VALTION TEKNILLINEN TUTKIMUSLAITOS
THE STATE INSTITUTE FOR TECHNICAL RESEARCH, FINLAND

TIEDOTUS. SARJA III — RAKENNUS. 87

NUMERICAL SOLUTION OF DIFFUSION EQUATION WITH DIFFUSIVITY CONCENTRATION DEPENDENT

Nomograms for the drying of slabs with moisture conductivity linearly moisture dependent, and surface at constant moisture

S. E. PIHLAJAVAARA
M. VÄISÄNEN

HELSINKI 1965

VALTION TEKNILLINEN TUTKIMUSLAITOS

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FOREWORD

Since 1961, one of the present authors, S.E. Pihlajavaara, has been engaged in a research project termed the "Drying of Concrete" (1). The diffusion theory has formed the mathematical basis for the theoretical treatment of the drying of concrete. The diffusion-type equation with diffusivity concentration dependent has been of special interest, and this report presents the numerical solutions of the diffusion equation with diffusivity linearly concentration dependent. The main aim was to find nomograms for the drying of a concrete slab with the moisture conductivity (diffusivity) linearly moisture dependent, and a surface at constant moisture. The task of finding the required solutions was entrusted to the other author Mr. M. Väisänen, M. Sc., who calculated the numerical examples with the aid of an electronic computer, Elliot 803, using the difference method.

The final linquistic revision was made by Mrs. Lorna Sundström, M. Sc. Nomograms were drawn by Irja Tennberg.

It is a pleasure to make acknowledgement to Prof. Arvo Nykänen, Director of this laboratory, whose support made this report possible.

STATE INSTITUTE FOR TECHNICAL RESEARCH

Laboratory of Concrete Technology
Physics Section

S.E. Pihlajavaara

CONTENTS

1 PHYSICS AND MATHEMATICAL FORMULATION

It is a widely accepted fact that fields involving molecular transfer of heat and mass can be analytically described with the aid of the so-called diffusion equation

$$\partial u/\partial h = \mathrm{div}(k \,\mathrm{grad}\, u)$$

where u is concentration, h time and k diffusivity. In the mathematical treatment of the drying of a porous solid the diffusion equation has successfully been the governing equation, although the drying of a porous solid also involves molar or macroscopic fluid transfer. However, these macroscopic fluid movements are of random character, thus being phenomenologically similar to the diffusion process, which term in physical science, strictly speaking, is usually applied to random molecular motion.

Most earlier works on the drying of concrete, which is a very fine-porous material, have shown that the diffusion type equation "sufficiently" depicts the phenomenon. Although there is no reason to believe that the moisture conductivity or diffusivity k is constant, even the usage of a constant moisture conductivity, reasonably chosen and used, has given "sufficient" solutions when applied. However, experimental studies on concrete clearly indicate that the assumption of a constant k may lead to completely erroneous results. The improvement in the adoption of the diffusion theory seems to be the assumption that k is linearly dependent on the moisture content u.

In spite of the fact that the problem is very old and has been treated in Crank's well-known book [2], there seemed to be not enough solutions for our simple case:

The problem consists of finding, during drying, moisture contents u in a wide slab or wall, and average moisture contents $\bar{u}$ of a wide

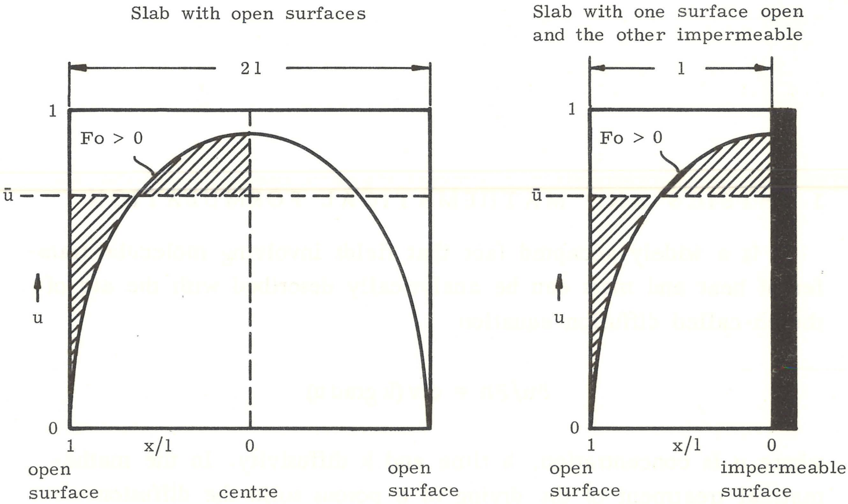


Figure 1. Showing the meaning of the symbols and giving examples of distribution of moisture content u and corresponding average moisture content ū in an infinite slab or wall with finite thickness (2 l or l), at a fixed time or Fourier number $Fo = k_e h/l^2$.

(infinite) slab or wall, whose two surfaces (or one surface) are kept at constant moisture (or the other surface is impermeable), and whose initial moisture content in the interior is uniform and unity; the moisture conductivity k depends linearly on moisture content u; gravity is neglected. (Fig. 1).

Mathematically, the problem consists first of finding solutions of the dimensionless partial differential equation

$$\partial u/\partial Fo = (\partial/\partial z)[(1 + bu)\partial u/\partial z] \quad (1)$$

under boundary conditions

$$u = 0, \text{ for } z = 1, \ Fo > 0$$

$$\partial u/\partial z = 0, \text{ for } z = 0, \ Fo > 0$$

and with the initial condition

$$u = 1, \text{ for } Fo = 0, \ 0 \leq z < 1$$

when

$$k = k_e(1 + bu)$$

In the above equations (see Fig. 1)

$$u = \text{variable relative moisture content} = \frac{C - C_0}{C_0 - C_e}$$

$$z = x/l$$

$$Fo = k_e h/l^2 \text{ (Fourier number)}$$

$$b = \text{parameter}$$

$$C = \text{variable moisture content (kg/m}^3\text{)}$$

$$C_0 = \text{initial moisture content (kg/m}^3\text{)}$$

$$C_e = \text{equilibrium moisture content (kg/m}^3\text{)}$$

The b-range selected was 0 ··· 100.

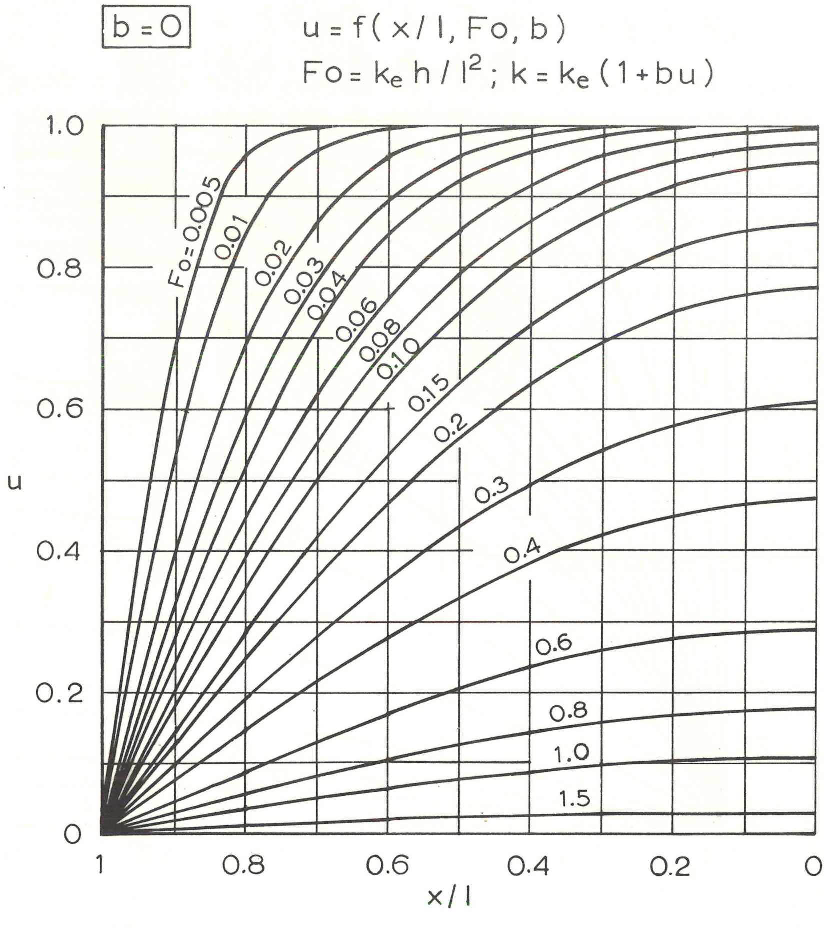
When the solutions u have been found, the problem then consists of calculating the integral

$$\bar{u} = \int_0^1 u dz$$

The basis of the drying problem presented in this report is more deeply treated, as regards concrete, in the Monograph (1). Further, the problem is also examined from a practical point of view, especially concerning the estimation of C_0 and C_e values, in paper (3).

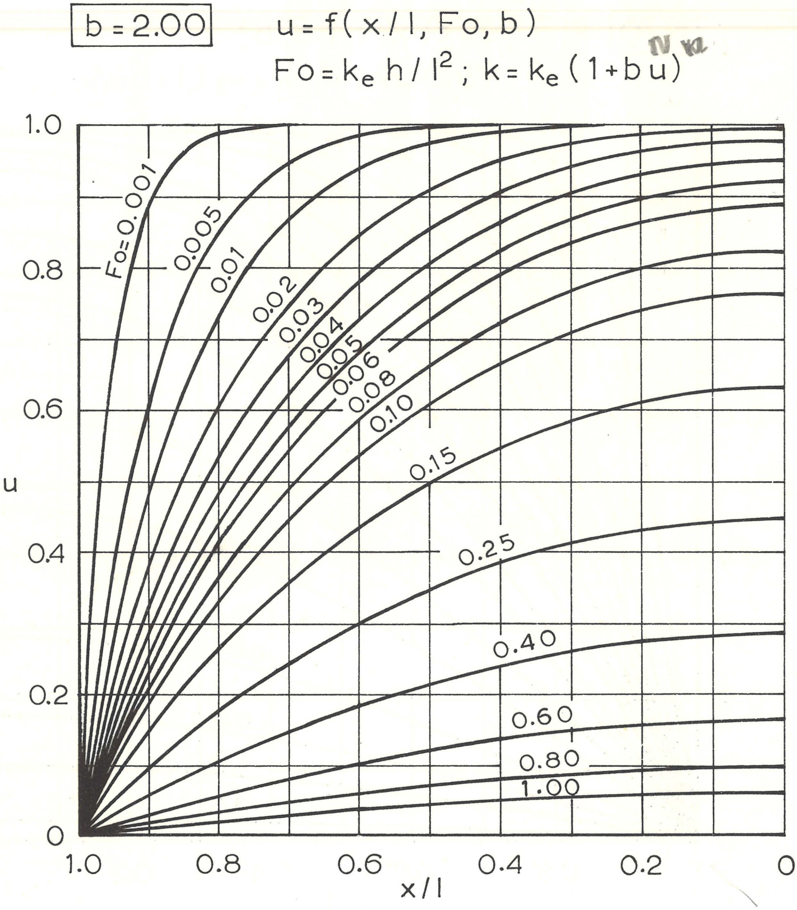
2 NOMOGRAMS

The solutions of Equations (1) and (2) with the specified boundary and initial conditions were calculated by the method of finite differences (4) using an electronic computer. The task led to a special method which will be published at a future date. The numerical accuracy of the curves in the figures or nomograms 1 ··· 11 are sufficient enough for practical work, and the accuracy of u and $\bar{u}$ in the numerical tables, on the basis of which the nomograms have been drawn, is correct to at least three decimal places.

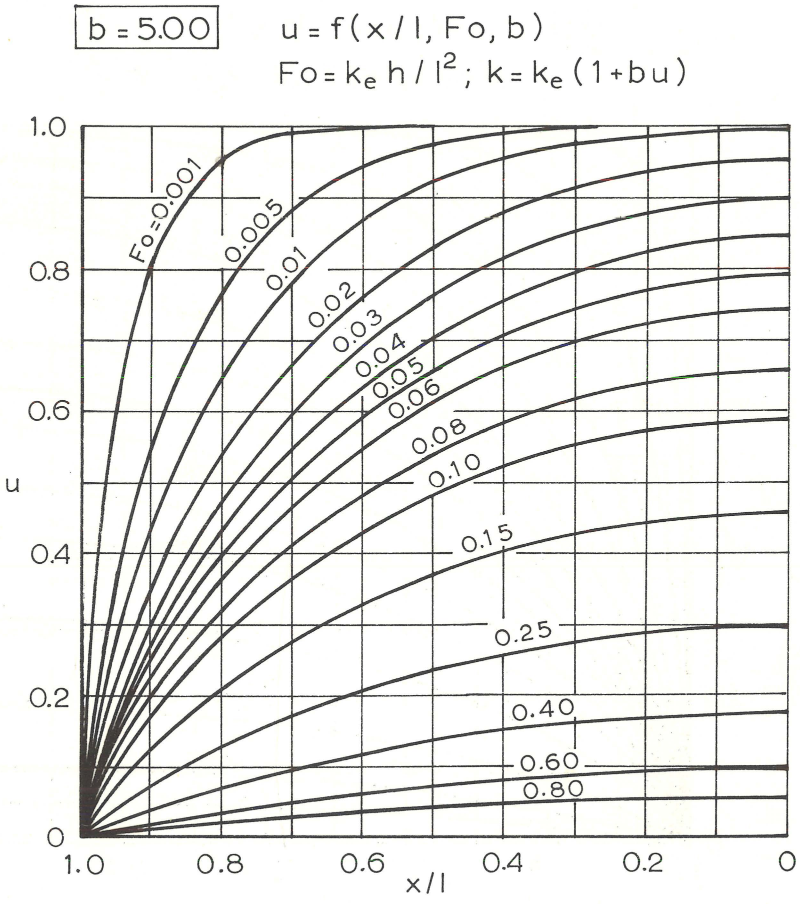


b = 0
u = f(x / l, Fo, b)
Fo = k_e h / l^2 ; k = k_e (1 + bu)
1.0
0.8
0.6
u
0.4
0.2
0
1
0.8
0.6
0.4
0.2
0
x / l
Fo=0.005
0.01
0.02
0.03
0.04
0.06
0.08
0.10
0.15
0.2
0.3
0.4
0.6
0.8
1.0
1.5

Nomogram 1.

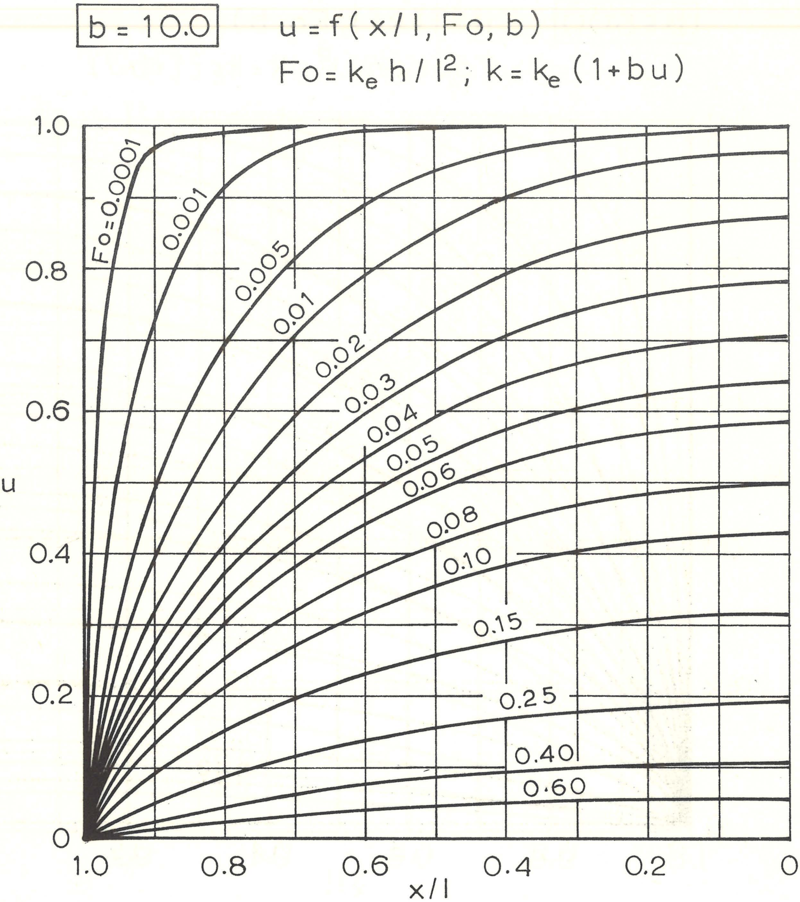

b = 2.00
u = f(x/l, Fo, b)
Fo = k_e h/l²; k = k_e (1+bu)
Fo=0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
0.40
0.60
0.80
1.00
u
1.0
0.8
0.6
0.4
0.2
0
x/l
1.0
0.8
0.6
0.4
0.2
0

Nomogram 2.



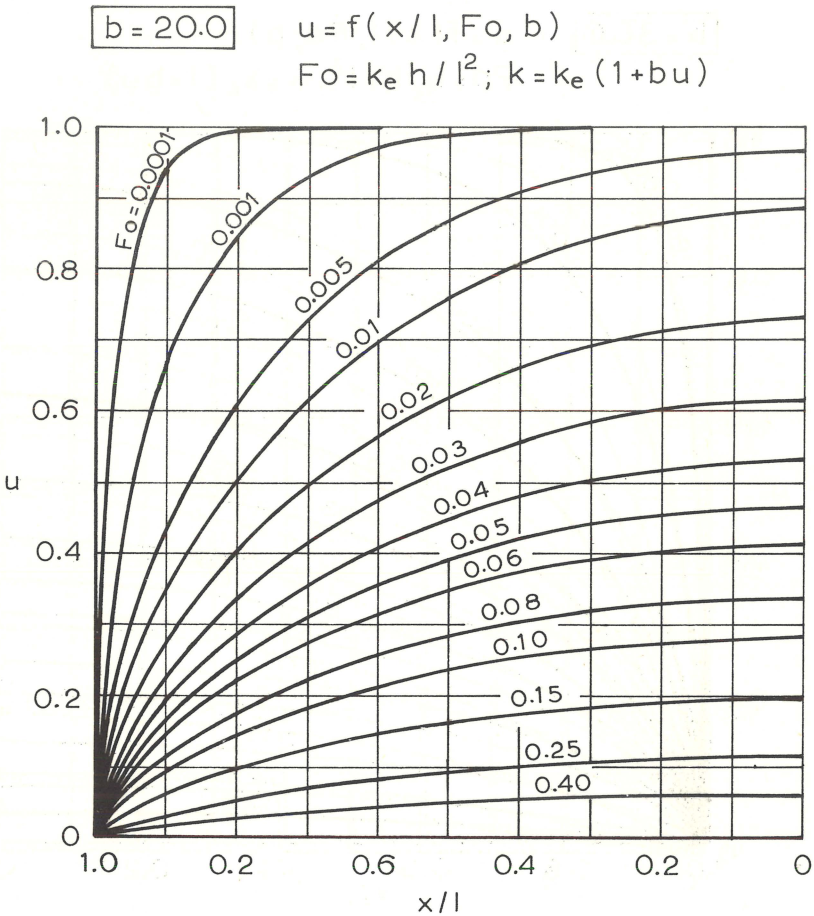
b = 5.00
u = f(x / l, Fo, b)
Fo = k_e h / l^2 ; k = k_e (1 + bu)
1.0
0.8
0.6
u
0.4
0.2
0
1.0
0.8
0.6
0.4
0.2
0
x / l
Fo = 0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
0.40
0.60
0.80

Nomogram 3.

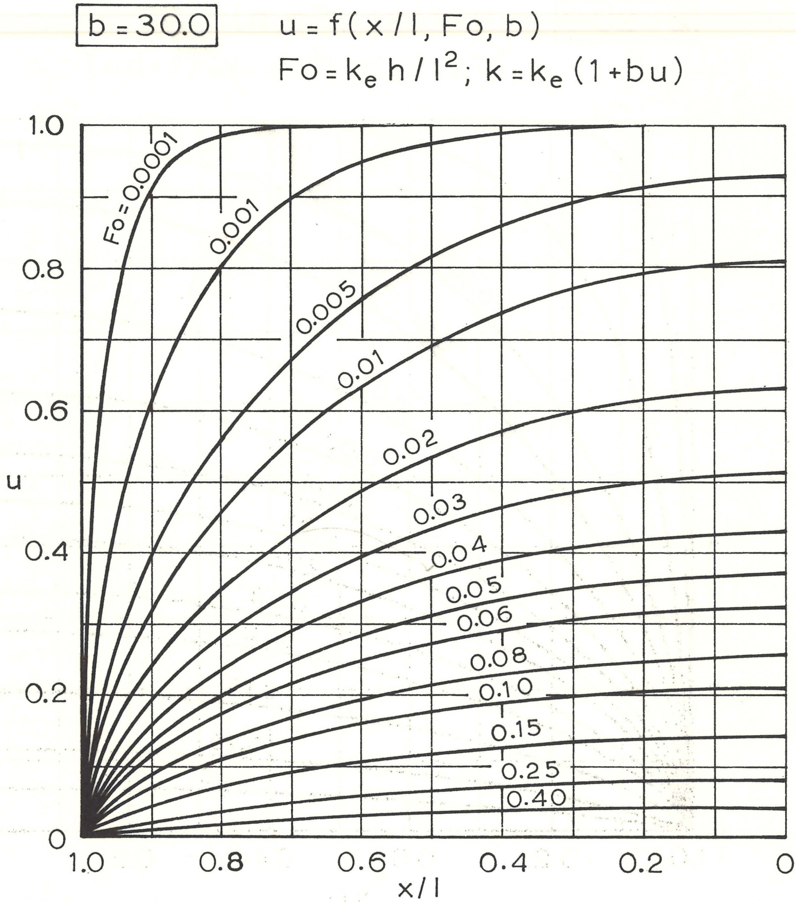


b = 10.0
u = f(x / l, Fo, b)
Fo = k_e h / l²; k = k_e (1 + b u)
1.0
0.8
0.6
0.4
0.2
0
u
Fo=0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
0.40
0.60
1.0
0.8
0.6
0.4
0.2
0
x / l

Nomogram 4.

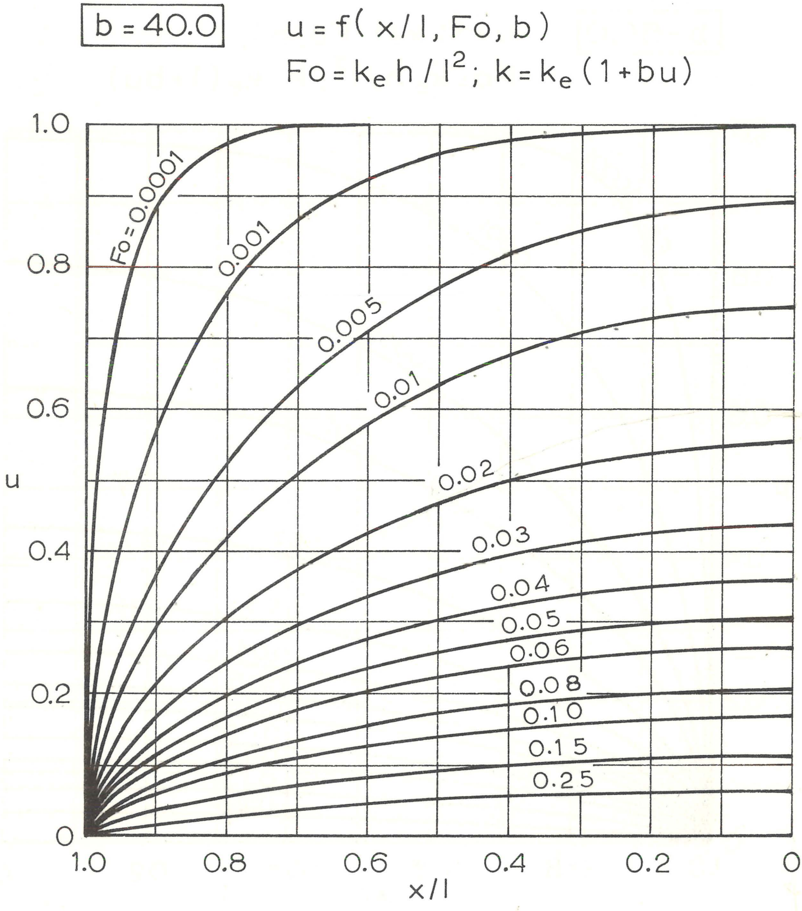

b = 20.0
u = f(x / l, Fo, b)
Fo = k_e h / l^2; k = k_e (1 + b u)
1.0
0.8
0.6
u
0.4
0.2
0
1.0
0.2
0.6
0.4
0.2
0
x / l
Fo=0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
0.40

Nomogram 5.

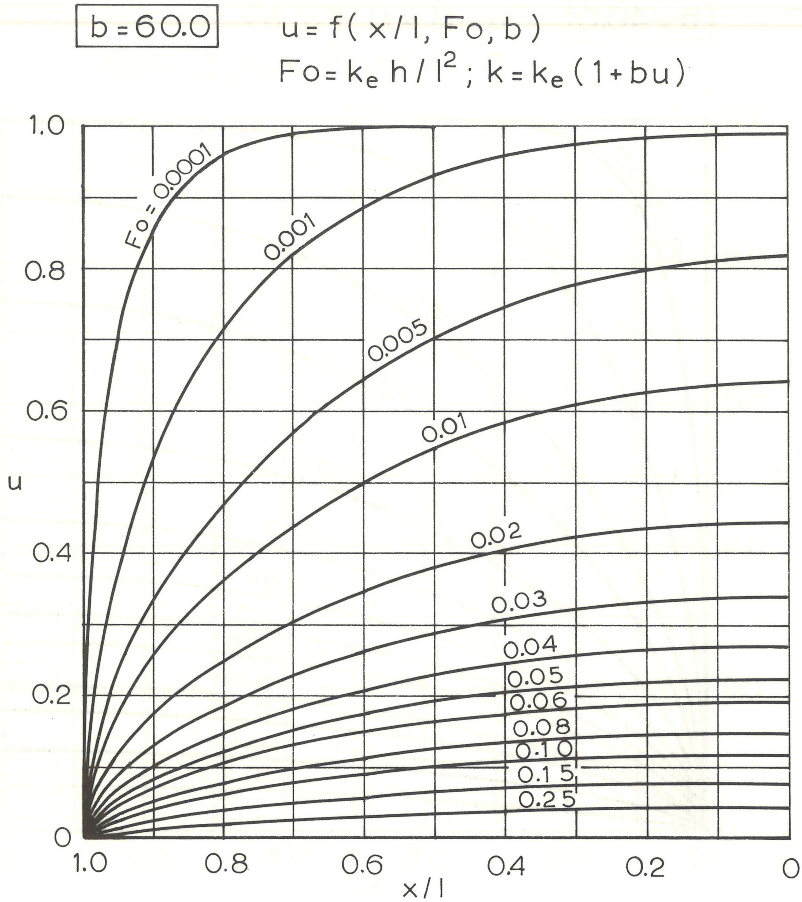


b = 30.0
u = f(x/l, Fo, b)
Fo = k_e h/l²; k = k_e (1+bu)
1.0
0.8
0.6
u
0.4
0.2
0
Fo=0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
0.40
1.0
0.8
0.6
0.4
0.2
0
x/l

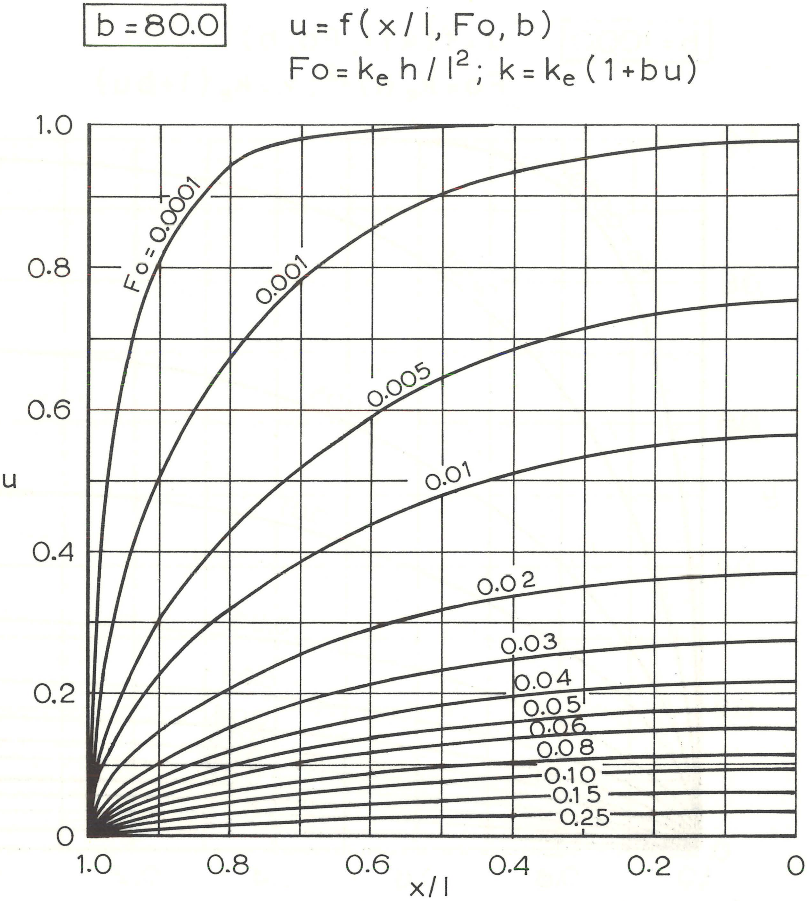
Nomogram 6.


b = 40.0
u = f(x / l, Fo, b)
Fo = k_e h / l^2; k = k_e (1 + bu)
1.0
0.8
0.6
u
0.4
0.2
0
Fo = 0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
1.0
0.8
0.6
0.4
0.2
0
x / l

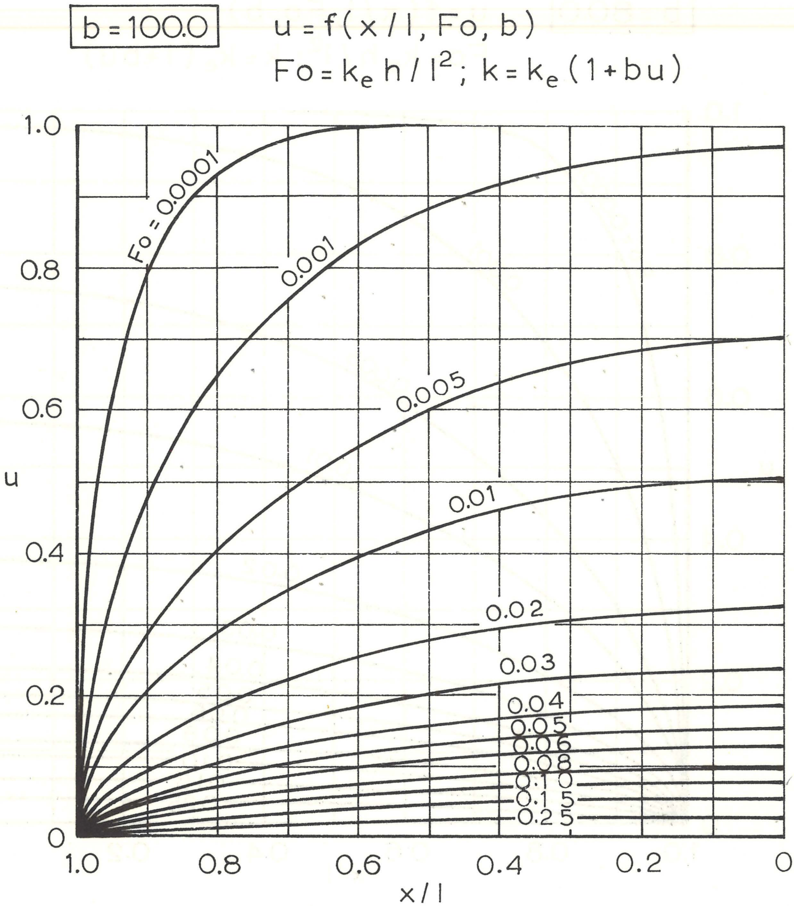
Nomogram 7.


b = 60.0
u = f(x / l, Fo, b)
Fo = k_e h / l^2 ; k = k_e (1 + bu)
Fo = 0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
1.0
0.8
0.6
0.4
0.2
0
u
x / l

Nomogram 8.


b = 80.0
u = f (x / l, Fo, b)
Fo = k_e h / l^2 ; k = k_e (1 + b u)
1.0
0.8
0.6
u
0.4
0.2
0
Fo = 0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
1.0
0.8
0.6
0.4
0.2
0
x / l

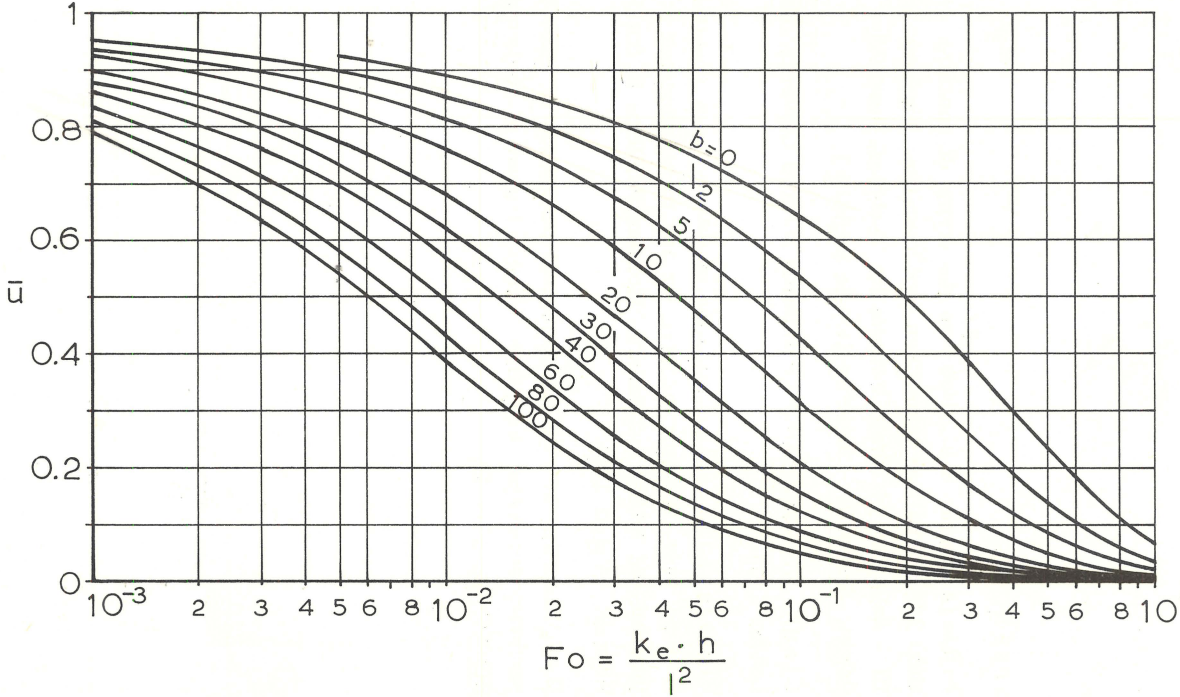
Nomogram 9.



b = 100.0
u = f(x / l, Fo, b)
Fo = k_e h / l²; k = k_e (1 + bu)
1.0
0.8
0.6
u
0.4
0.2
0
Fo = 0.0001
0.001
0.005
0.01
0.02
0.03
0.04
0.05
0.06
0.08
0.10
0.15
0.25
1.0
0.8
0.6
0.4
0.2
0
x / l

Nomogram 10.

$\bar{u} = f(Fo, b);\ k = k_e(1 + bu)$



Nomogram 11.

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