8043	.8143	.8175
.03000	.58880	8.183 6.294	.0604	.1368	.2276	.2913	.3404	.3805	.4144	.4700	.5511	.6088	.6516	.6836	.7069	.7228	.7321	.7349
.04000	.53389	6.175 4.771	.0483	.1126	.1941	.2534	.2996	.3374	.3695	.4219	.4984	.5526	.5928	.6228	.6446	.6595	.6682	.6714
.06000	.45293	4.893 3.373	.0335	.0807	.1466	.1981	.2393	.2736	.3027	.3505	.4202	.4694	.5059	.5330	.5527	.5662	.5740	.5766
.08000	.39480	3.373 2.465	.0250	.0612	.1149	.1595	.1964	.2277	.2545	.2988	.3636	.4094	.4432	.4683	.4866	.4991	.5063	.5086
.12000	.31482	2.514 1.623	.0160	.0396	.0768	.1102	.1397	.1657	.1886	.2273	.2849	.3259	.3563	.3788	.3951	.4063	.4128	.4150
.20000	.22136	1.602 .8831	.0086	.0215	.0424	.0626	.0816	.0994	.1160	.1455	.1922	.2268	.2527	.2722	.2863	.2959	.3015	.3036
.30000	.15564	.8608 .5155	.0050	.0126	.0251	.0373	.0492	.0608	.0719	.0926	.1281	.1562	.1780	.1946	.2069	.2153	.2203	.2220
.50000	.08643	.5037 .2443	.0024	.0059	.0118	.0176	.0234	.0292	.0348	.0457	.0658	.0832	.0978	.1096	.1185	.1248	.1286	.1296
.80000	.03951	.2374 .1039 .0986	.0010	.0025	.0050	.0075	.0099	.0124	.0148	.0196	.0286	.0369	.0441	.0502	.0550	.0585	.0606	.0615

NOMOGRAM 11



$u = u(z, Fo)$

$z = x/l$

$Fo = k_e h/l^2$

$b = 20$
$n = 2$

Concentration distribution in slab $D = k/k_e = 1 + bu^n$	TABLE 12 b = 20 n = 5

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) ← z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00010	.98342	82.89	.6082	.8047	.9122	.9585	.9812	.9921	.9970	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.97656	58.61	.5075	.7393	.8623	.9198	.9527	.9726	.9846	.9958	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.96293	37.07	.3577	.6369	.7846	.8541	.8971	.9264	.9472	.9735	.9944	.9992	.9999	1.000	1.000	1.000	1.000	1.000
.00100	.94758	26.21	.2590	.5419	.7162	.7953	.8446	.8793	.9052	.9408	.9777	.9926	.9979	.9995	.9999	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000269																		
.00250	.91713	16.67 16.59	.1654	.3944	.6084	.7050	.7636	.8047	.8360	.8810	.9348	.9644	.9812	.9906	.9956	.9980	.9991	.9994
.00500	.88273	11.25 11.71	.1168	.2882	.5074	.6234	.6919	.7391	.7746	.8259	.8894	.9277	.9525	.9690	.9799	.9867	.9905	.9915
.00900	.84277	8.706 8.740	.0673	.2170	.4112	.5418	.6218	.6759	.7160	.7732	.8439	.8874	.9169	.9375	.9519	.9613	.9667	.9684
.01500	.79710	6.746 6.734	.0673	.1677	.3278	.4586	.5490	.6109	.6562	.7196	.7964	.8433	.8751	.8975	.9132	.9236	.9296	.9316
.02500	.73895	4.972 5.081	.0508	.1267	.2508	.3651	.4586	.5287	.5808	.6528	.7371	.7871	.8205	.8440	.8603	.8711	.8773	.8792
.04000	.67351	3.788 3.790	.0379	.0946	.1880	.2783	.3614	.4326	.4903	.5731	.6682	.7226	.7582	.7827	.7997	.8109	.8173	.8194
.06000	.60798	2.898 2.856	.0286	.0713	.1421	.2115	.2784	.3408	.3969	.4861	.5946	.6555	.6944	.7207	.7387	.7505	.7572	.7594
.08000	.55681	2.347 2.302	.0230	.0575	.1146	.1710	.2261	.2791	.3291	.4157	.5328	.6004	.6429	.6713	.6905	.7030	.7100	.7123
.12000	.47882	1.645 1.669	.0167	.0417	.0832	.1244	.1650	.2047	.2434	.3159	.4326	.5097	.5595	.5924	.6144	.6286	.6366	.6391
.20000	.37217	1.072 1.082	.0108	.0270	.0540	.0808	.1074	.1337	.1596	.2099	.3016	.3766	.4329	.4728	.5002	.5179	.5278	.5311
.30000	.28278	.7414 .7441	.0074	.0186	.0372	.0556	.0740	.0922	.1103	.1456	.2123	.2717	.3216	.3611	.3904	.4103	.4218	.4256
.40000	.21868	.5577 .5523	.0055	.0138	.0276	.0413	.0550	.0685	.0820	.1084	.1589	.2049	.2451	.2786	.3047	.3233	.3343	.3380
.50000	.17025	.4280 .4233	.0042	.0106	.0211	.0317	.0422	.0526	.0629	.0832	.1222	.1579	.1897	.2165	.2379	.2535	.2628	.2660
.70000	.10376	.2588 .2562	.0026	.0064	.0128	.0192	.0255	.0318	.0381	.0504	.0740	.0958	.1153	.1319	.1452	.1550	.1609	.1629
.90000	.06333	.1578 .1563	.0016	.0039	.0078	.0117	.0156	.0194	.0232	.0307	.0452	.0585	.0703	.0805	.0886	.0946	.0982	.0995

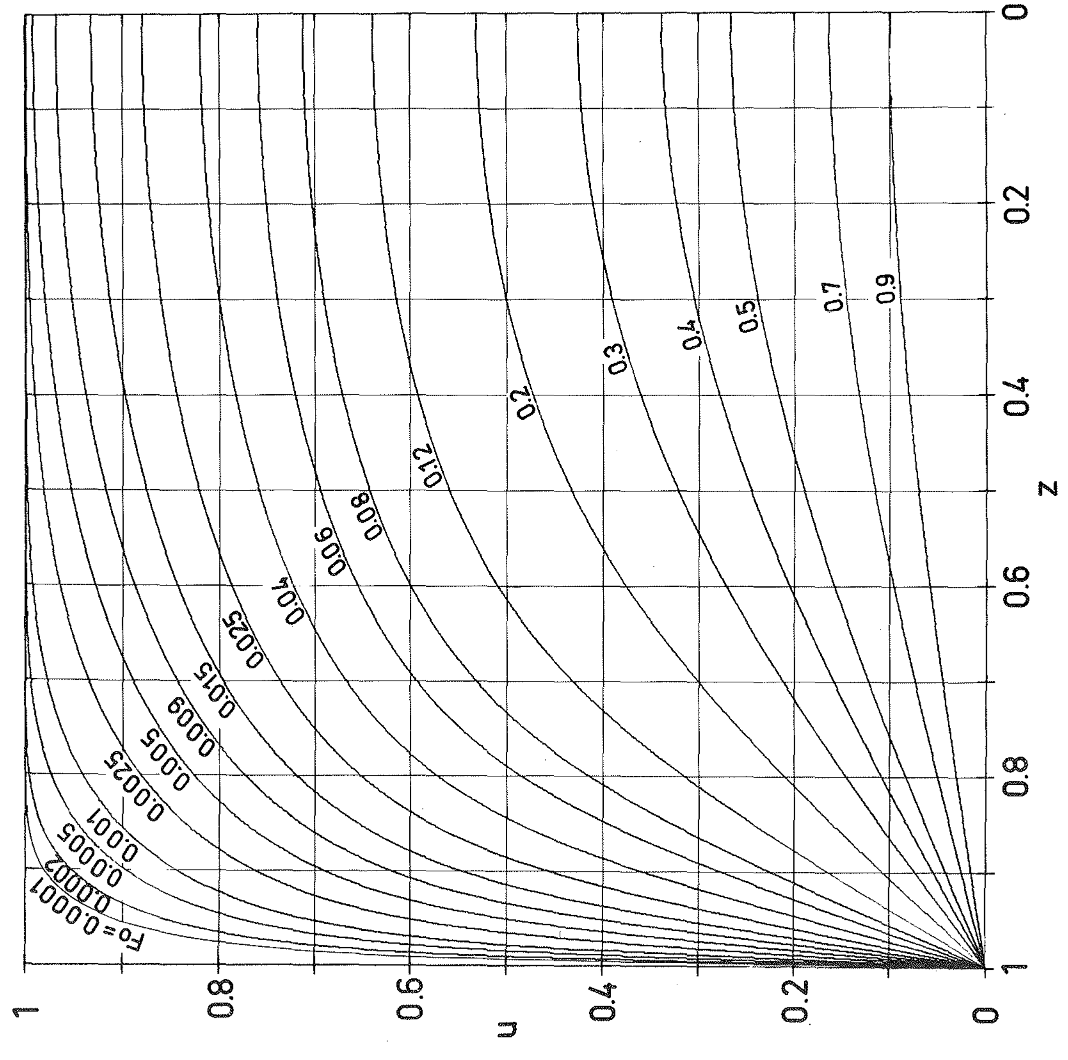
NOMOGRAM 12

$\underline{u = u(z, Fo)}$

$z = x/l$

$Fo = k_e h/l^2$

$b = 20$
$n = 5$



Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 13
$b = 40$
$n = 1$

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) z = .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00005	.96921	307.9	.3649	.5757	.7755	.8866	.9473	.9779	.9918	.9992	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00010	.95646	217.7	.3043	.4871	.6734	.7926	.8718	.9239	.9569	.9883	.9996	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.93842	154.0	.2528	.4093	.5757	.6907	.7755	.8390	.8866	.9473	.9918	.9992	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.90263	97.37	.1967	.3227	.4608	.5616	.6412	.7062	.7600	.8423	.9378	.9791	.9942	.9987	.9998	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000739																		
.00090	.86918	70.31 72.71	.1621	.2760	.3969	.4868	.5596	.6208	.6731	.7579	.8717	.9366	.9714	.9884	.9958	.9987	.9996	.9996
.00150	.83110	58.00 56.26	.1403	.2404	.3478	.4284	.4946	.5510	.6003	.6825	.8017	.8799	.9304	.9617	.9801	.9902	.9950	.9961
.00250	.78215	44.34 43.57	.1212	.2092	.3041	.3760	.4356	.4869	.5322	.6094	.7268	.8109	.8714	.9144	.9439	.9628	.9734	.9765
.00400	.72449	33.71 34.33	.1056	.1835	.2681	.3325	.3861	.4326	.4739	.5452	.6567	.7403	.8039	.8519	.8869	.9107	.9246	.9295
.00600	.66294	28.13 27.78	.0931	.1629	.2390	.2971	.3456	.3878	.4255	.4907	.5941	.6732	.7345	.7818	.8169	.8413	.8556	.8606
.00800	.61190	23.59 23.53	.0842	.1481	.2181	.2716	.3163	.3552	.3900	.4504	.5464	.6202	.6778	.7224	.7558	.7790	.7927	.7971
.01000	.56814	19.86 20.34	.0770	.1361	.2011	.2508	.2923	.3285	.3609	.4171	.5066	.5755	.6294	.6712	.7025	.7243	.7372	.7410
.01500	.48161	15.06 14.79	.0631	.1130	.1683	.2106	.2460	.2768	.3044	.3524	.4288	.4877	.5338	.5696	.5965	.6151	.6262	.6294
.02000	.41723	11.39 11.23	.0529	.0960	.1441	.1809	.2117	.2386	.2626	.3044	.3711	.4225	.4627	.4940	.5174	.5337	.5433	.5468
.02500	.36738	8.643 8.830	.0452	.0830	.1254	.1579	.1852	.2090	.2303	.2674	.3264	.3720	.4077	.4354	.4561	.4706	.4791	.4824
.04000	.26850	4.929 4.924	.0301	.0573	.0885	.1126	.1328	.1505	.1663	.1939	.2379	.2718	.2984	.3191	.3346	.3454	.3517	.3538
.06000	.19496	2.669 2.747	.0195	.0386	.0613	.0790	.0940	.1071	.1188	.1393	.1720	.1973	.2171	.2325	.2441	.2521	.2569	.2580
.09000	.13541	1.420 1.438	.0116	.0242	.0398	.0523	.0629	.0722	.0805	.0952	.1187	.1369	.1512	.1623	.1707	.1765	.1799	.1809
.15000	.08006	.5765 .5898	.0053	.0119	.0208	.0281	.0345	.0402	.0454	.0545	.0692	.0807	.0898	.0969	.1022	.1059	.1081	.1090
.30000	.03397	.1614 .1570	.0015	.0037	.0069	.0098	.0125	.0150	.0172	.0214	.0283	.0339	.0383	.0418	.0445	.0463	.0474	.0480

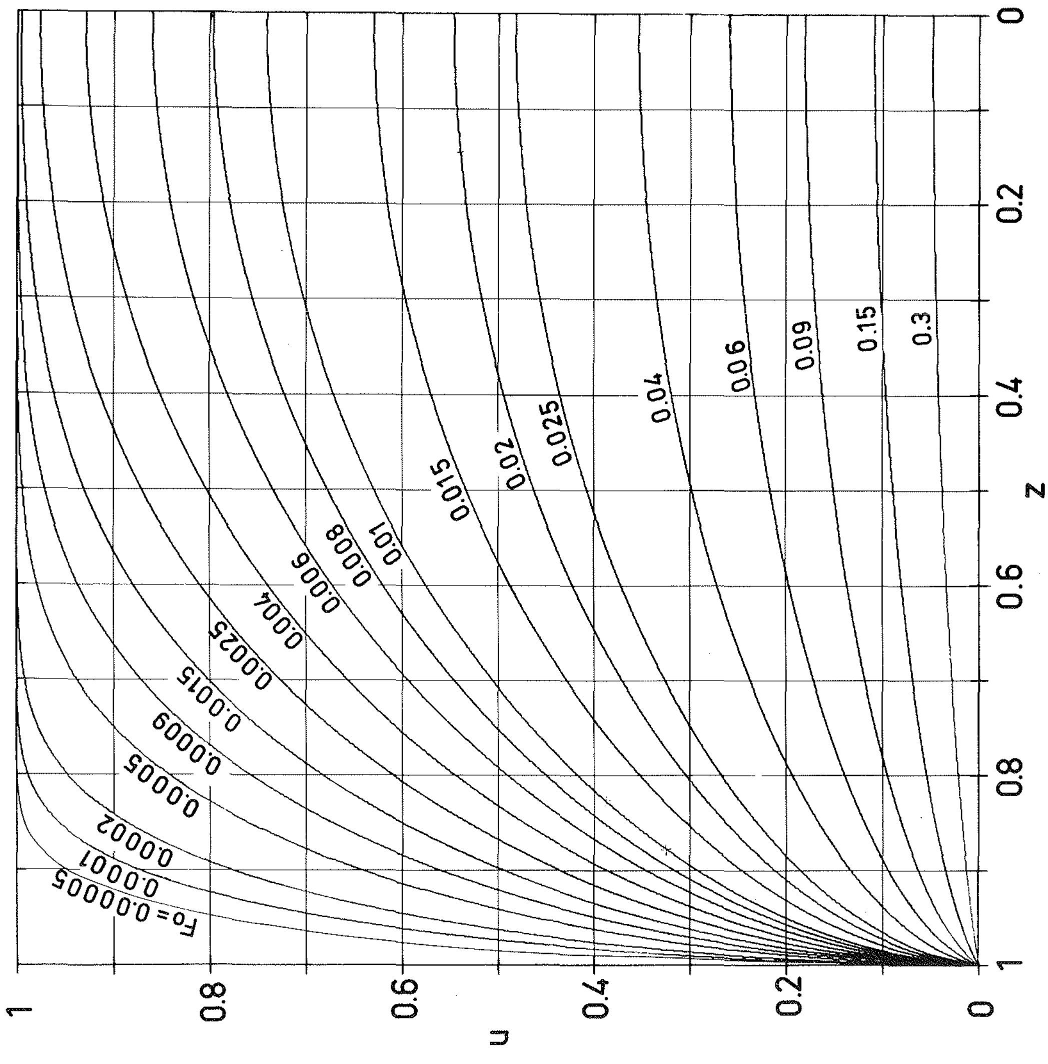
NOMOGRAM 13

$\underline{u = u(z, Fo)}$

$z = x/l$

$Fo = k_e h/l^2$

$b = 40$
$n = 1$



Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 14
$b = 40$
$n = 2$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) z = .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00005	.97726	227.4	.5037	.6944	.8480	.9255	.9659	.9859	.9948	.9995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00010	.96784	160.8	.4402	.6188	.7721	.8603	.9155	.9505	.9722	.9925	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.95451	113.7	.3817	.5472	.6944	.7853	.8480	.8929	.9255	.9659	.9948	.9995	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.92808	71.92	.3115	.4601	.5952	.6827	.7471	.7970	.8368	.8952	.9597	.9866	.9963	.9992	.9999	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.001924																		
.00100	.89809	52.06 50.90	.2549	.4000	.5251	.6075	.6696	.7192	.7602	.8240	.9060	.9521	.9772	.9901	.9961	.9986	.9995	.9999
.00200	.85596	34.52 35.98	.2132	.3445	.4598	.5364	.5949	.6424	.6824	.7467	.8365	.8950	.9339	.9596	.9760	.9859	.9911	.9924
.00400	.79652	24.96 25.43	.1750	.2937	.3999	.4706	.5249	.5694	.6072	.6692	.7595	.8227	.8686	.9021	.9261	.9421	.9514	.9546
.00700	.73114	19.44 19.00	.1454	.2540	.3529	.4187	.4693	.5109	.5463	.6046	.6907	.7523	.7981	.8324	.8573	.8743	.8843	.8878
.01000	.68003	15.26 15.41	.1259	.2272	.3211	.3835	.4314	.4707	.5042	.5594	.6410	.6996	.7433	.7761	.8000	.8164	.8260	.8292
.01500	.61327	11.65 11.64	.1019	.1934	.2808	.3389	.3834	.4199	.4509	.5020	.5773	.6314	.6717	.7019	.7241	.7392	.7480	.7509
.02500	.51970	7.776 7.563	.0711	.1465	.2242	.2764	.3162	.3488	.3764	.4218	.4884	.5360	.5715	.5981	.6175	.6308	.6385	.6414
.04000	.43053	4.724 4.729	.0461	.1032	.1695	.2156	.2511	.2800	.3045	.3446	.4033	.4450	.4760	.4991	.5160	.5275	.5343	.5365
.06000	.35533	3.090 3.010	.0298	.0704	.1240	.1639	.1953	.2211	.2430	.2788	.3310	.3680	.3954	.4158	.4306	.4408	.4467	.4484
.08000	.30468	2.075 2.140	.0213	.0516	.0950	.1296	.1577	.1811	.2011	.2340	.2820	.3159	.3409	.3595	.3731	.3824	.3878	.3893
.12000	.23348	1.318 1.293	.0129	.0319	.0613	.0873	.1097	.1293	.1463	.1749	.2171	.2471	.2692	.2857	.2976	.3057	.3105	.3119
.20000	.16438	.6525 .6636	.0066	.0165	.0326	.0480	.0624	.0758	.0882	.1100	.1442	.1693	.1881	.2021	.2123	.2193	.2233	.2247
.30000	.11489	.3844 .3791	.0038	.0095	.0188	.0280	.0369	.0456	.0538	.0692	.0952	.1156	.1314	.1434	.1523	.1583	.1619	.1630
.50000	.06338	.1731 .1746	.0017	.0044	.0087	.0130	.0173	.0215	.0257	.0337	.0484	.0612	.0718	.0803	.0868	.0914	.0941	.0950

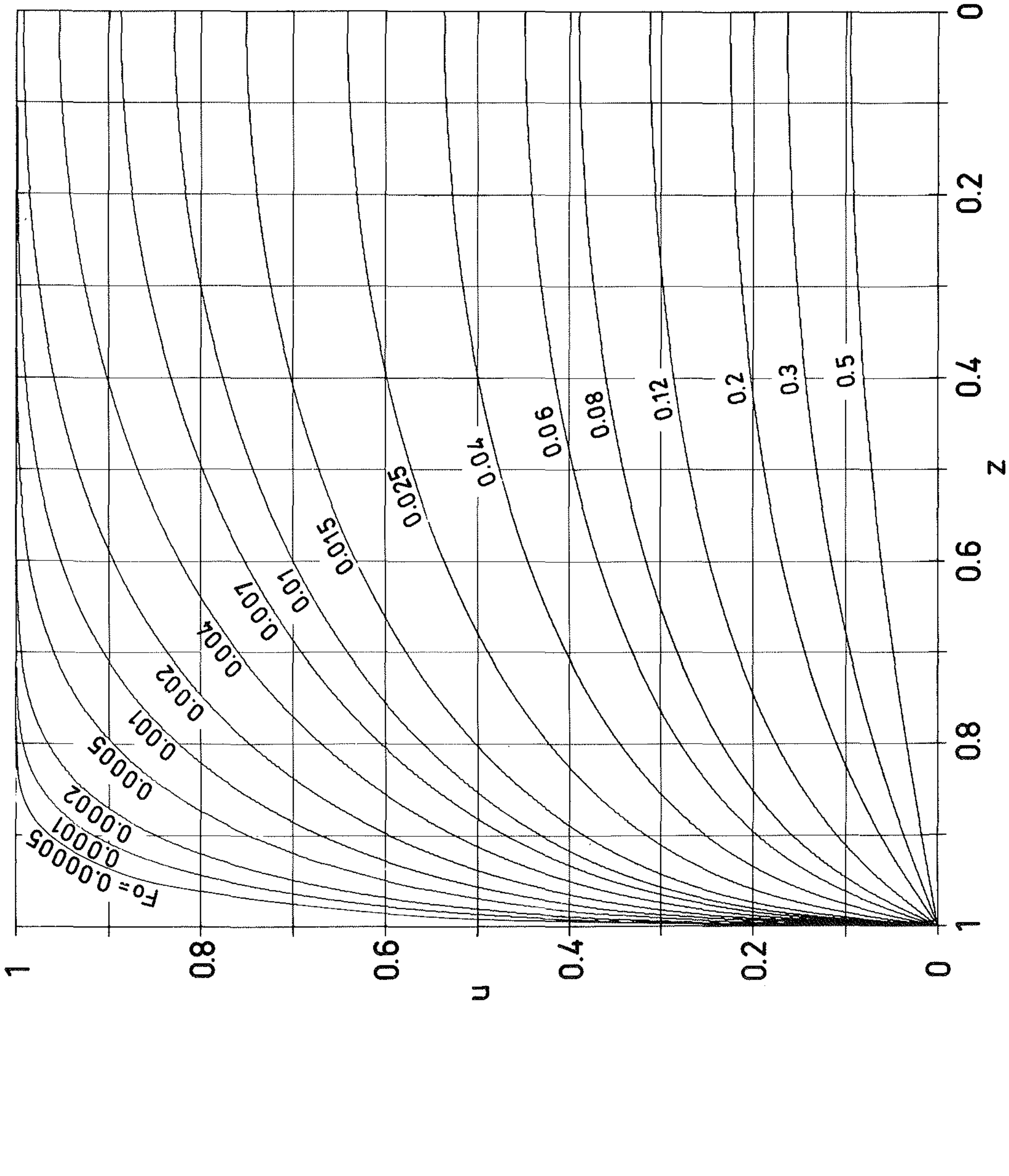
NOMOGRAM 14

$u = u(z, Fo)$

$z = x/l$

$Fo = k_e h / l^2$

$b = 40$
$n = 2$



Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 15
$b = 40$
$n = 5$

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000 ← z
.00005	.98570	143.0	.6781	.8275	.9207	.9625	.9831	.9931	.9974	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00010	.97977	101.1	.6117	.7743	.8767	.9275	.9572	.9753	.9863	.9963	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.97139	71.52	.5353	.7173	.8275	.8846	.9207	.9453	.9625	.9831	.9974	.9998	1.000	1.000	1.000	1.000	1.000	1.000
.00050	.95477	45.23	.4125	.6339	.7564	.8197	.8613	.8915	.9144	.9465	.9800	.9934	.9982	.9996	.9999	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.000627																		
.00120	.93004	29.31 35.79	.2953	.5406	.6818	.7512	.7969	.8305	.8568	.8957	.9432	.9695	.9843	.9924	.9966	.9986	.9994	.9996
.00300	.88918	17.97 17.29	.1853	.4187	.5928	.6721	.7227	.7596	.7886	.8322	.8887	.9243	.9482	.9646	.9757	.9828	.9868	.9880
.00600	.84329	12.62 12.19	.1298	.3164	.5123	.6043	.6605	.7005	.7316	.7781	.8388	.8779	.9052	.9247	.9385	.9477	.9530	.9546
.01200	.77919	9.275 8.931	.0894	.2220	.4084	.5181	.5836	.6286	.6626	.7123	.7756	.8160	.8441	.8642	.8784	.8879	.8934	.8951
.02000	.71861	6.681 6.506	.0648	.1615	.3136	.4301	.5063	.5581	.5962	.6501	.7166	.7579	.7862	.8063	.8205	.8300	.8354	.8371
.03000	.66285	4.849 4.862	.0485	.1211	.2391	.3449	.4269	.4861	.5296	.5899	.6612	.7042	.7332	.7535	.7678	.7773	.7827	.7845
.05000	.58403	3.206 3.258	.0326	.0813	.1616	.2394	.3111	.3730	.4234	.4958	.5791	.6268	.6579	.6793	.6942	.7039	.7095	.7113
.07000	.52761	2.390 2.465	.0246	.0616	.1226	.1826	.2404	.2945	.3431	.4209	.5157	.5689	.6029	.6259	.6416	.6518	.6577	.6595
.09000	.48338	1.905 1.990	.0199	.0497	.0991	.1479	.1956	.2415	.2847	.3601	.4622	.5212	.5583	.5831	.5998	.6107	.6169	.6190
.12000	.43104	1.538 1.553	.0155	.0388	.0774	.1157	.1533	.1902	.2258	.2920	.3951	.4609	.5028	.5305	.5491	.5610	.5678	.5699
.15000	.38867	1.203 1.273	.0127	.0318	.0635	.0950	.1260	.1566	.1865	.2435	.3402	.4091	.4550	.4856	.5061	.5193	.5267	.5292
.20000	.33317	.9605 .9831	.0098	.0246	.0491	.0734	.0976	.1214	.1449	.1904	.2729	.3393	.3882	.4226	.4460	.4610	.4695	.4722
.30000	.25232	.7013 .6676	.0067	.0167	.0333	.0499	.0664	.0827	.0989	.1306	.1903	.2432	.2875	.3223	.3478	.3652	.3751	.3785
.40000	.19500	.5120 .4936	.0049	.0123	.0247	.0369	.0491	.0613	.0733	.0969	.1419	.1830	.2188	.2485	.2716	.2880	.2977	.3010
.50000	.15175	.3645 .3776	.0038	.0094	.0189	.0283	.0376	.0469	.0561	.0742	.1090	.1409	.1691	.1930	.2121	.2258	.2342	.2371
.70000	.09249	.2351 .2284	.0023	.0057	.0114	.0171	.0227	.0284	.0339	.0449	.0660	.0854	.1028	.1176	.1294	.1381	.1434	.1453
1.0000	.04410	.1120 .1088	.0011	.0027	.0054	.0081	.0108	.0135	.0162	.0214	.0315	.0407	.0490	.0560	.0617	.0659	.0684	.0693

NOMOGRAM 15

$\underline{u = u(z, Fo)}$

$z = x/l$

$Fo = k_e h/l^2$

$b = 40$
$n = 5$

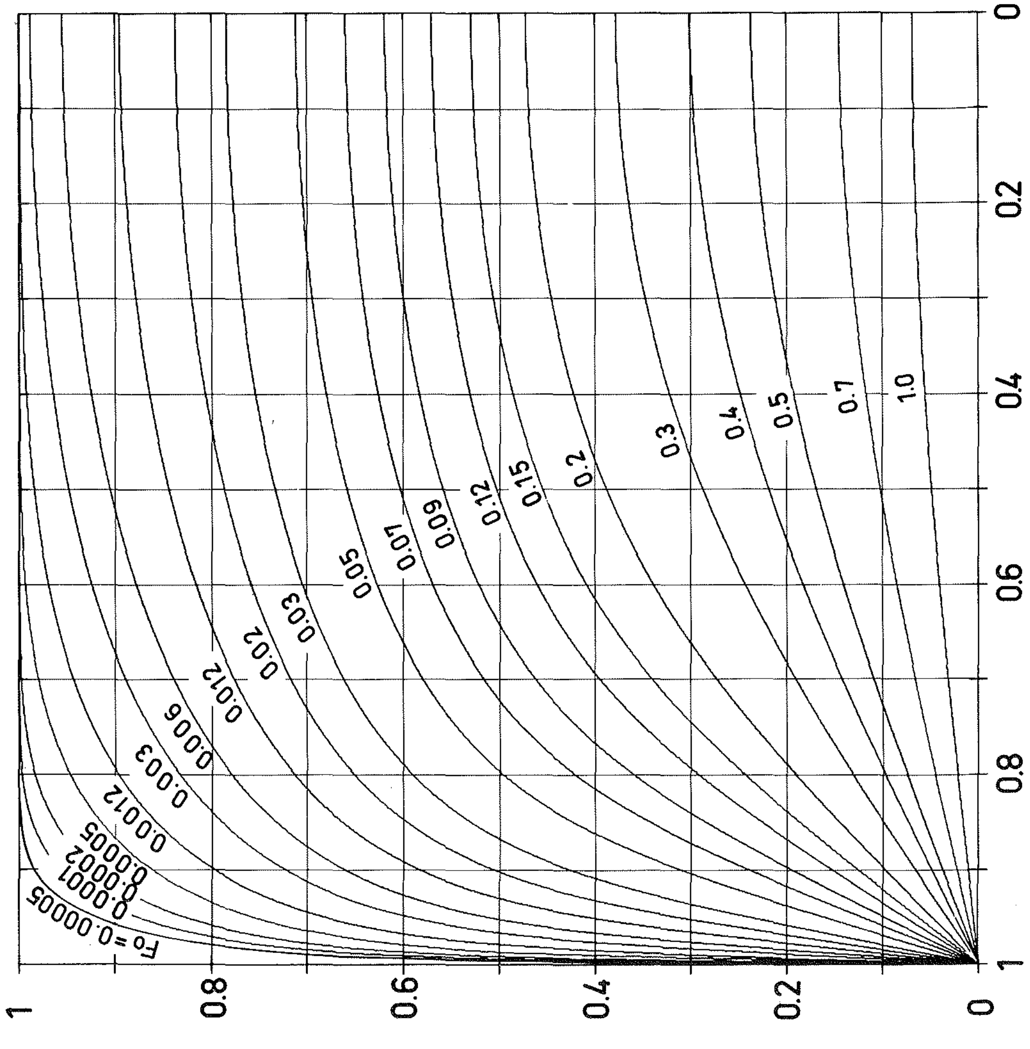


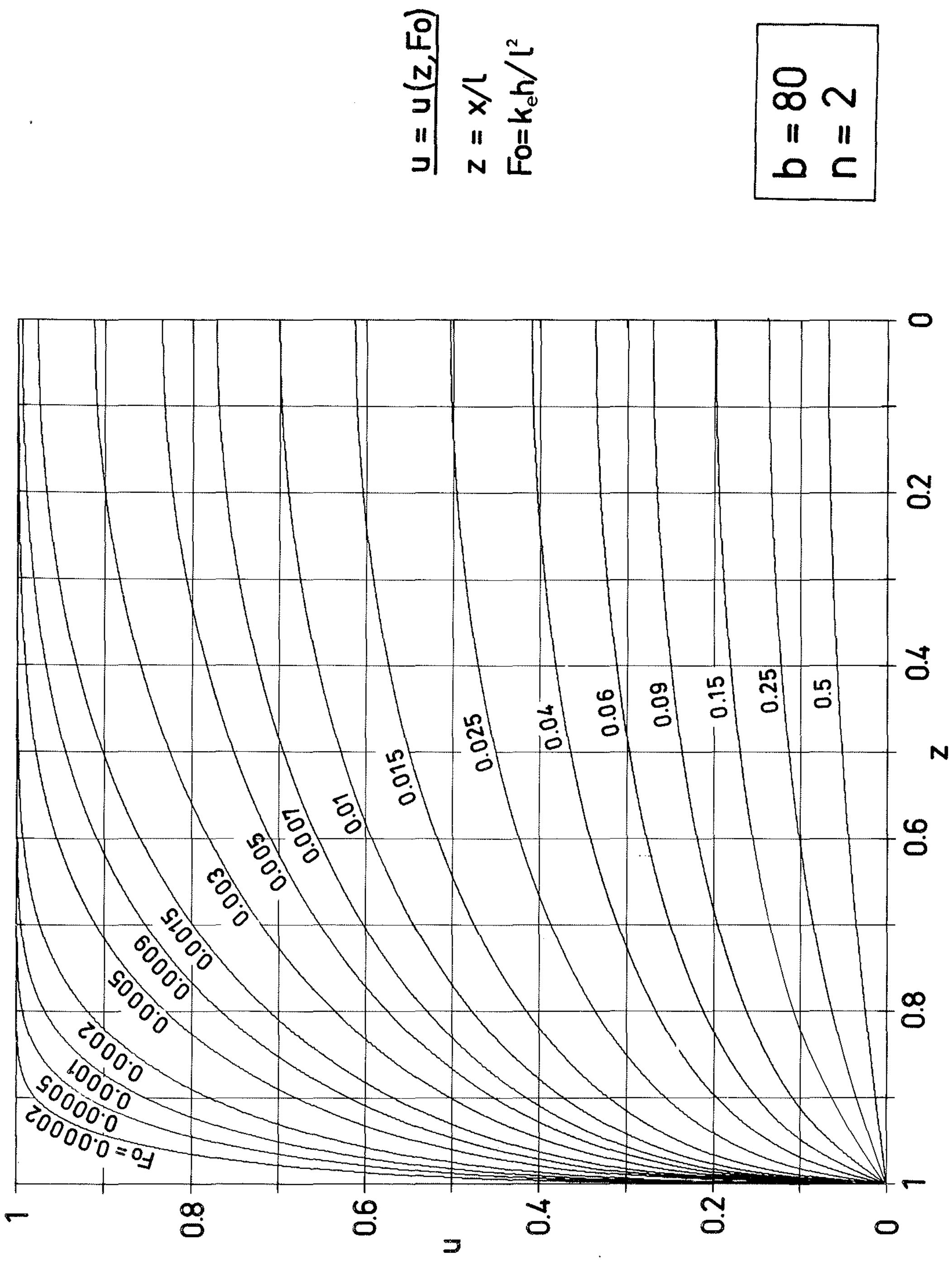
TABLE 17

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

$b = 80$

$n = 2$

Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) z = .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00002	.98035	491.4	.5405	.7276	.8750	.9451	.9782	.9924	.9978	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00005	.96892	310.8	.4597	.6304	.7788	.8646	.9183	.9523	.9734	.9929	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00010	.95605	219.7	.4046	.5616	.7034	.7916	.8526	.8964	.9281	.9673	.9951	.9996	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.93785	155.4	.3545	.4979	.6304	.7162	.7788	.8269	.8646	.9183	.9734	.9929	.9985	.9998	1.000	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.001820																		
.00050	.90133	100.1 / 98.34	.2824	.4218	.5404	.6195	.6794	.7275	.7673	.8293	.9091	.9539	.9782	.9906	.9963	.9987	.9996	.9999
.00090	.86735	71.14 / 73.24	.2499	.3775	.4874	.5614	.6182	.6644	.7033	.7658	.8525	.9080	.9441	.9673	.9815	.9898	.9941	.9955
.00150	.82945	54.33 / 56.65	.2234	.3418	.4444	.5138	.5674	.6114	.6488	.7100	.7981	.8586	.9015	.9319	.9530	.9668	.9746	.9774
.00300	.75920	38.73 / 39.87	.1898	.2965	.3899	.4530	.5020	.5425	.5771	.6344	.7192	.7799	.8250	.8586	.8831	.8997	.9094	.9130
.00500	.69069	29.03 / 29.77	.1641	.2619	.3481	.4063	.4514	.4886	.5205	.5734	.6520	.7087	.7512	.7831	.8064	.8224	.8317	.8351
.00700	.63775	23.24 / 23.68	.1451	.2365	.3176	.3721	.4143	.4491	.4789	.5282	.6016	.6545	.6942	.7240	.7458	.7608	.7696	.7722
.01000	.57632	17.90 / 17.83	.1231	.2070	.2822	.3326	.3715	.4035	.4309	.4762	.5434	.5918	.6281	.6554	.6754	.6890	.6970	.6997
.01500	.50266	12.61 / 12.24	.0965	.1710	.2394	.2849	.3199	.3487	.3732	.4137	.4736	.5167	.5490	.5732	.5909	.6031	.6102	.6128
.02500	.41007	6.972 / 7.101	.0640	.1249	.1844	.2239	.2542	.2790	.3001	.3347	.3858	.4224	.4497	.4702	.4851	.4954	.5013	.5035
.04000	.32988	3.971 / 4.073	.0393	.0851	.1355	.1699	.1962	.2177	.2359	.2658	.3095	.3406	.3638	.3811	.3938	.4024	.4075	.4090
.06000	.26703	2.386 / 2.448	.0241	.0563	.0971	.1268	.1498	.1688	.1848	.2110	.2493	.2764	.2965	.3115	.3224	.3299	.3342	.3355
.09000	.21092	1.449 / 1.446	.0144	.0349	.0647	.0886	.1082	.1245	.1385	.1614	.1950	.2187	.2362	.2492	.2587	.2652	.2689	.2702
.15000	.14979	.7431 / .7297	.0073	.0181	.0352	.0508	.0647	.0771	.0882	.1069	.1349	.1550	.1698	.1808	.1888	.1942	.1974	.1985
.25000	.09891	.3583 / .3599	.0036	.0090	.0178	.0264	.0346	.0423	.0497	.0629	.0845	.1008	.1131	.1223	.1291	.1337	.1364	.1373
.50000	.04568	.1258 / .1263	.0013	.0032	.0063	.0094	.0125	.0156	.0185	.0243	.0350	.0441	.0518	.0579	.0625	.0658	.0677	.0684


u = u(z,Fo)
z = x/l
Fo=k_eh/l²
b = 80
n = 2
u
z
0
0.2
0.4
0.6
0.8
1
Fo = 0.00002
0.00005
0.0001
0.0002
0.0005
0.0009
0.0015
0.003
0.005
0.007
0.01
0.015
0.025
0.04
0.06
0.09
0.15
0.25
0.5

Concentration distribution in slab $D = k/k_e = 1 + bu^n$

TABLE 18
$b = 80$
$n = 5$

Fo	$\bar{u}$	$\frac{d\bar{u}}{dFo}$ Ø	u(z,Fo) ← z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
.00002	.98840	290.1	.7261	.8536	.9370	.9731	.9894	.9964	.9989	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00005	.98165	183.5	.6582	.7913	.8839	.9314	.9595	.9767	.9871	.9966	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.00010	.97406	129.7	.6039	.7422	.8388	.8912	.9250	.9482	.9645	.9841	.9976	.9998	1.000	1.000	1.000	1.000	1.000	1.000
.00020	.96331	91.73	.5451	.6917	.7913	.8466	.8839	.9110	.9314	.9595	.9871	.9966	.9993	.9999	1.000	1.000	1.000	1.000
DERIVATE QUANTITY ON SURFACE - INTEGRAL OF CONC. CHANGE = -.009042																		
.00060	.93645	50.56 / 53.33	.4240	.6070	.7127	.7709	.8110	.8413	.8654	.9015	.9461	.9711	.9852	.9929	.9968	.9987	.9995	.9996
.00150	.89969	33.60 / 33.99	.3250	.5286	.6443	.7052	.7468	.7785	.8039	.8430	.8951	.9284	.9510	.9665	.9771	.9839	.9877	.9889
.00400	.83609	20.58 / 20.48	.2061	.4245	.5628	.6290	.6729	.7057	.7320	.7724	.8271	.8634	.8891	.9077	.9210	.9299	.9351	.9368
.00800	.77042	13.62 / 13.52	.1349	.3207	.4850	.5600	.6071	.6414	.6683	.7091	.7635	.7993	.8245	.8428	.8558	.8646	.8696	.8713
.01500	.69564	8.412 / 8.595	.0858	.2126	.3808	.4725	.5272	.5652	.5941	.6368	.6917	.7270	.7517	.7694	.7820	.7904	.7953	.7969
.02500	.62639	5.562 / 5.669	.0566	.1411	.2739	.3754	.4417	.4867	.5198	.5667	.6246	.6606	.6853	.7028	.7151	.7234	.7281	.7298
.04000	.55760	3.713 / 3.771	.0377	.0941	.1863	.2718	.3426	.3961	.4361	.4913	.5559	.5942	.6198	.6377	.6502	.6585	.6632	.6648
.06000	.49508	2.650 / 2.627	.0263	.0656	.1305	.1936	.2529	.3054	.3495	.4144	.4896	.5322	.5599	.5788	.5919	.6005	.6054	.6070
.08000	.44910	2.033 / 2.027	.0203	.0506	.1009	.1503	.1982	.2434	.2847	.3523	.4368	.4845	.5147	.5350	.5489	.5580	.5632	.5648
.12000	.38207	1.427 / 1.403	.0140	.0351	.0699	.1045	.1385	.1716	.2036	.2626	.3528	.4093	.4451	.4688	.4847	.4950	.5008	.5027
.20000	.29427	.8855 / .8759	.0088	.0219	.0437	.0654	.0869	.1081	.1290	.1695	.2424	.3006	.3432	.3728	.3930	.4059	.4132	.4156
.30000	.22263	.5980 / .5915	.0059	.0148	.0295	.0442	.0588	.0733	.0876	.1156	.1684	.2151	.2539	.2843	.3066	.3217	.3303	.3331
.40000	.17190	.4360 / .4359	.0044	.0109	.0218	.0326	.0434	.0541	.0647	.0855	.1253	.1614	.1930	.2191	.2393	.2537	.2622	.2650
.60000	.10435	.2606 / .2582	.0026	.0065	.0129	.0193	.0257	.0321	.0384	.0508	.0746	.0965	.1160	.1326	.1460	.1557	.1616	.1636
.80000	.06366	.1585 / .1571	.0016	.0039	.0078	.0118	.0156	.0195	.0233	.0309	.0454	.0588	.0707	.0809	.0891	.0951	.0987	.1000
1.2000	.02372	.0590 / .0585	.0006	.0015	.0029	.0044	.0058	.0073	.0087	.0115	.0169	.0219	.0263	.0301	.0332	.0354	.0368	.0372

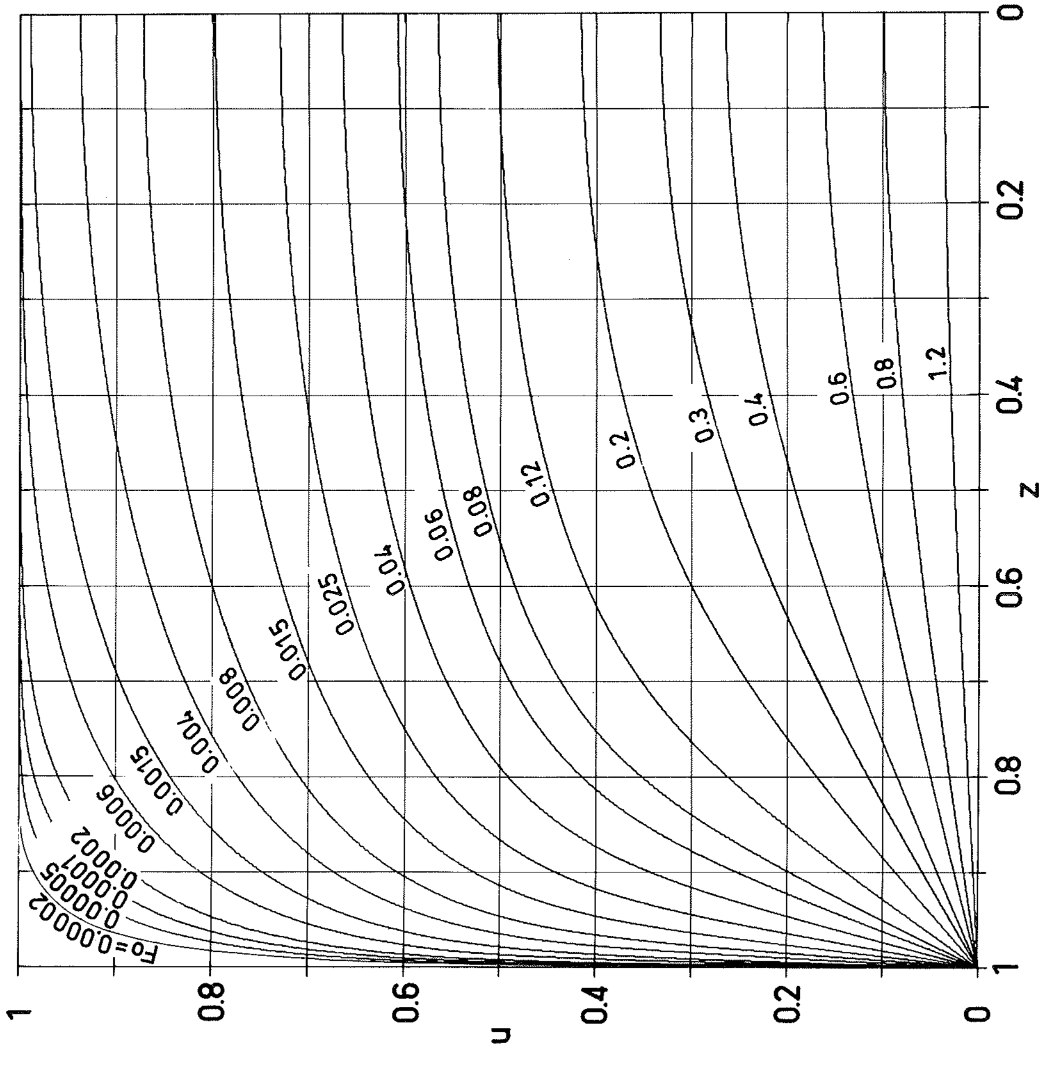
NOMOGRAM 18

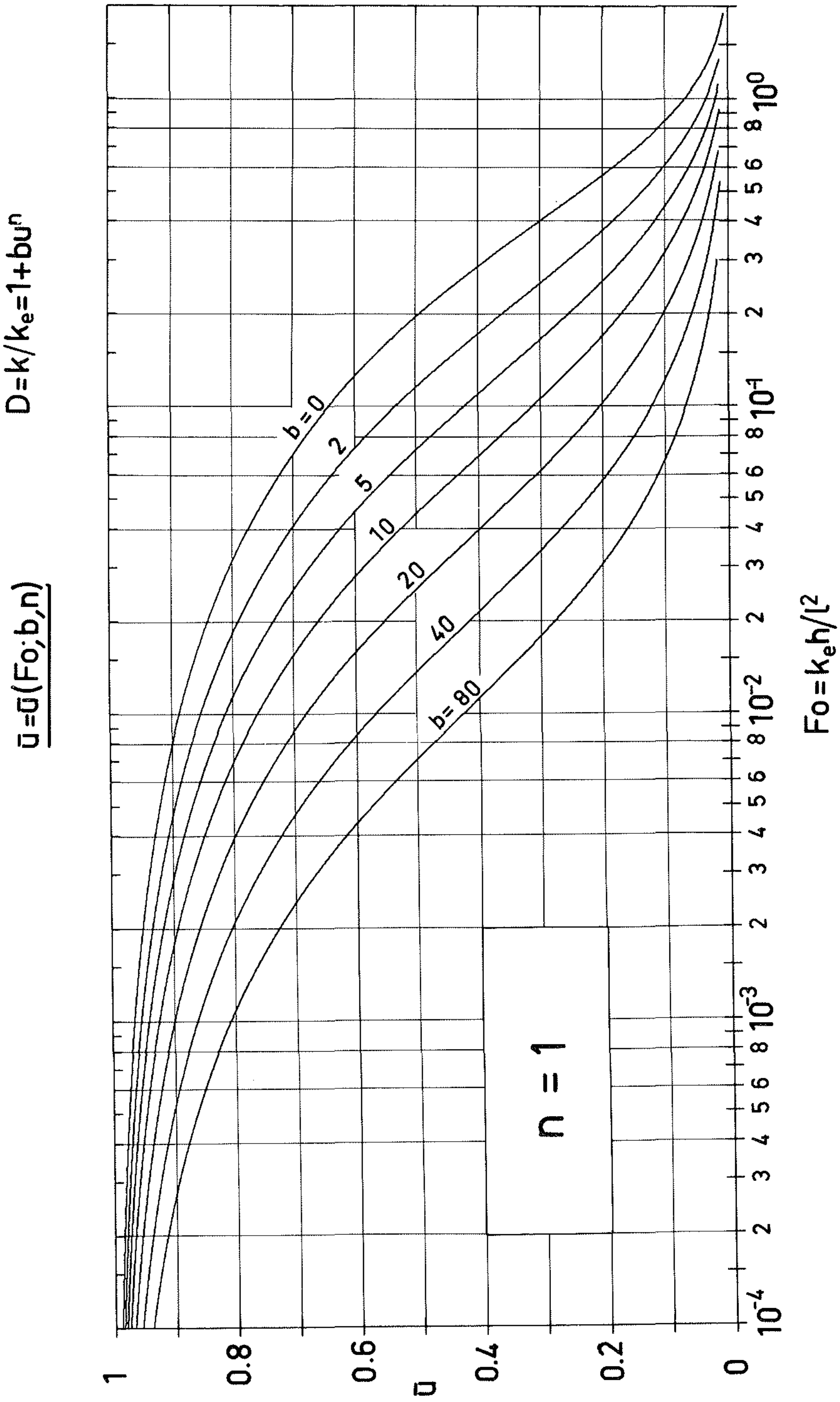
$\underline{u = u(z, Fo)}$

$z = x/l$

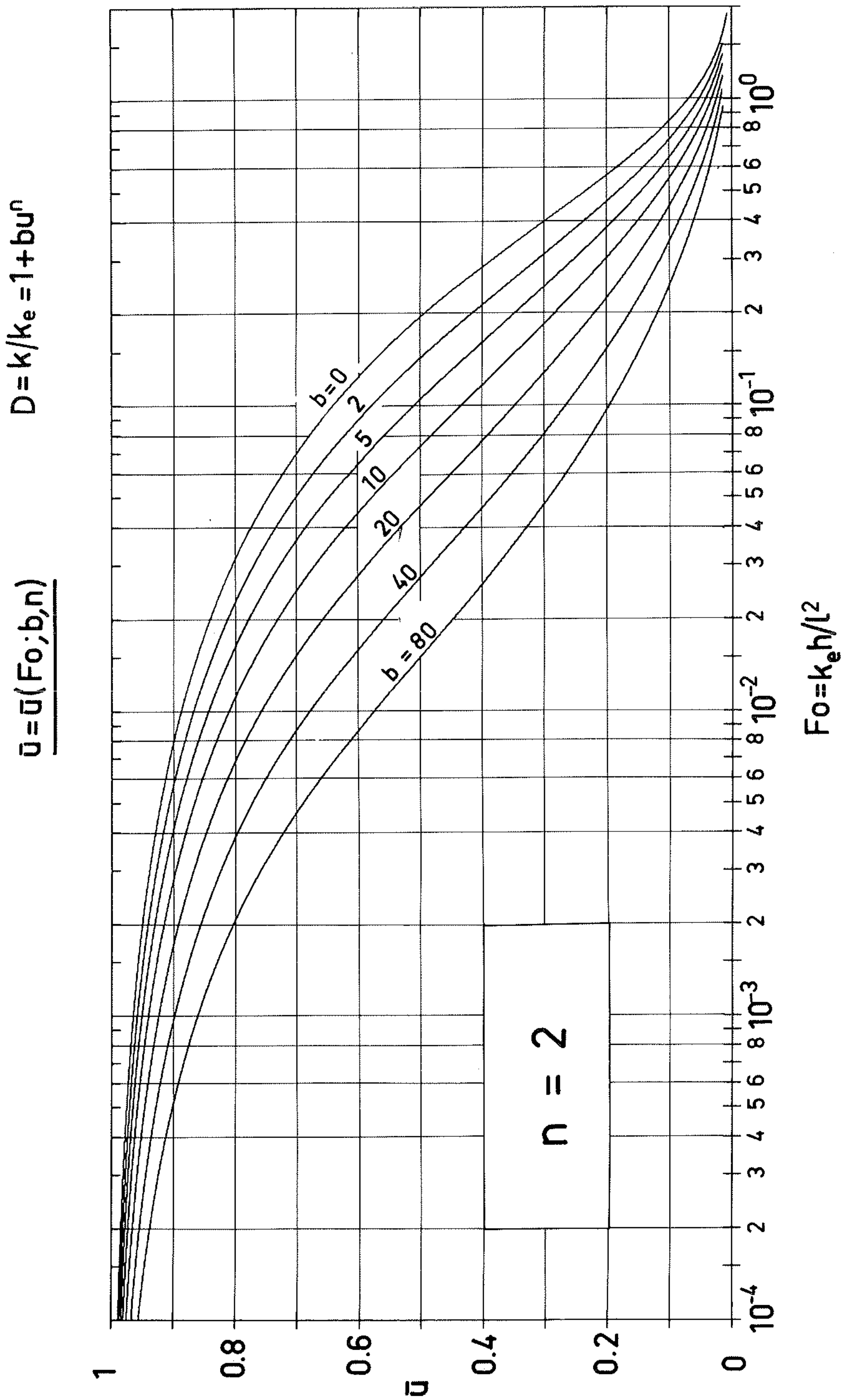
$Fo = k_e h/l^2$

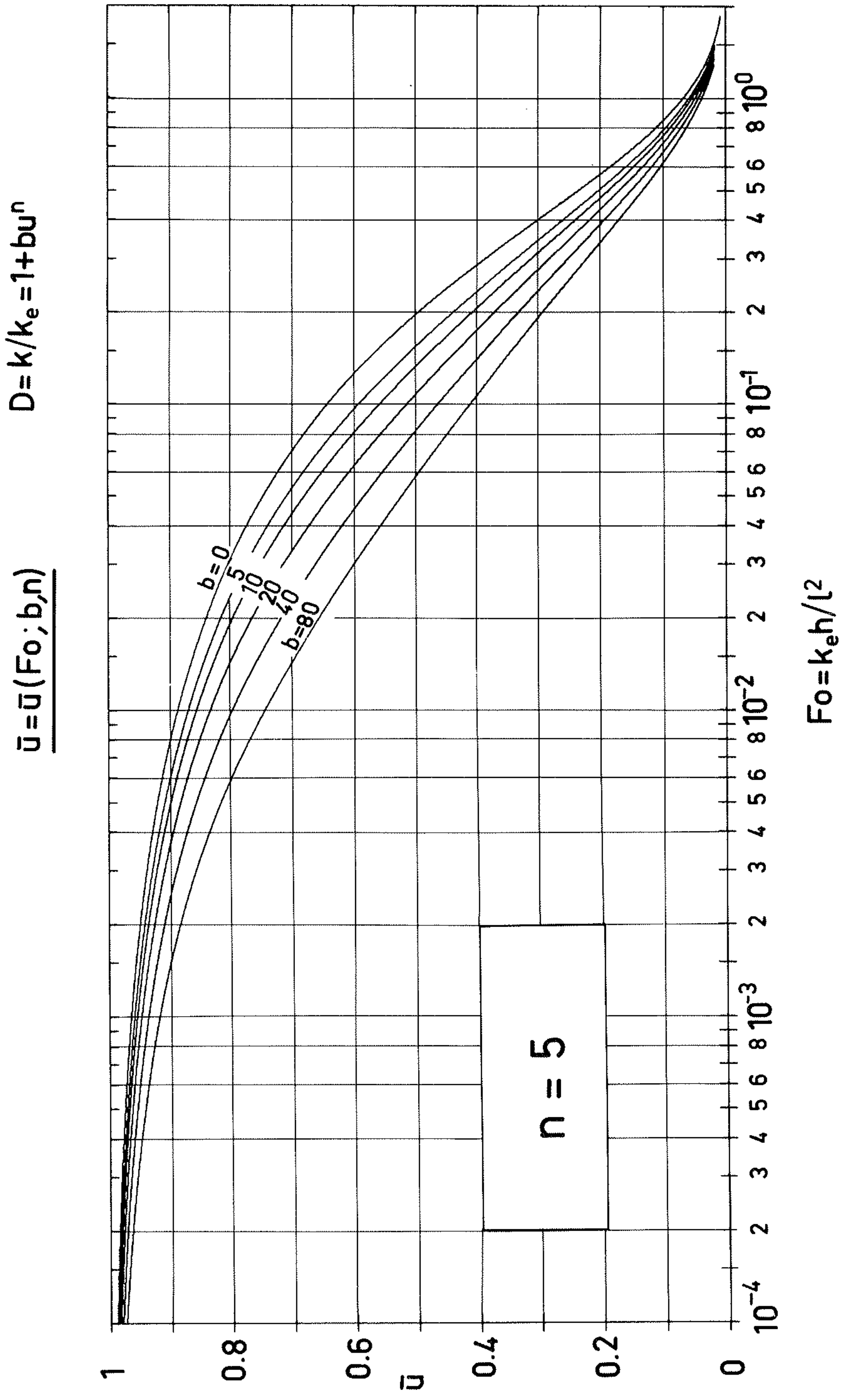
$b = 80$
$n = 5$

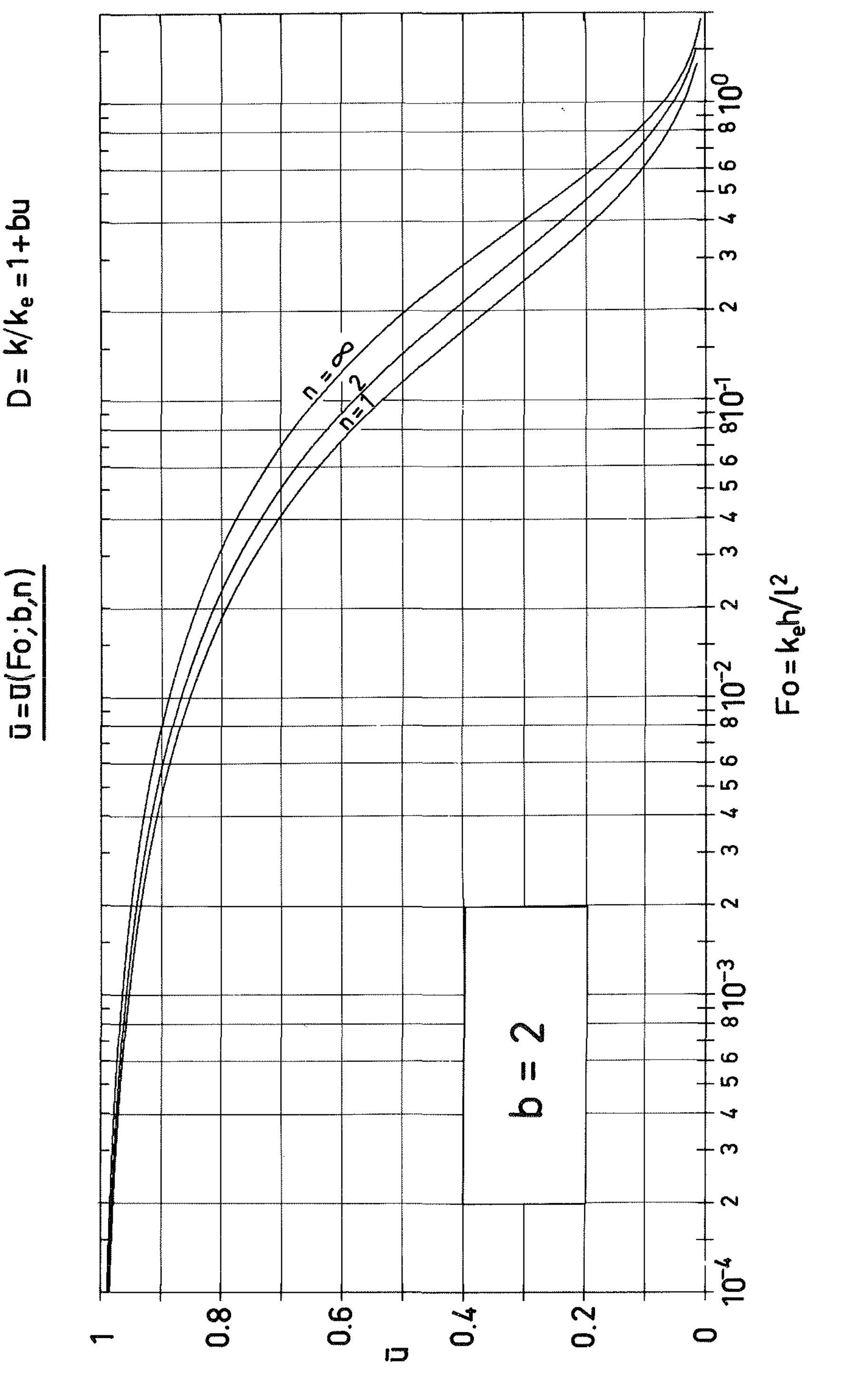


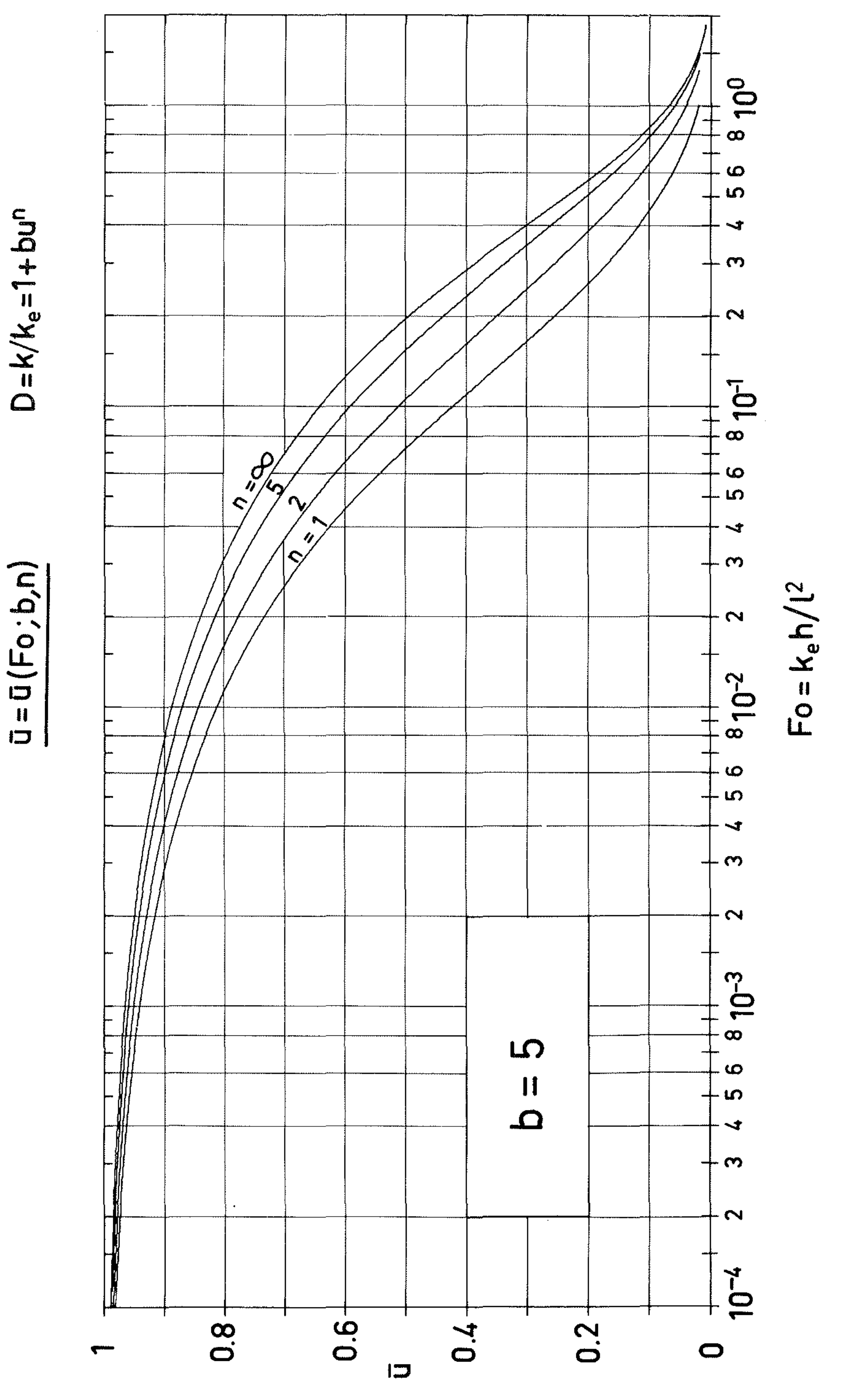


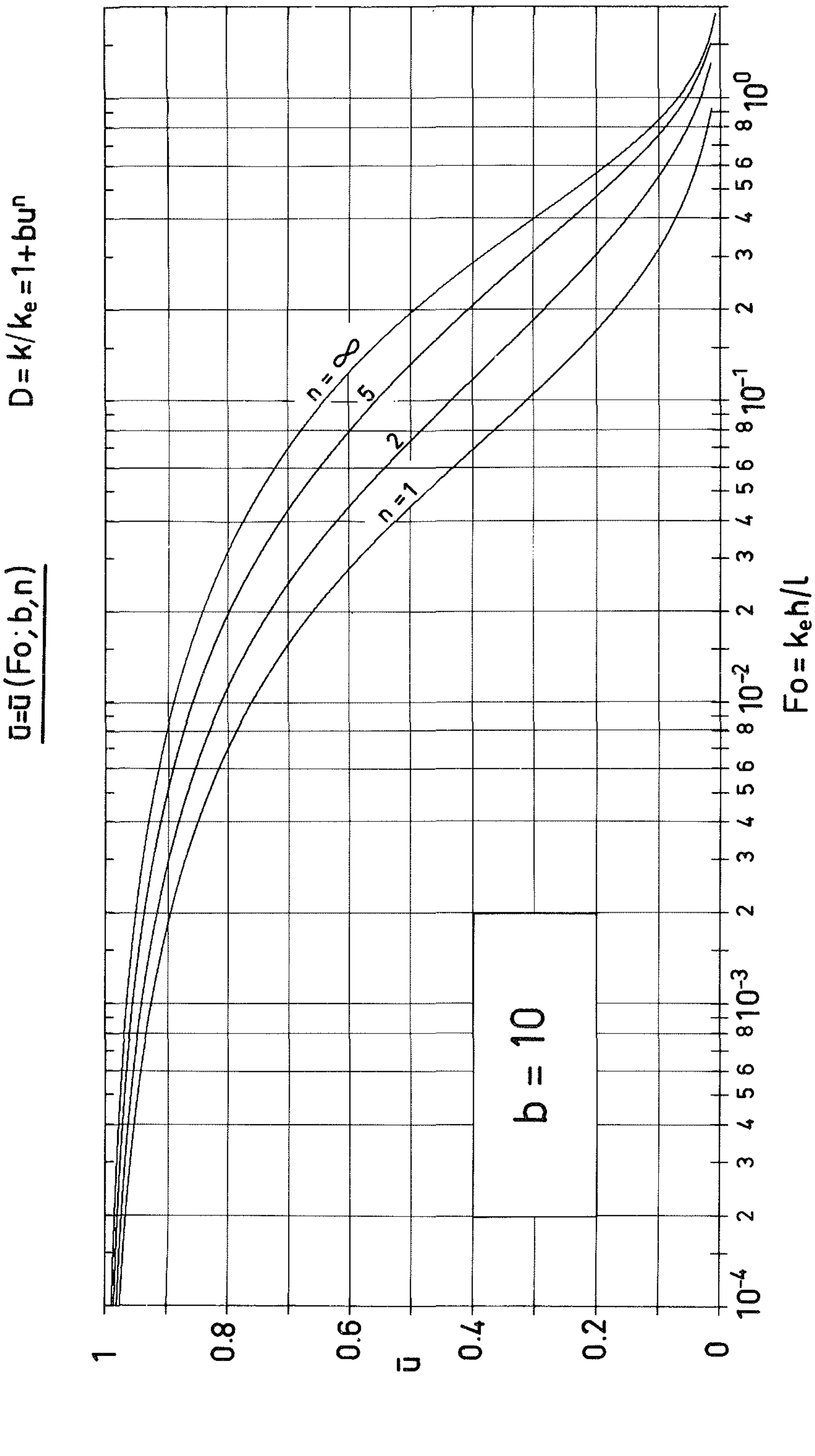
D=k/k_e=1+bu^n
ū=ū(Fo;b,n)
b=0
2
5
10
20
40
b=80
n = 1
Fo=k_eh/l²
ū
1
0.8
0.6
0.4
0.2
0
10^-4
10^-3
10^-2
10^-1
10^0

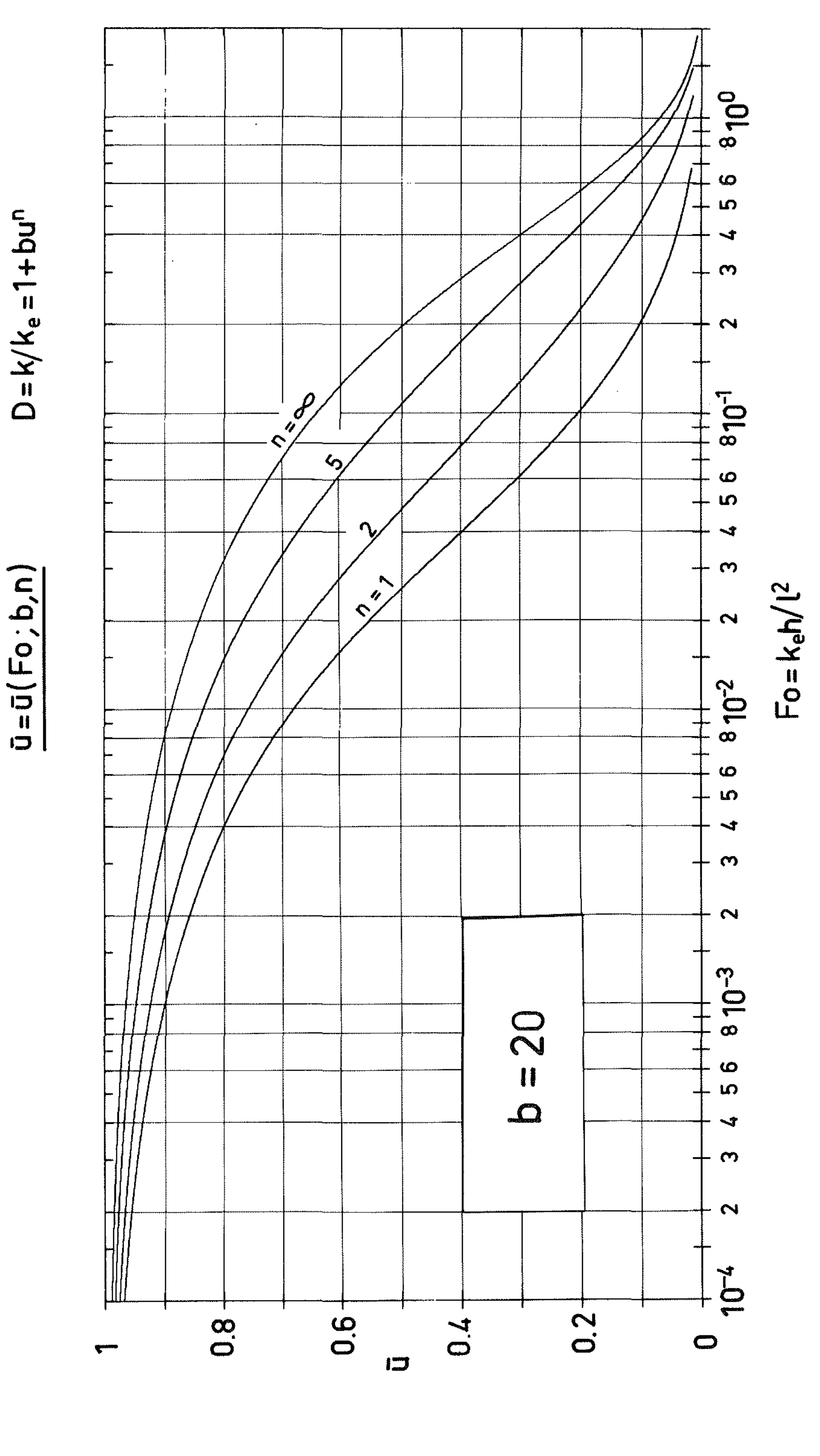

ū=ū(Fo;b,n)
D=k/kₑ=1+buⁿ
n = 2
b=0
2
5
10
20
40
b=80
Fo=kₑh/l²
ū
1
0.8
0.6
0.4
0.2
0
10⁻⁴
10⁻³
10⁻²
10⁻¹
10⁰

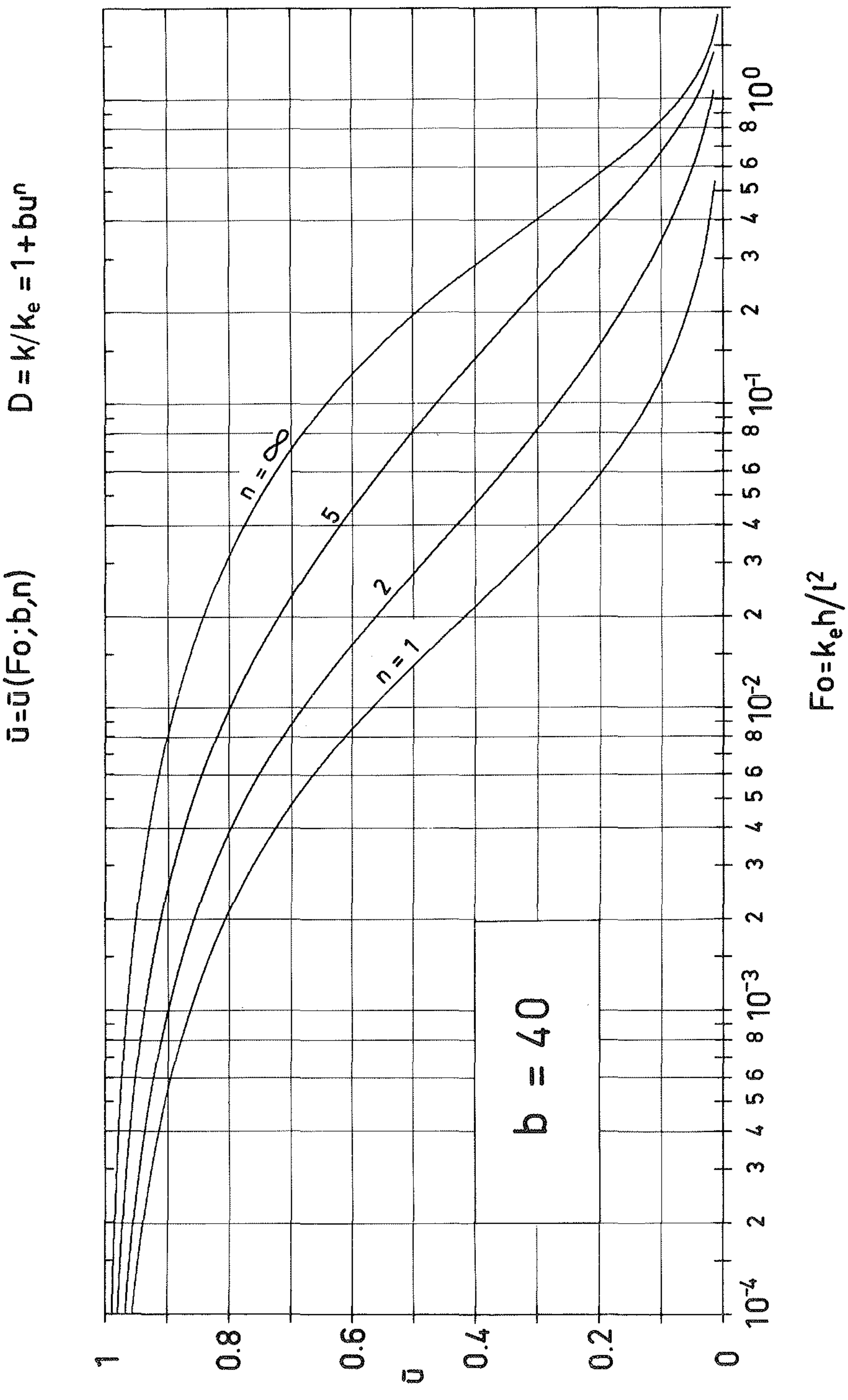

ū = ū(Fo; b,n)
D = k/k$_e$ = 1+bu^n
n = 5
b=0
5
10
20
40
b=80
Fo=k$_e$h/l^2
ū
1
0.8
0.6
0.4
0.2
0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}


ū = ū(Fo; b,n)
D = k/kₑ = 1+bu
Fo = kₑh/l²
b = 2
n = ∞
n = 2
n = 1
ū
1
0.8
0.6
0.4
0.2
0
10⁻⁴
10⁻³
10⁻²
10⁻¹
10⁰

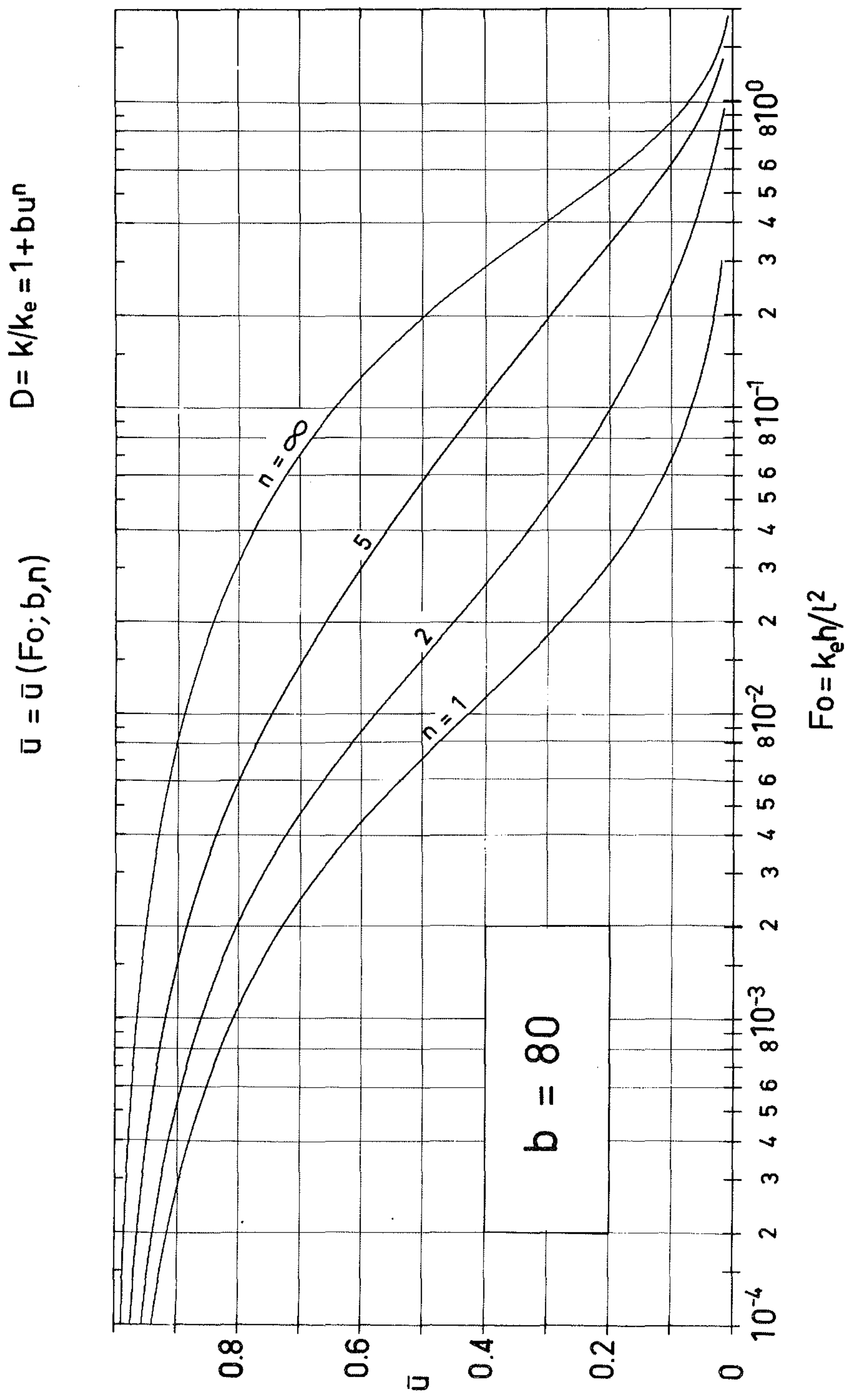

ū = ū(Fo; b,n)
D=k/kₑ=1+buⁿ
b = 5
n=∞
5
2
n=1
ū
1
0.8
0.6
0.4
0.2
0
Fo = kₑh/l²
10⁻⁴
10⁻³
10⁻²
10⁻¹
10⁰
2 3 4 5 6 8


ū=ū (Fo; b,n)
D = k/k_e = 1+bu^n
Fo = k_e h/l
b = 10
n = ∞
5
2
n = 1
ū
1
0.8
0.6
0.4
0.2
0
10^-4
2
3
4
5 6
8 10^-3
10^-2
10^-1
10^0


ū=ū(Fo; b,n)
D=k/kₑ=1+buⁿ
b = 20
n=∞
5
2
n=1
Fo=kₑh/l²
ū
1
0.8
0.6
0.4
0.2
0
10⁻⁴
10⁻³
10⁻²
10⁻¹
10⁰



ū=ū(Fo;b,n)
D = k/k_e = 1+bu^n
b = 40
n = ∞
5
2
n = 1
ū
Fo=k_e h/l^2
1
0.8
0.6
0.4
0.2
0
10^-4
10^-3
10^-2
10^-1
10^0


D= k/k_e =1+bu^n
ū = ū (Fo; b,n)
Fo= k_e h/l²
b = 80
n = ∞
5
2
n = 1
0.8
0.6
ū
0.4
0.2
0
10^-4
2
3
4 5 6
8 10^-3
2
3
4 5 6
8 10^-2
2
3
4 5 6
8 10^-1
2
3
4 5 6
8 10^0

Concentration distribution in slab

$D = k/k_e = 1 + bu^n$

TABLE 19
$b = 40$
$n = 5$

u(z,Fo)

	Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	z: .990	.975	.950	.925	.900	.875	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
K=200 f=0.25	.00120	.92993	29.29 29.19	.2865	.5403	.6818	.7512	.7968	.8305	.8568	.8957	.9432	.9695	.9843	.9924	.9966	.9986	.9994	.9996
K=200 f=2	.00120	.92992	29.63 29.20	.2865	.5402	.6817	.7511	.7968	.8305	.8568	.8957	.9432	.9695	.9843	.9924	.9966	.9986	.9994	.9997
	.00300	.88920	18.71 18.47	.1839	.4185	.5926	.6722	.7227	.7597	.7886	.8322	.8888	.9243	.9482	.9646	.9757	.9828	.9868	.9880
	.00600	.84333	13.18 13.03	.1301	.3163	.5123	.6043	.6605	.7006	.7317	.7781	.8388	.8780	.9053	.9248	.9385	.9477	.9530	.9546
	.01200	.77924	8.967 8.957	.0895	.2220	.4084	.5181	.5836	.6287	.6627	.7123	.7757	.8161	.8442	.8643	.8785	.8880	.8935	.8953

	Fo	ū	$\frac{d\bar{u}}{dFo}$ Ø	z: .990	.97	.94	.91	.88	.850	.800	.700	.600	.500	.400	.300	.200	.100	.000
K=100 f=2	.00120	.92992	28.34 29.64	.2871	.5815	.7137	.7803	.8244	.8567	.8956	.9431	.9694	.9843	.9924	.9966	.9986	.9994	.9995
	.00300	.88921	18.68 18.48	.1842	.4702	.6300	.7047	.7531	.7887	.8322	.8888	.9243	.9482	.9646	.9757	.9828	.9868	.9882
	.00600	.84331	12.69 13.03	.1301	.3693	.5559	.6406	.6934	.7316	.7781	.8388	.8780	.9053	.9248	.9385	.9477	.9530	.9549
	.01200	.77924	8.801 8.958	.0895	.2645	.4602	.5607	.6208	.6627	.7123	.7757	.8161	.8442	.8643	.8785	.8880	.8935	.8952
	.02000	.71859	6.618 6.486	.0648	.1933	.3656	.4796	.5491	.5961	.6501	.7166	.7579	.7862	.8063	.8205	.8300	.8354	.8373
	.03000	.66274	4.782 4.854	.0485	.1450	.2835	.3970	.4756	.5294	.5897	.6611	.7041	.7331	.7534	.7677	.7772	.7826	.7845
	.04000	.61941	3.893 3.891	.0389	.1164	.2298	.3323	.4134	.4727	.5395	.6167	.6619	.6918	.7126	.7271	.7367	.7422	.7440
	.06000	.55372	2.790 2.802	.0280	.0839	.1666	.2461	.3186	.3800	.4562	.5457	.5961	.6285	.6507	.6659	.6760	.6817	.6835
	.08000	.50421	2.203 2.201	.0220	.0659	.1312	.1949	.2557	.3114	.3886	.4878	.5440	.5795	.6034	.6196	.6301	.6361	.6381
	.12000	.43073	1.573 1.551	.0155	.0465	.0927	.1382	.1827	.2255	.2916	.3947	.4605	.5024	.5302	.5488	.5607	.5675	.5697
	.20000	.33290	.9841 .9823	.0098	.0294	.0588	.0879	.1166	.1448	.1902	.2726	.3389	.3879	.4222	.4456	.4607	.4692	.4719
	.30000	.25224	.6761 .6677	.0067	.0200	.0400	.0598	.0795	.0989	.1306	.1903	.2432	.2874	.3221	.3477	.3650	.3750	.3782
	.40000	.19484	.5045 .4933	.0049	.0148	.0295	.0442	.0588	.0732	.0968	.1418	.1828	.2186	.2483	.2714	.2877	.2974	.3006
	.60000	.11830	.2848 .2927	.0029	.0088	.0175	.0263	.0349	.0435	.0576	.0845	.1094	.1315	.1504	.1655	.1765	.1832	.1855
	.80000	.07217	.1734 .1782	.0018	.0053	.0107	.0160	.0212	.0265	.0350	.0515	.0666	.0802	.0917	.1010	.1078	.1120	.1134
	1.2000	.02689	.0646 .0664	.0007	.0020	.0040	.0060	.0079	.0099	.0131	.0192	.0248	.0299	.0342	.0376	.0402	.0417	.0423

Appendix I

```
comment Start of diffusion calc. when D = 1 + b u↑n;

begin real n,b,a,denom,Fo,M,difc0,dy,H,R,L; integer i,N,K; boolean A;
read b,n; denom:=1+b/(n+1); a:= n/(n+1); M:=1/(n+1);
punch(4); sameline;
difc0:= 1;
H:=80-71*exp(-b/235)-5.55*exp(-b/5.65); print punch(1),££14??,freepoint(5),b,££s3??,n,££12??,H,££1???;

print punch(2),££1q?b = ?,b,££s4?n = ?,n,££13u??;
print ££16s26?C O N C E N T R A T I O N   D I S T R I B U T I O N   I N   S L A B££15s30?D = 1 + b*u↑n,£s17??,
£b =?,freepoint(4),b,££s11?n =?,n,££15s54?u = u(z,Fo),£s7?1 > z > 0£1??,££s4?Fo?,££s5?uav?,
££s?-duav/dFo?, ££1s24??;

begin real Focr,Fp; array slink[0:80], uold[0:10];
      real procedure conctr(s); value s; real s;
      begin own real u; real du,R; switch ss:=again;
            R:=s*denom; if A then begin u:=s; A:=false end;
  again:    du:= u*M + (u*a-R)/(1+b*u↑n); u:=u-du;
            if abs(du)>₁₀-5 then goto again; conctr:= u
     end of concentration;
     real procedure difc(s); value s; real s; difc:= 1+b*conctr(s)↑n;comment diffusion coefficient;
     procedure both(s,u,d); value s; real s,u,d; begin u:=conctr(s); d:=1+b*u↑n end of conctr and difc;
     real procedure Dmax(s); array s; Dmax:=difc(s[80]);comment maximum of diff.coefficient;
     real procedure Ders(s); value s; real s;
     begin real u,d; both(s,u,d);
           Dders:= denom*n*(1-1/d)/u
     end of derivate of D = D(s);
```

Appendix II

```
comment Unchangeable part of diffusion calc;

      real procedure lagrange(x,dx,i,v); value x,dx,i; integer i; real x,dx; array v;
      begin real x0,x1,x2,y0,y1,y2,y3; x0:= i*dx; x1:= x0+dx; x2:= x1+dx;
            y0:= v[i]; y1:= v[i+1]; y2:= v[i+2]; y3:= v[i+3];
            lagrange:= y0 + (x-x0)/dx*(y1-y0 + (x-x1)/(2*dx)*(y2-2*y1+y0+(x-x2)/(3*dx)*(y3-3*y2+3*y1-y0)))
      end of procedure lagrange;

      begin integer i; penraise; plotn(1500); plotw(1200);
            plote(100); abscissae:= ordinate:= 0; penlower;
            for i:= 1 step 1 until 10 do begin line(i,0); plots(10); plotn(20); plots(10) end;
            for i:= 1 step 1 until 10 do begin line(10,i); plotw(10); plote(10) end;
            for i:= 9 step -1 until 0 do begin plots(10); plotn(10); line(i,10) end;
            for i:= 9 step -1 until 0 do begin plotw(10); plote(20); plotw(10); line(0,i) end;
            plotw(10); plote(10); plots(10); plotn(10); penraise
      end drawing of axes;

      begin comment of Boltzmann-calc.; real error; array s[0:1280]; switch st:=new;
      new:  N:=5;
            for N:=4*N while N<1281 do
            begin array S[0:N]; switch sk:=start,more,back; dy:= H/N;
                  if N=20 then
                  begin real kr; dy:=H/N; kr:=-5.5*dy/denom;
                        for i:=0 step 1 until N do S[i]:=s[i]:=1-exp(kr*i)
                  end else
                  begin for i:=N step -4 until 0 do s[i]:=s[i/4];
                        goto more; back: for i:=0 step 1 until N do S[i]:=s[i];
                  end;
                  K:=N;
       start:     begin real dI,ds,c,d,e,f,g,h,k;integer j;array I[-2:0];
                        I[-2]:=c:=s[0]:=0;
                        e:=1;k:=-2*dy*dy/3;h:=2*dy/3; A:=true;
                        for i:=4 step 4 until N do
                        begin for j:=-2,0 do
                              begin g:=(j+i-1)/difc(s[j+i-1]); d:=(j+i)/difc(s[j+i]);
                                    dI:=k*(c+4*g+d); I[j/2]:=I[j/2-1]+dI; c:=d
                              end;
                              f:=exp(I[0]); ds:=h*(e+4*exp(I[-1])+f);
                              s[i]:=s[i-4]+ds; e:=f; I[-2]:=I[0]
                        end;
                        for i:=4 step 4 until N do s[i]:=s[i]/s[N]
                  end of integration;
```

```
more: s[1]:=231*(s[4]/384-s[8]/896+s[12]/4224);
      s[2]:=15*(s[4]/16-s[8]/48+s[12]/240);
      s[3]:=135*(s[4]/128-s[8]/640+s[12]/3456);
      s[N-1]:=231*(s[N]/384+s[N-4]/384-s[N-8]/896+s[N-12]/4224);
      s[N-2]:=15*(s[N]/48+s[N-4]/16-s[N-8]/48+s[N-12]/240);
      s[N-3]:=135*(s[N]/1152+s[N-4]/128-s[N-8]/640+s[N-12]/3456);
      for i:=6 step 4 until N-6 do
      begin s[i]:=(-s[i-6]-s[i+6]+9*(s[i-2]+s[i+2]))/16;
            s[i-1]:=(-7*s[i-6]-5*s[i+6]+105*s[i-2]+35*s[i+2])/128;
            s[i+1]:=(-5*s[i-6]-7*s[i+6]+35*s[i-2]+105*s[i+2])/128
      end;
      if N>K then goto back;
      if N=320 then
      begin if 1-s[7*N/10]<_10-4 then begin H:=7*H/10; print punch(1),££12??,H,££12??; goto new end;
            if 1-s[9*N/10]>_10-4 then begin H:=5*H/4; print punch(1),££12??,H,££12??; goto new end
      end of checking range H;

      begin real err,a,c,f; err:=0;
            for i:=0 step 1 until N do
            begin a:=abs(S[i]-s[i]); S[i]:=s[i]; if a>err then err:=a
            end;
            print punch(1),err,££1??; if err>1280/N*_10-4 then goto start
      end of accuration of iteration
end of iteration;
N:= 1280;
begin real f,d;
      begin real int,der,di,ero,c,k;
            int:=2*H; k:=2*dy/3; c:=0; A:= true;
            for i:= 1 step 2 until N-1 do
            begin f:=conctr(s[i]);
                  d:=conctr(s[i+1]);
                  di:=k*(c+4*f+d);c:=d;int:=int-di
            end;
            der:= denom*(4*s[1]-3*s[2]+4*s[3]/3-s[4]/4)/dy;
            error:=int-der;
      end of checking if integral of conc. change is equal with derivate on surface;
      begin integer N1; switch sk:=forw;
            for i:=0 step 1 until N do
            begin d:=1-s[i]; if d<_10-4 then
                  begin L:=i*dy; N1:=i; goto forw end
            end;
  forw:     R:=1/(2*L); Focr:= R*R
      end of shortening of conc. range H
end;
```

```
for H:=.99,.975 step -.025 until .849, .8 step -.1 until .09, 0 do
      print freepoint(3),H,££s??; print ££12??;
begin real uav,uk,k,d,dz,a,dx,x,z,f; integer j,r,l,m,P; array u[0:880];
      P:= 880; dx:=2*R*dy; dz:= .00125; u[0]:=0; j:=2; A:= true;
      for i:=1 step 1 until P do
      begin z:=i*dz;
            for r:=j while z>j*dx do j:=j+1;
            f:= lagrange(z,dx,j-2,s);
            if i≤800 and not i div 10 * 10 < i then slink[i/10]:= f;
            u[i]:= conctr(f)
      end of change of argument;
      uav:=0; a:= dz/3;
      for i:=1 step 2 until 799 do
      begin d:=a*(u[i+1]+4*u[i]+u[i-1]); uav:=uav+d
      end of calc. of uaver;
      begin real dzFo,sqrFo; array U[0:16];
            for l:=-6 step 1 until 1 do
            begin d:=10↑l;
                  for Fo:=d,2*d,5*d do
                  begin sqrFo:=sqrt(Fo); f:=2*L*sqrFo;
                        if f>.2 and f<1 then
                        begin i:=0; Fp:= Fo; dzFo:= f*dz; k:= 1/f;
                              for z:=.01,.025 step .025 until .151,.2 step .1 until 1.01 do
                              begin i:=i+1; j:=entier(800*z*k);
                                    U[i]:= if j<99*P/100 then lagrange(z,dzFo,j-1,u) else 1
                              end of interpolation of result;
                              dx:= 10*dzFo; f:= 99/100*P*dx;
                              curve(0,10,.01, if a<f then 10*u[a/dx] else 10, a);
                              if Fo<10-5 then print freepoint(6),Fo else print freepoint(5),Fo,££s??;
                              print freepoint(5),1-(1-uav)/k,££s??,freepoint(4),(1-uav)/sqrFo*L,££s??;
                              for i:=1 step 1 until 16 do print freepoint(4),U[i];
                              print ££l??
                        end
                  end
            end;
            uold[0]:= 0;
            for i:= 1 step 1 until 10 do
            begin z:= i/10; j:= entier(800*z*k); uold[i]:= if j<99*P/100 then lagrange(z,dzFo,j-1,u) else 1
            end
       end of listing;
      print £Derivative quantity on surface - integral of conc. change = ?,freepoint(6),error,££l??;
      print punch(2), scaled(4),Focr, freepoint(4),uav
  end
end of Boltzmann-calc.;
```

```
begin comment of netanalysis; real ai,f,dz,r,e,g,h,um,uma,uav,t,dF,dFO,dFp,Difaam; integer l,m;
      array s[0:81],Dif[0:80]; switch ss:= calc,time,new,list;
      s[0]:= 0; for i:= 1 step 1 until 80 do s[i]:= slink[i];
      Fo:= Focr; dz:= .0125; A:=true;
      for i:=1 step 1 until 80 do Dif[i]:= difc(s[i]);
new:  Difaam:=0; t:= Dmax(s); A:= true;
      for i:=1 step 2 until 79 do
      begin ai:= Ddens(s[i])*(s[i+1]+s[i-1]-2*s[i]);
            f:= if ai>0 then Dif[i]*ai else -t*ai;
            if f>Difaam then Difaam:= f
      end for stability consideration;
      if Difaam<₁₀-5 then begin dFp:=Fp/7; goto calc end;
      h:= 2*dz*dz/sqrt(Difaam);
      dFp:= if h<Fo/7 then Fp/Fo*h else Fp/7; comment prelimenary Fo-step selected;

calc: begin array b[0:80],c[0:80];
            dFO:= Fo*dFp/Fp; if dFO>.025 then dFO:=.025; Fo:= Fo+dFO; r:= dFO/(dz*dz); c[0]:=b[0]:=0; A:=true;
            for i:=1 step 1 until 79 do
            begin h:=r*Dif[i]; e:=s[i]; g:=s[i-1]+s[i+1]; t:=(1-h)*e+h*g/2;
                  h:=2/(r*difc(t));
                  c[i]:=t:=1/(-2-h-c[i-1]);
                  b[i]:=t*(-g+(2-h)*e-b[i-1])
            end auxliary quantities c and b got;
            t:=r*Dif[80]; h:=1/(r*difc((1-t)*s[80]+t*s[79]));
            s[80]:=t:=(-s[79]+(1-h)*s[80]-b[79])/(-1-h-c[79]);
            for i:= 79 step -1 until 1 do s[i]:=t:=b[i]-c[i]*t;
            s[81]:= s[79]
      end of direct solution to following row;

      uma:= um; um:= 0; e:= dz/3; g:=0; A:=true;
      begin array conc[0:10]; for l:=0 step 1 until 9 do
            begin for i:= 8*l+1 step 2 until 8*l+7 do
                  begin t:=g; both(s[i],h,Dif[i]); both(s[i+1],g,Dif[i+1]);
                        f:= e*(g+t+4*h); um:= um + f
                  end;
                  conc[l+1]:= g
            end averaged concentration, diff.coeff. and test.q. conc for concentr.change calculated;
            print punch(2), scaled(4), Fo, freepoint(4),um;
            if um<.02 then begin print punch(2), £*?; stop end;
            for i:=1 step 1 until 10 do if abs(uold[i]-conc[i])>.06 then goto time
      end if conc.change somewhere is large enough for listing;
      goto calc;
```

```
time: dF:=dF0; for l:= -5 step 1 until 1 do
      begin t:=10↑l;
            for g:= .1, .12, .15 step .05 until .31, .4 step .1 until .91 do
            begin f:=g*t; h:= f - Fo;
                  if abs(h)<abs(dF) then dF:=h;
                  if abs(h)>abs(dF) and abs(dF)< 5*dF0/9 then
                  begin Fp:= Fo+dF; goto list end
      end
      end ready for listing if Fo is most near by any selected listing value;
      goto calc:

list: begin array sfolc[0:80],result[0:80];
            A:= true;
            for i:= 1 step 1 until 80 do
            begin real hfolc;
                  hfolc:= dF*Dif[i]/(dz*dz);
                  sfolc[i]:= (1-hfolc-hfolc)*s[i]+hfolc*(s[i-1]+s[i+1]);
                  result[i]:= conctr(sfolc[i])
            end concentration values calculated;
            uav:= 0; e:= dz/3; g:= 0;
            for l:= 0 step 1 until 9 do
            begin for i:= 8*l+1 step 2 until 8*l+7 do
                  begin t:=g; h:= result[i]; g:= result[i+1];
                        f:=e*(t+4*h+g); uav:=uav+f
                  end;
                  uold[l+1]:=g
            end of mean value of concentration;
            curve(0,10,.01, if t<.25 then 10*(lagrange(t/10,dz,0,result) + t*(t-.125)*(t-.25)*(t-.375)*
                  (result[4]-4*result[3]+6*result[2]-4*result[1])/.0058594) else 10*lagrange(t/10,dz,if t<9.8
                  then 8*t-1.5 else 77,result),t); comment conc.curve drawn;
            print freepoint(5),Fp,££s??,uav,££s??;
            if abs(dF/Fp)<_10-5 then begin uav:= uma; dF:= -dF0 end;
            print freepoint(4),(um-uav)/dF,££s??;
            freepoint(4);
            print 1.056*result[1] - .176*result[2] + .032*result[3];
            for i:= 2 step 2 until 12, 16 step  8 until 80 do
                  print result[i]:
            print ££ls16??, denom*(4*sfolc[1] - 3*sfolc[2] + 4/3*sfolc[3] - .25*sfolc[4])/dz,££l??;
            comment last printed flux on surface of slab;
            goto new
      end of listing
   end
end end end end end end end end end end;
```

Appendix III

```
comment curve of averages ;

begin comment axes; real x; integer i;
      penraise; plote(1500); plots(1200); plotn(200); penlower; abscissae:= -1381.56; ordinate:= 0;
      for i:= -4 step 1 until -1 do
      begin for x:= 1.5, 2 step 1 until 10.1 do
            begin line(1.5*(ln(x)+i*2.3026), 0); plots(10); plotn(20); plots(10) end
      end;
      for x:= 1.5, 2 do begin line(1.5*ln(x),0); plots(10); plotn(20); plots(10) end;
      for x:= .7 step .7 until 7.01 do
      begin line(1.5*ln(2),x); plotw(10); plote(10) end;
for x:= 1.5, 1 do begin line(1.5*ln(x),7); plots(10); plotn(10) end;
      for i:= -1 step -1 until -4 do
      begin for x:= 9 step -1 until 1.99, 1.5, 1 do
            begin line(1.5*(ln(x)+i*2.3026),7); plots(10); plotn(10) end
      end;
      for x:= 6.3 step -.7 until -.1 do
      begin plote(10); plotw(20); plote(10); line(-13.8156,x) end;
      plotw(10); plote(10); plots(10); plotn(10)
end of axes;

begin real Focr,sqrFoc,lnFocr,uavcr,lnFoM,t; integer i,M; array Fo[0:1000],uav[0:1000];
      boolean A; switch st:= start;
      real procedure parabola(Y,X,i,x);      array Y,X; integer i; real x;
      begin real x1,x2,x3,y1,y2,y3;
            x1:=X[i]; y1:=Y[i];
            x2:=X[i+1];y2:=Y[i+1];
            x3:=X[i+2];y3:=Y[i+2];
            parabola:=y1+(x-x1)/(x3-x2)*((y3-y1)/(x3-x1)*(x-x2)-(y2-y1)/(x2-x1)*(x-x3))
      end of parabola;

      real procedure avconc(x); value x; real x;
      begin own integer j;
            if A then begin j:= 0; A:= false end
            else if j<M-2 then
                  begin for i:= j+1 while x>(Fo[i]+Fo[i+1])/2 and j<M-2 do j:= j+1 end;
            avconc:= parabola(uav,Fo,j,x)
      end of avconc;
```

```
start: i:= 0; read instring(Fo,i),Focr,uavcr; lnFocr:= 1.5*ln(Focr); sqrFoc:= sqrt(Focr);
       Fo[0]:= Focr; uav[0]:= uavcr;
       i:= 0;
       for i:= i+1 while not buffer(1,£*?) do read Fo[i],uav[i];
       M:= i-1; lnFoM:= 1.6*ln(Fo[M]);
       A:= true;
       curve(-13.8156, lnFoM, .01, if t<lnFocr then 7*(1-(1-uavcr)*sqrt(exp(.666667*t))/sqrFoc) else 7*avconc(exp(.666667*t)), t);
       wait; goto start
end of curve of averages
 end end end end end end
```

Appendix IV

```
comment Start of diffusion-calc. when D(u) discrete;

begin integer i,N,K,M; read M;
begin real duu,f,denom,Fo,dy,H,R,L,difc0; array D[0:M],ss[0:M]; boolean A;
       for i:= 0 step 1 until M do read D[i]; duu:= 1/M; ss[0]:= 0;
       for i:= 1 step 1 until M do if 2*(idiv2) = i or i = M
              then ss[i]:= ss[i-1] + duu*(-D[i-2] + 8*D[i-1] + 5*D[i])/12
              else ss[i]:= ss[i-1] + duu*(5*D[i-1] + 8*D[i] - D[i+1])/12;
       denom:= ss[M];
       difc0:= D[0];
       for i:= 1 step 1 until M do ss[i]:= ss[i]/denom;
       for i:= 2 step 1 until M-1 do if ss[i+1]-ss[i] > 2*(ss[i]-ss[i-1]) and ss[i-1]-ss[i-2] > 2*(ss[i]-ss[i-1])
              then begin print £In data concentration-division must be made denser, that is M larger£15??; stop end;
       punch(4); sameline;
       f:= 0; for i:= 0 step 1 until M do if D[i]>f then f:= D[i];
       H:= 80 - 70*exp(-f/235) - 5.5*exp(-f/5.6); print H,££12??;
```

```
begin real Focr,Fp; array slink[0:80], uold[0:10];
      real procedure conctr(s); value s; real s;
      begin own integer j; real s0,s1,s2,s3,t,t0,dt; integer i;
            if s>ss[M] then s:= ss[M]-.00001;
            if A then begin j:= 0; A:= false end;
            i:= j; for i:= i+1 while s>ss[i] do j:= i-1;
            s0:= ss[j]; s1:= ss[j+1]; s2:= ss[j+2]; s3:= if j<M-2 then ss[j+3] else 10; t:= 0;
            if s1-s0<s3-s2 then
            begin t0:= dt:= (s-s1)/(s2-s1);
                  for t:= t+dt while abs(duu*dt)>_10-5 do dt:= -t + t0 + t*(1-t)/2*(s2-2*s1+s0)/(s2-s1)
            end else
            begin t0:= dt:= (s-s2)/(s2-s1);
                  for t:= t+dt while abs(duu*dt)>_10-5 do dt:= -t + t0 - t*(1+t)/2*(s3-2*s2+s1)/(s2-s1)
            end;
            conctr:= (j+1+t)*duu
      end concentration;
      procedure both(s,u,d); real s,u,d;
      begin real d0,d1,d2,u0,u1,u2; integer j;
            u:= conctr(s); j:= entier(u/duu);
            if u/duu-j>.5 then j:= j+1; if j>M-1 then j:= M-1;
            d0:= D[j-1]; d1:= D[j]; d2:= D[j+1]; u0:= (j-1)*duu; u1:= u0+duu; u2:= u1+duu;
            d:= d0 + (u-u0)/duu*((d2-d0)/(2*duu)*(u-u1)-(d1-d0)/duu*(u-u2))
      end both conotr and difc;
      real procedure difc(s); real s;
      begin real uf,ddd; both(s,uf,ddd); difc:= ddd
      end diffusion coefficient;
      real procedure Dmax(s); array s;
      begin integer i,j; real smax,dmax; smax:= 0;
            for i:= 0 step 1 until 80 do if s[i]>smax then smax:= s[i];
            i:= j:= 0; if smax>ss[M] then smax:= ss[M]-.00001;
            for i:= i+1 while smax>ss[i] do j:= i; dmax:= 0;
      for i:= 0 step 1 until j do if D[i]>dmax then dmax:= D[i]; Dmax:= dmax
      end Diff.coeff.maximum;
      real procedure Dders(s); value s; real s;
      begin own integer j; integer i;
            if A then begin j:= 0; A:= false end;
            for i:= j while s>ss[i] do j:= j+1;
            Dders:= (D[j]-D[j-1])/(ss[j]-ss[j-1])
      end derivative of D to s;
```

VALTION TEKNILLINEN TUTKIMUSLAITOS
THE STATE INSTITUTE FOR TECHNICAL RESEARCH

JULKAISUJA - PUBLICATIONS

139. Visapää, Asko, Tape preparation programs for Mathatron digital computer systems. Part III. X-ray fluorescence calculations. 171 p. 1969.

140. Hyttinen, Rainer - Falcke, Caj, Asuinkerrostalojen käyttökustannukset. Tutkimus kaukolämmitettyjen arava-asunto-osakeyhtiökerrostalojen käyttökustannuksista Helsingissä v. 1960 ··· 1964. English summary. Running costs of dwelling houses. Investigation of the running costs of Arava Housing Company long-distance heated multi-story houses in Helsinki 1960 ··· 1964. 90 s. 1969.

141. Hyyppä, J.M.I., Survey of the skid resistance properties of asphalt surfacings. Tutkimuksia asfalttipäällysteiden kitkaominaisuuksista. 56 p. 1969.

142. Visapää, Asko, Tape preparation programs for Mathatron digital computer systems. Part IV. Reaction kinetics calculations. 125 p. 1969.

143. Änkö, Olavi, Maan hintataso Helsingin seudulla v. 1967. English summary. The level of site prices in the region of Helsinki in 1967. 76 s. 1969.

144. Heiskanen, Ossi, Viljelmän sisäisen ja ulkoisen liikenneaseman vaikutuksesta maatalousliikenteen kustannuksiin. Zusammenfassung. Über den Einfluss der inneren und äusseren Verkehrslage eines Landwirtschaftsbetriebes auf die Transport- und Wegekosten. 185 s. 1969.

145. Vuorinen, Jukka, On the behaviour of hardened concrete during freezing. Experimental studies on the application of the one-cycle freezing tests in combination with porosity determinations for the evaluation of the freezing-and-thawing resistance of hardened concrete. 148 p. 1969.

146. Rechardt, T., Effect of reinforcement on the bending moment distribution in fixed-end reinforced concrete beam. 63 p. 1969.

147. Heleskivi, J. - Salo, T. - Stubb, T., The conduction mechanism in monocrystalline selenium. 16 p. 1969.

148. Graeffe, Ralf, Explanation of the properties of the thin-film transistor. 38 p. 1969.

149. Räsänen, Reino, Toimistohuoneistojen vuokrat Helsingissä 1968 ··· 1969. English summary: Rents of office premises in Helsinki in 1968 ··· 1969. 43 s. 1969.

150. Rechardt, T., On the deformations and stiffness properties of reinforced concrete members in flexure under working conditions. 15 p. 1969.

151. Suokko, Seppo, Marknadspriser på nya radhuslägenheter i Helsingfors, Esbo och Grankulla 1967. English summary: Price of new terrace houses in Helsinki, Espoo, and Kauniainen in 1967. 44 s. 1969.