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Sampling and Reconstruction of Transient Signals by Parallel Exponential Filters

H. Olkkonen and J. T. Olkkonen

Abstract—This brief introduces a new method for sampling of transient analog waveforms based on the parallel exponential filters. The signal is fed to the parallel network consisting of resistor–capacitor (RC) circuits, outputs of which are simultaneously sampled. We show that N previous samples of the input signal can be reconstructed from single output samples of N parallel RC circuits. The parallel sampling method increases the sampling rate of the data acquisition system by a factor of N. In particular, the method is useful in increasing the sampling rate of the Flash-type analog-to-digital VLSI circuits. We present the parallel RC network, develop the reconstruction algorithm, and briefly describe a variety of applications such as measurement and reconstruction of pulses produced by ultrawideband transmitters, radiation detectors, and pulse lasers.

Index Terms—Analog-to-digital converters (ADCs), compressed sensing, sampling and interpolation, VLSI.

I. INTRODUCTION

THE sampling methods mostly rely on the Shannon's famous theorem [1], which concerns the sequential sampling of the band-limited signals at equidistant time intervals. In many areas of science and technology, the measurement of impulses and short pulses such as exponential waveforms from radiation detectors gives an extended demand on the data acquisition system. The Flash-type analog-to-digital converters (ADCs) have a limited conversion time, which is not sufficient for sequential sampling of the short-term transients. The sequential sampling scheme based on the finite rate of innovation (FRI) has been an object of vivid research in the signal processing society for reconstruction of the Diracs, pulse edges, and other discontinuities [2]–[6]. In the FRI methods, the transient signal is fed to the analog network, which broadens the signal for sampling with an ADC. The reconstruction algorithm is based on the ad hoc knowledge of the signal waveform. In our recent work [7], we have introduced a method for reconstruction of the amplitudes and appearance times of the impulse train using parallel exponential filters. In [8], a simple two-channel method for sampling FRI signals is described.

In this brief, we introduce a new method for sampling and reconstruction of continuous transient waveforms. The signal is
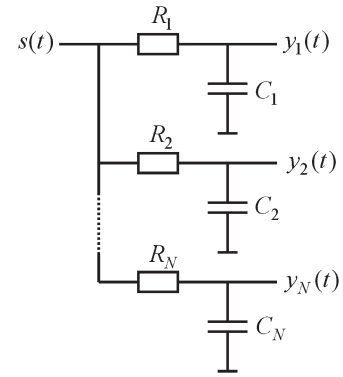
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Fig. 1. Parallel RC sampling filters.

fed to the parallel network consisting resistor–capacitor (RC) circuits. The outputs of the RC circuits are simultaneously sampled by an ADC. We show that N signal samples can be reconstructed from the single samples of N parallel RC circuits. In the following, we describe the parallel RC network, develop the reconstruction algorithm, and describe a variety of applications of the parallel sampling scheme.

II. THEORY

A. Parallel Sampling Scheme for Transient Signals

Let us consider a causal analog signal $s(t)$, which is defined as

$$s(t) = \begin{cases} s(t), & \text{for } t \geq 0 \\ 0, & \text{for } t < 0. \end{cases} \tag{1}$$

The signal $s(t)$ is fed to the network consisting of N parallel RC filters shown in Fig. 1. For $t \geq 0$, the exponential impulse responses of the RC filters are

$$h_i(t) = A_i e^{-\alpha_i t}, \qquad i = 1, 2, \ldots, N \tag{2}$$

where $\alpha_i = 1/(R_i C_i)$. In an ideal case, $A_i = \alpha_i$, but in practice, they may slightly differ from each other. The outputs of the RC filters are obtained as

$$y_i(t) = s(t) * h_i(t) = \int_0^t s(\tau) h_i(t - \tau) d\tau \tag{3}$$

where $*$ denotes convolution, and τ is the dummy integration variable.

Next, we move from continuous signals to discrete-time signals. We assume that the signals $y_i(t)$ and $s(t)$ are sampled
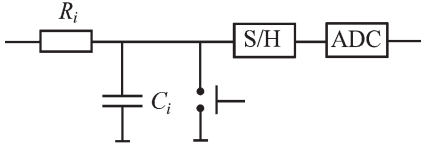
Fig. 2. Parallel RC filter network equipped with the FET switch and the S/H circuit for sampling of continuous-time waveforms.

at the time instants $t = n\Delta t$, $n \in \mathbb{N}_0$, and denoted by $y_i[n]$ and $s[n]$, respectively. By replacing the convolution integral (3) by the convolution sum, we obtain

$$y_i[n] = \sum_{k=0}^{n-1} s[k]h_i[n-k]\Delta t = \sum_{k=0}^{n-1} s[k]A_i e^{-\alpha_i(n-k)\Delta t}\Delta t. \tag{4}$$

Using the short notation

$$\lambda_i = e^{-\alpha_i \Delta t} \tag{5}$$

we have

$$z_i[n] = y_i[n]/(A_i \Delta t) = \sum_{k=0}^{n-1} s[k]\lambda_i^{n-k} \tag{6}$$

and we obtain the following matrix–vector representation:

$$\begin{bmatrix} z_1[n] \\ z_2[n] \\ \vdots \\ z_N[n] \end{bmatrix} = \begin{bmatrix} \lambda_1^n & \lambda_1^{n-1} & \cdots & \lambda_1 \\ \lambda_2^n & \lambda_2^{n-1} & \cdots & \lambda_2 \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_N^n & \lambda_N^{n-1} & \cdots & \lambda_N \end{bmatrix} \begin{bmatrix} s[0] \\ s[1] \\ \vdots \\ s[n-1] \end{bmatrix}$$
$$\Leftrightarrow \mathbf{z} = \boldsymbol{\lambda}\mathbf{s}. \tag{7}$$

By setting $n = N$, where N is the number of RC circuits, we notice that (7) includes a nonsingular Vandermonde matrix having rank N. This enables us to solve the input signal $s[n]$, $n = 0, 1, \ldots, N-1$, from the outputs of parallel RC circuits y_i, $i = 1, 2, \ldots, N$, sampled at the time instant $t = N\Delta t$.

The signal vector $\mathbf{s}$, containing N discrete values, can simply be reconstructed as

$$\mathbf{s} = \boldsymbol{\lambda}^{-1}\mathbf{z}. \tag{8}$$

Since the inverse matrix $\boldsymbol{\lambda}^{-1}$ only depends on the properties of RC circuits and the sampling rate, not on the input signal, the signal reconstruction in later measurements is obtained by a single matrix–vector multiplication with the predetermined $\boldsymbol{\lambda}^{-1}$.

B. Measurement and Reconstruction of Continuous Waveforms

The above formulation is only valid for the reconstruction of the N consecutive samples from the causal analog waveform. For measurement of the next sequence of N samples, the parallel RC network has to be modified by adding a FET switch and a sample-and-hold (S/H) circuit, as illustrated in Fig. 2. The switch resets the output at the beginning of the measurement period at the time instant $t = 0$, and the S/H circuit samples the signal at $t = N\Delta t$. The next measurement period follows

the same procedure. With this arrangement, the continuous waveform can be reconstructed without any discontinuities.

Alternatively, the effect of the previous measurement sequence can numerically be eliminated. At the time instant $t = 2N\Delta t$, (6) attains a value

$$z_i[2N] = \sum_{k=0}^{2N-1} s[k]\lambda_i^{2N-k}$$
$$= \sum_{k=0}^{N-1} s[k, N]\lambda_i^{2N-k} + \sum_{k=0}^{N-1} s[k, 2N]\lambda_i^{N-k}$$
$$= \lambda_i^N z_i[N] + \sum_{k=0}^{N-1} s[k, 2N]\lambda_i^{N-k} \tag{9}$$

where two consecutive signal sequences are denoted by $s[k, N]$ and $s[k, 2N]$. Now, we obtain a difference

$$z_i[2N] - \lambda_i^N z_i[N] = \sum_{k=0}^{N-1} s[k, 2N]\lambda_i^{N-k} \tag{10}$$

which can be written as the Vandermonde matrix structure permitting the solution of the signal sequence $s[k, 2N]$, $k = 0, 1, \ldots, N-1$. In a similar manner, we may write (6) for the signal sequence $s[k, 3N]$, etc.

C. Reconstruction at Nonequidistant Intervals

The reconstruction method via (7) and (8) reconstructs the signal at constant time intervals. However, with a slight modification, the parallel sampling method permits the reconstruction of the signal at nonequidistant time intervals. Instead of (4), we may write

$$y_i(N\,\Delta t) = \sum_{k=0}^{N-1} s(t_k)h_i(N\,\Delta t - t_k)\Delta t_k$$
$$= \sum_{k=0}^{N-1} r(t_k)h_i(N\,\Delta t - t_k) \tag{11}$$

where $\Delta t_k = t_{k+1} - t_k$ denotes the time interval between the signal samples $s(t_k)$ and $s(t_{k+1})$. The time intervals are selected so that $t_0 = 0$ and $t_N = N\Delta t$. In a similar manner as in (4)–(8), the sequence $r(t_k)$ is reconstructed, and then, the signal sequence is computed as $s(t_k) = r(t_k)/\Delta t_k$.

D. Selection of the Discretization Interval

For perfect reconstruction, the measured analog waveform must be band limited to the frequency $f_{\max}$. Due to Shannon's sampling theorem [1], the sampling frequency f_s should obey the criteria $f_s > 2f_{\max}$ and $\Delta t < 1/(2f_{\max})$. If the measurement signal contains very high frequency components such as spikes, edges, or other discontinuities, the antialiasing filter (cutoff frequency f_c) has to be installed in front of the parallel RC network. Then, the discretization time should be selected as $\Delta t < 1/(2f_c)$.
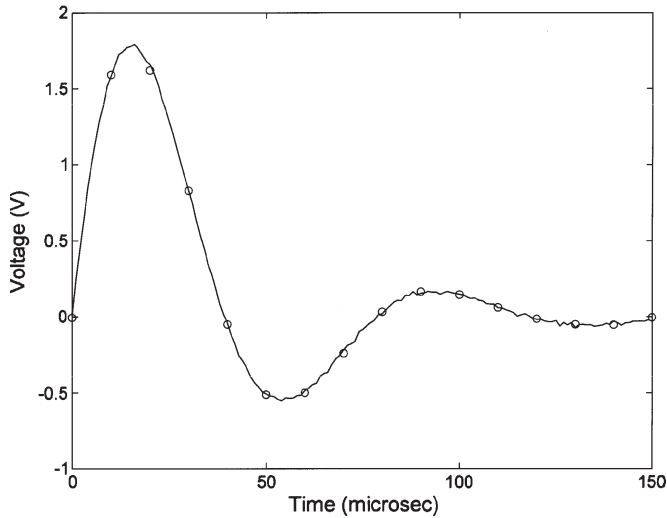
Fig. 3.   Measurement and reconstruction of the exponentially damped sinusoidal waveform. The dots denote 16 reconstruction points, and the continuous line depicts the signal measured with the memory oscilloscope.
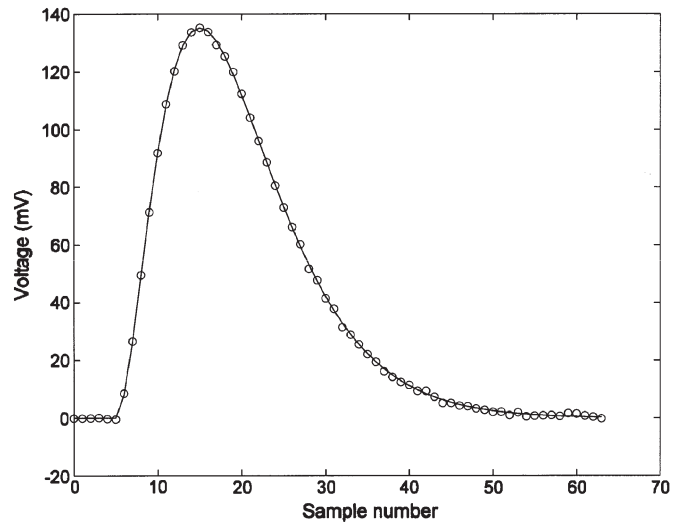

Fig. 4.   Measurement and reconstruction of the four sequences of the semi-Gaussian-shaped pulse. The effect of the previous measurement sequence was numerically eliminated. The dots denote the reconstruction points. The continuous line depicts the signal obtained with the memory oscilloscope.

III. EXPERIMENTAL

The experimental arrangement consisted of the transient pulse generator, which was programmed to yield short-term semi-Gaussian, exponential, and sinusoidal waveforms. The measurement system consisted of 16 parallel RC circuits equipped with FET switches and S/H circuits. The outputs of the S/H's were fed to the 16-channel 12-bit ADC unit, which had the ± 5-V measurement range. Using this arrangement, the 16 signal samples were reconstructed corresponding to the measurement interval $t \in [0, N\Delta t]$. The reference measurements were carried out with a memory oscilloscope. All experimental tests were performed in a Faraday cage environment.

The first measurement signals were the exponentially decaying sinusoids. Fig. 3 shows a typical example, where the 16 points were reconstructed. The average difference between reconstructed and measured signals was 1.2 mV.

The reconstruction of four sequences of the typical semi-Gaussian waveform is described in Fig. 4. The average difference between reconstructed and measured signals was 2.2 mV when the RC circuits were reset with the FET switches. The average difference was 1.4 mV when the measurements were performed without resetting the RC circuits, and the effect of the previous measurement sequence was numerically subtracted via (10). Fig. 5 shows the reconstruction of the four sequences of the sinusoidal waveform, which is disturbed by the high-frequency noise. In this experiment, an antialiasing filter was used as a main amplifier in front of the RC circuits to cut the high-frequency noise components above the Nyquist frequency.

IV. DISCUSSION

In this preliminary work, we have described a new measurement and sampling concept, which permits the measurement of short-term transient signals having extreme high-frequency content. The key feature of the method is the use of the parallel exponential filters, whose outputs are simultaneously measured.
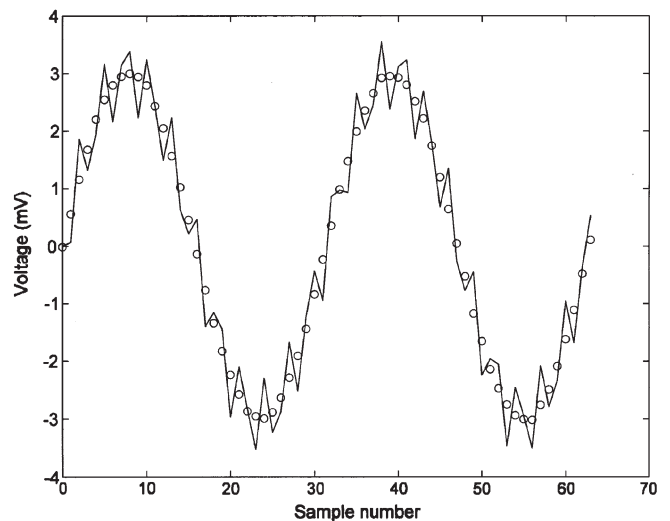

Fig. 5.   Measurement and reconstruction of the sinusoidal waveform, which is corrupted by the high-frequency noise. The continuous line depicts the signal obtained with the memory oscilloscope. The dots denote the reconstruction points. The effect of the previous measurement sequence was numerically eliminated.

The reconstruction algorithm is based on the replacement of the convolution integral (3) by the convolution sum (4), which yields a Vandermonde matrix–vector equation (7). The reconstruction of the N signal samples needs only one matrix–vector multiplication (8). The analytical formulas for the inversion of the Vandermonde matrix in (8) are well known [9]–[14]. The explicit solutions are preferred since they are more accurate than the general matrix inversion algorithms.

The conversion time of the Flash-type ADCs is limited to the through output time of the comparator chain. In VLSI design, the most effective configuration would be to use $t \in [0, N\Delta t]$ individual ADCs equipped with S/H circuits to separately measure the parallel RC filters. By using N parallel exponential filters, the conversion time of each ADC is prolonged to $N\Delta t$.

This makes it possible to increase the sampling rate of the data acquisition system by a factor of N.

In this brief, we tested two alternative methods for elimination of the effect of previous signal sequences. The FET switch method is preferable in the measurement of continuous waveforms. The reconstruction of the signal sequences by the numerical method (10) yielded slightly better results. The primary reason is obviously the interference due to the operation of the FET switches. It is obvious that also other types of linear networks could be used in place of the RC filters. However, the transfer function of the RC filter has only a single pole. On the other hand, the RC filter has a low-pass filter characteristic, which effectively reduces the high-frequency noise imposed on the signal. This is clearly displayed in Fig. 5, where the original sinusoidal waveform measured with a memory oscilloscope has an elevated noise level.

The competing method to the present parallel sampling scheme is the use of N parallel ADCs, which sequentially sample the signal [15]. This would also increase the sampling rate by a factor of N. However, in high-speed sampling, the timing error in synchronization of the ADCs increases. In the present method, all ADCs are simultaneously triggered with one timing pulse. Another advantage of the parallel sampling scheme is that the reconstruction can be carried out at non-equidistant intervals. An example would be the measurement of the exponentially decaying short pulses, whose amplitude and decay constant carry information. The short discretization interval in the close vicinity of the amplitude peak would yield an accurate value for the amplitude. In the descending part of the pulse, only a few signal values would give an accurate value for the decay constant.

The present method has plenty of applications, including the measurement of pulses yielded by radiation and optical detectors and the recovery of ultrawideband (UWB) pulse waveforms. Usually, the UWB pulses are short transients (Diracs), and the information is coded to the appearance time of the pulses. The FRI-like methods [2]–[6] are based on the ad hoc knowledge of the signal. In this aspect, the present sampling and reconstruction method clearly differs from the FRI reconstruction. The parallel sampling scheme recovers the transient signals without any knowledge of the signal waveform. The information may be coded to the shape of the UWB pulses. Using a single receiver, several transmitters can simultaneously be measured, e.g., in multisensor applications.

The present parallel sampling scheme can be seen as a special case of the setup called multichannel or compressive sampling [16]–[21], which can be adapted to recover certain signals and images from far fewer measurements than conventional sampling methods need.

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