Comparison of three sampling methods in the context of probabilistic fracture mechanic analyses of NPP piping welds: case study

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Summary

Assessment of the probability of failure due to crack growth constitutes one part of the risk informed in-service inspection methodology. In Nuclear Power Plant (NPP) piping systems, failure of a component due to crack growth generally occurs when the crack has penetrated through the pipe wall. Most of the factors contributing to the crack growth are not strictly deterministic. It can only be estimated how probable it is that these factors will have certain values. As a result, we cannot usually say (without inspections) that there is a crack on some particular weld and that this crack will propagate through the pipe wall in some given time, for example exactly in 4 years. What we can try to estimate, however, is the probability that there exists a crack in some particular weld and the probability that this crack will propagate through the pipe wall in some given time, for example within the following 4 years.

Computation of these probabilities requires usually numerous deterministic crack growth analyses with different sets of input values assigned for the parameters which govern the crack growth. The process of assigning input value sets for the parameters is called sampling. The main aim in sampling is to obtain adequate balance between accuracy of the estimates for the probabilities and the number of the different input value sets. How successfully this aim is achieved, depends on the sampling method chosen.

Three commonly used ways to perform the sampling were studied in this project. First, both Monte Carlo (MC) simulation and Latin Hypercube Sampling (LHS) method were performed so that the space of possible values for each variable was considered as a whole entity. Additionally, the LHS procedure was also used so that sampling was performed separately in numerous smaller subcategories. The results given by different methods with the same total number of samples were then compared against each other in order to rank these methods in relation to each other.

The study was performed as a case study in which it was assumed that stress corrosion (SC) causes a circumferential, semi-elliptical crack under given axial tensile loading to grow from the inner surface of a NPP pipe at the location of a weld. Sampling of input values was concentrated on two variables while assuming deterministic values for the other variables. Depth of a crack and its length were chosen as parameters to be varied in the analyses. No correlation was assumed to exist between these two variables and each value pair was considered as equally probable. Sampling was performed using 10 different sample sizes ranging from 50 to 1000 samples.

In addition to continuous growth of a particular crack, crack growth was also treated at more general level of discrete crack growth states, called also as system degradation states. Six system degradation states were used in this study. Depth of the crack was used as a parameter which determines into which state the crack in question belongs to. It follows that, when a crack grows, it eventually experiences transition from one system state into the following state; unless it is detected during inspections and repaired. The probabilities sought after in this study were the ones for a postulated crack, initially in one system state, to experience transition into one of the following system states during one year.

It was detected that the LHS method did not consistently increase accuracy of the estimates for the state transition probabilities when compared to the ones obtained using MC method. This was contrary to expectations. However, the LHS method, when performed separately for each system degradation state, did yield somewhat better results. However, coincidence cannot be ruled out as a cause of this result. For estimation of small state transition probabilities, none of the investigated methods yielded satisfying results when using a reasonably large sample size.

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Preface

This report has been prepared under the research project PURISTA, which concerns Risk Informed In-service Inspection (RI-ISI) analyses of Nuclear Power Plant (NPP) piping systems. The project is a part of SAFIR2010, which is a national nuclear energy research program. PURISTA project work in 2009 was funded by the State Nuclear Waste Management Fund (VYR) and the Technical Research Centre of Finland (VTT). This funding is gratefully acknowledged.

The work described in this report was carried out during 2008 and 2009 in the premises of VTT. The purpose of the study was to obtain quantitative means to assess the goodness of different sampling methods when applied to probabilistic analysis of crack growth. The work described in the report was done by the authors. When it comes to input values used in the analysis, this paper is based on work done by Mr. Cronvall, as reported in earlier yearly reports of the PURISTA project.

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Contents

List of symbols and abbreviations

Latin symbols

A	constant used in computation of stress intensity factors
A, A_i, A_{ij}	different types of areas
a	crack depth
B	constant used in computation of stress intensity factors
C	material parameter to be used in stress corrosion crack growth formula
c	crack length
D	outer diameter of a pipe
d_f	probability of detecting a flaw
d_l	probability of detecting a leak
$\mathbf{M_d}$	transition probability matrix
$\mathbf{M_i}$	inspection matrix
N	total number of samples
N_i	number of samples falling into category i
n	material parameter to be used in stress corrosion crack growth formula, total number of system states used in the analyses
n_{ij}	number of samples falling into category i's subcategory j
K_I	mode I stress corrosion factor
P	probability
t	wall thickness of a pipe, time
p	pressure
p_{ij}	state transition probability
$\mathbf{p}_m$	probability vector

Greek symbols

σ	stress, far field stress

Abbreviations

API	American Petroleum Institute
EPRI	Electric Power Research Institute
LHS	Latin Hypercube Sampling
MC	Monte Carlo (simulation)
NPP	Nuclear Power Plant
PFM	Probabilistic Fracture Mechanics
RI-ISI	Risk Informed In-Service Inspection
SIF	Stress Intensity Factor
SC(C)	Stress Corrosion (Cracking)
TVO	Teollisuuden Voima Oyj.
VTT	Technical Research Centre of Finland
VYR	State Nuclear Waste Management Fund (VYR)

1   Introduction

Much attention has been paid on the risk-informed in service inspection strategies during the last decade and a half. For example, a qualitative RI-ISI matrix procedure has been developed by EPRI /1/. This qualitative matrix procedure has been the starting point for the study performed at VTT. In this study, the qualitative EPRI matrix procedure was modified into a semi-quantitative procedure by changing the degradation category from qualitative to quantitative /2/, /3/, /4/. This refined version of the procedure has the benefit that it enables quantified comparison of different inspection strategies of welds.

In this method, postulated cracks at welds are divided into different states. Basis of this categorisation is the depth of the crack and the probability of its detection in inspections. During one time step, these cracks either stay in their initial state or make transition into one of the successive states. Whether this transition happens, is dependent on the values of the variables in the equations that describe the behaviour of the crack during the time step as well as the loading conditions.

Many of these variables have scattered characteristics. This can be observed for example from the results of laboratory tests. Such variables obviously include initial crack depth and length, as well as formation frequency of initial cracks, material properties like fracture toughness and tensile strength and frequencies of different load cycles.

It is assumed that the future state of the crack is independent of its past states given its present state. I. e. it has the Markov property. In this case, growth of a crack can be thought of as a discrete time Markov chain in which each crack either jumps from one state to another within the investigated period or stays in its initial state. Probabilities of these jumps can be either computed analytically for the simplest cases or estimated using probabilistic fracture mechanics (PFM). Once estimated, the probabilities are collected into a transition probability matrix. These transition probabilities are assumed to be stationary.

Estimation of these transition probabilities with at least reasonable accuracy requires numerous deterministic analyses to be performed with different input value sets. Method of choosing these input values was the topic of this study. Two commonly used methods of Latin hypercube sampling (LHS) and Monte Carlo sampling were investigated and compared against each other. Additionally, the LHS method was also applied within each crack state separately and the results were compared with the ones obtained using MC method and simple LHS method.

2 Goal of the study

The goal of the study was to assess the goodness of the LHS sampling method in relation to MC sampling method when applied to computation of state transition probabilities.

3 Description of the subject of the study

As stated earlier, the subject of the study was the efficiency of the LHS method when applied to probabilistic crack growth analysis. The study was performed as a case study using only one weld section of the auxiliary feed water system (327) piping of the utility Teollisuuden Voima Oyj.'s (TVO) NPP unit OL1. For this particular weld, the loading consisted of normal static operational pressure of 7.0 MPa. The material, environment and temperature specific parameter values used in the stress corrosion (SC) induced crack growth analyses were $C_{SCC}=1.42*10^{-4}$ and $n_{SCC}=3.00$ /5/, /6/ and these values were assumed to be applicable in water environment. At the studied location, the nominal values for the wall thickness and the outer diameter are 6 and 115.0 mm, respectively /6/.

4 Limitations and assumptions

The study was limited to SC driven growth of semi-elliptical circumferential cracks, with mode I stress intensity factor, K_I as the governing crack growth parameter. Parameters to be varied in the study were limited to initial length, c, and initial depth, a, of a crack. Using only two variables actually reduce the latin hypercube sampling method merely to latin squares method.

The study was additionally limited to contain only one case. This means that only one cross section was studied, one set of material values used and one loading profile applied to the cross section. It was assumed that cross sectional values and loading as well as material parameters contain no scatter. In other words, the nominal values given for the cross section and the material apply. For example, if the nominal wall thickness is 6 mm, it is not possible that in reality it is 6.01 mm. These limitations were made due to time restrictions.

It was also assumed that, if there is an existing crack, all possible length-depth value combinations for this crack have equal probabilities. This assumption is obviously not true. It was made in order to facilitate and speed up the computation and analysis process. Based on the judgement of the author, it was also assumed that this would not affect the assessment of the goodness of the sampling methods too much.

Related to the aforementioned issue, it was assumed that there does not exist any correlation between the initial values of the length and the depth of the crack. In reality, based on the documented crack cases in Swedish NPPs /7/, there might exist some linear correlation between these parameters. Correspondingly, the sampling was made so that no correlation was intended to be achieved. Due to small sample sizes, some correlation between the parameters was encountered with small sample sizes of 50 and 75. This correlation decreased quite rapidly as the sample size was increased.

5 Methods

5.1 Deterministic crack growth analysis

SCC crack growth is mathematically expressed by the equation /8 /

$$\frac{da}{dt} = C_{SCC} K_I^{n_{scc}} \tag{1}$$

where a [mm] is the crack depth, K_I [MPa$\sqrt{m}$] is the stress intensity factor at the crack tip, t [s] is time, C_{SCC} is a constant characterizing the material, environment and temperature and n_{SCC} is an exponent characterizing the same physical qualities.

The stress intensity factor K_I is given by the equation /9/

$$K_I = A\sigma\sqrt{\frac{\pi a}{B}} \tag{2}$$

where A and B are constants depending on geometry and σ [MPa] is the far field tensile stress which depends on the loading conditions.

As can be seen, the parameters affecting the crack growth are initial crack size (parameters a & c), material parameters (C_{SCC} & n_{SCC}), and the magnitude and distribution of the loading.

5.2 Analysis code VTTBESIT

The deterministic crack growth analyses in this study were performed with the analysis code VTTBESIT. This code comprises parts developed by the Fraunhofer Institut fur Werkstoffmechanik (IWM), Germany and by VTT. Computation of the stress intensity factors is performed by program BESIT60 developed by IWM. The theoretical background and analysis procedures of BESIT60 are presented in references /10/, /11/ and /12/. Computed values of SIF are used as starting values for the crack growth analysis performed with the VTTBESIT. It should be kept in mind that the solution method used in the analyses is less accurate when the crack depth is over 80% of the wall thickness. Figure 1 shows a screenshot of the graphical user interface of the VTTBESIT with one set of initial values for the parameters a and c.
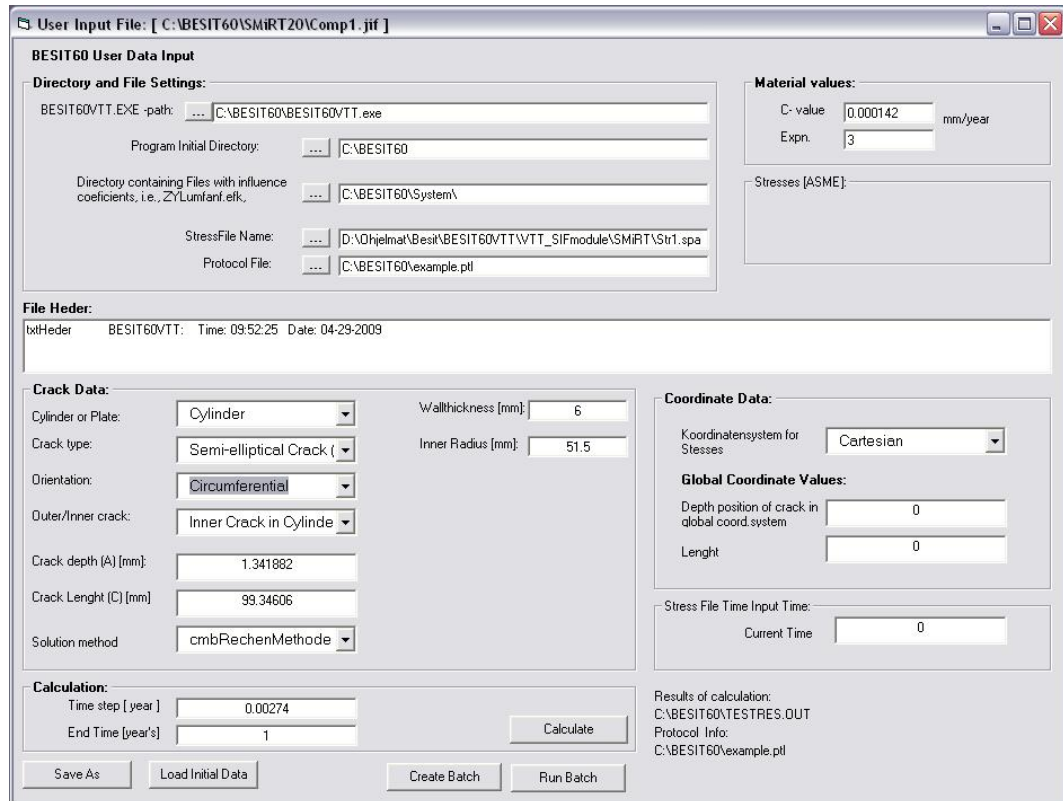
Figure 1.   Screenshot of the VTTBESIT graphical user interface.

5.3   Loading

The loading for the deterministic analyses was considered to be static. Although some time dependent variation takes place in the loading due to transients, these transients were judged to be short enough so that they do not have a considerable effect on the SC crack growth. Considered loadings are internal process pressure during operation and welding residual stresses. Stresses due to dead weight of the pipe and its contents as well as those due to internal process temperature during operation were not taken into account.

The axial membrane stress due to internal pressure was computed by the following equation:

$$\sigma = \frac{p \cdot (D/2 - t)}{2t} \tag{3}$$

where *p* [MPa] is the internal pressure, *D* [mm] is the outer diameter of the pipe and *t* [mm] is the wall thickness of the pipe.

The weld residual stress distribution was computed according to American Petroleum Institute (API) Recommended Practice 579 procedure /13/. More thorough representation of this procedure, as well as other loadings can be found in the work by Cronvall et. al. /6/. The used axial stress distribution due to welding residual stresses and axial membrane stress due to pressure is shown in Figure 2.
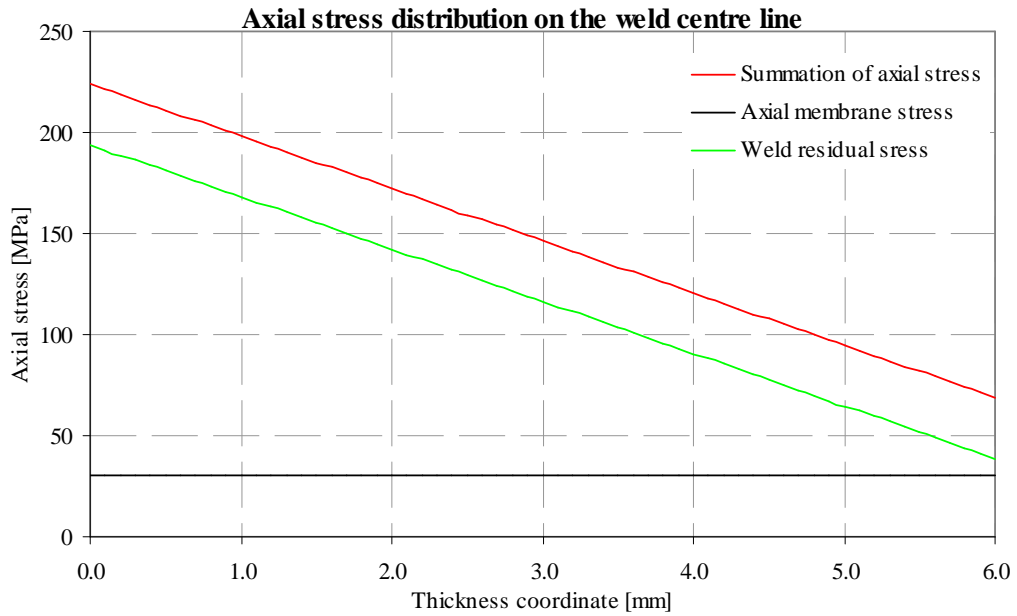
Figure 2. Axial weld residual stresses according to API-579 procedure, axial membrane stress and their summation over the thickness of the pipe wall. The origin of the thickness coordinate is on the inner surface of the pipe wall.

5.4   Modelling of crack growth as Markov process

In the PURISTA project, and also in its predecessor, crack growth has been studied on probabilistic basis using concepts of Markov process and Markov chain. The Markov process is a mathematical model for the random evolution of what is called "memoryless" system. "Memoryless" system is a system for which the likelihood of a given future state, at any given moment, depends only on its present state, and not on any past states. The term Markov chain in turn is often used to describe a Markov process which has a discrete state-space.

Assuming inspection interval of k years, utilizing the Markov property of crack growth and assuming stationary transition probabilities, the probabilities for a crack to be at one particular state after m years can be computed as

$$\mathbf{p}_m = \mathbf{p}_0^T \cdot \underbrace{\mathbf{M}_d}_{\text{Year no.1}} \cdot \underbrace{\mathbf{M}_d}_{\text{Year no.2}} \cdot \underbrace{\mathbf{M}_d}_{\text{Year no.3}} \cdot \llcorner \cdot \underbrace{\mathbf{M}_d}_{\text{Year no.k}} \cdot \underbrace{\mathbf{M}_d}_{\text{Year no. k+1}} \cdot \llcorner \cdot \underbrace{\mathbf{M}_d}_{\text{Year no.m}} \tag{4}$$

where

$$\mathbf{p}_m = \left\{ \begin{matrix} p_0 \\ p_1 \\ p_2 \\ \mathbb{M} \\ p_n \end{matrix} \right\}_m \tag{5}$$

is a probability vector with each component p_i [-] defining probability for a crack to be at state i after m years,

$$\mathbf{M}_d = \begin{bmatrix} p_{00} & p_{01} & p_{02} & \llcorner & p_{0n} \\ 0 & p_{11} & p_{12} & & \mathsf{M} \\ 0 & 0 & p_{22} & & \\ \mathsf{M} & & & \bigcirc & \mathsf{M} \\ 0 & \llcorner & & \llcorner & p_{nn} \end{bmatrix} \tag{6}$$

is a transition probability matrix where each component p_{ij} [-] is the computed probability for a crack to grow in one years time from state i to state j, defining probabilities of transition from one system state i to another system state j, and

$$\mathbf{M}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ d_f & 0 & 0 & 1-d_f & 0 & 0 \\ d_l & 0 & 0 & 0 & 1-d_l & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

is an inspection matrix, where d_f [-] is the probability of detecting the flaw and d_l [-] is the probability of detecting the leak during the inspections.

The term $\mathbf{p}_0$ in equation (4) represents the condition at the beginning of the study. To be more specific, its values represent probabilities that there is a crack in this weld belonging to one of the states 0, 1, …, m. Of course, it is probable that there is no crack (crack belongs to state 0).

Additionally it holds that

$$\sum_{j=i}^{n} p_{ij} = 1. \tag{8}$$

The terms p_{01}, p_{02}, p_{03} $\cdot\cdot$ p_{0n} [-] in equation (6) are associated with cracks that do not exist at the beginning of the time step but which initiate during the step. These probabilities include the formation frequency of initial cracks and growth of these initial cracks during the time step. These terms were excluded from the study.

5.5   System degradation states

Each system degradation state is defined by the depth of a crack. Limiting values for the depth of a crack in each state are based on the probabilities of detection of the crack and repairing of it. The system degradation states concerning this study are listed in Table 1.

Table 1.   The complete set of used system degradation states.

State number	Crack depth	Description
0	0	Undamaged piping cross sections
1	0 - 1 mm	Very unlikely to detect
2	1 mm – 50 % of wall thickness	Possibility of detection. No repair.
3	50 - 99 % of wall thickness	Possibility of detection + repair
4	99 – 100 % of wall thickness	Leak-before-break. Repaired if detected
5	100 %	Rupture

An illustration of the various considered crack states is shown in Figure 3. In the figure, these states are divided with the vertical lines at locations a_1, a_2, a_3 and t. Each crack state is further divided into subregions with curves, l_{ij}. These subregions represent cracks that fall into different subcategories within the given state; those which will stay in their initial state after the examined period and those which will undergo transition into one of the subsequent states. The areas between these curves, the straight vertical lines, and the straight horizontal lines at $c = 0$ and $c = c_{max}$ represent the state transition probabilities, p_{ij}, given that each pair of values a and c are equally probable:

$$p_{ij} = \frac{A_{ij}}{A_i}. \tag{9}$$

If digital approximation of each curve, l_{ij}, is known (meaning value pairs a and c at the curves) the accurate probability can be approximated using Riemann sums. In this study, the curves were digitally approximated using unequal subinterval widths.

The probabilities were then assessed numerically by using certain selected sampling methods to define initial values for the crack half length, c, and depth, a. These values are then used in deterministic SC induced crack growth analysis. As a result of this analysis, one obtains length and depth of the crack at different time instants, for example after one year. As the number of samples in each state, n_i, increases, the resulting estimates for the state transition probabilities are likely to converge nearer to their true values. The sampling was made using 10 different sample sizes (50, 75, 100, 200, 300, 400, 500, 600, 700, 1000).
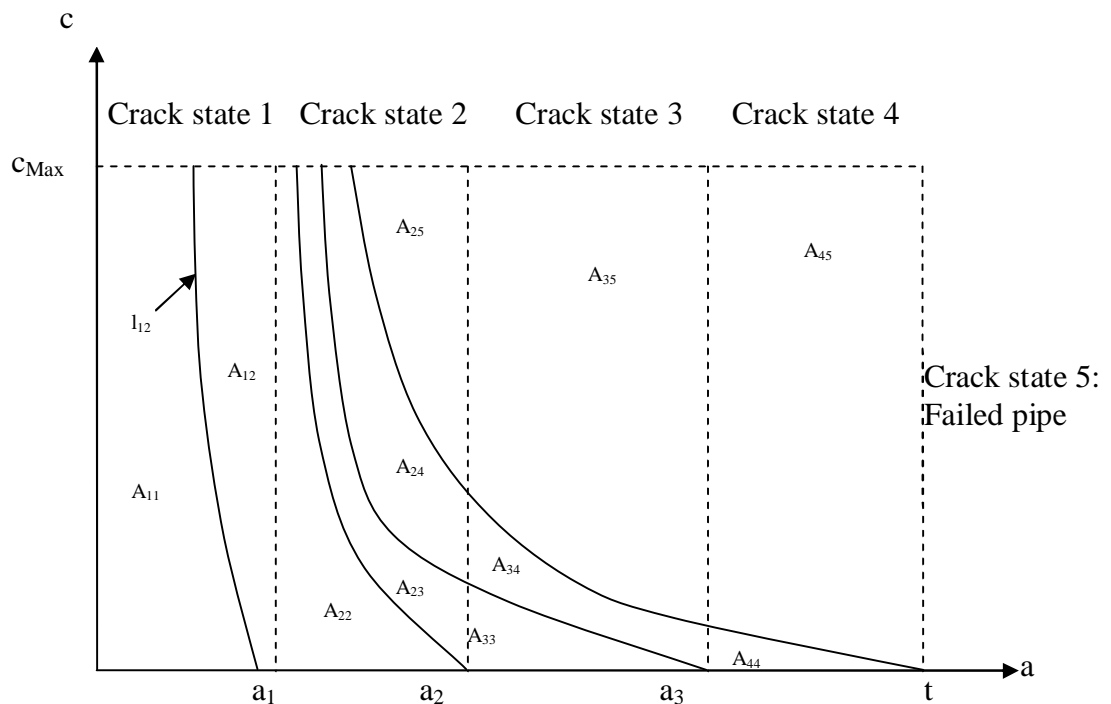

Figure 3. Possible spaces for half length, c, and depth, a, of a semi-elliptical surface crack divided into system degradation states and into further subregions of area A_{ij}.

Let the total number of samples in analyses for one weld be N. These N samples are divided into n subspaces. If the number of samples in each subspace is N_i, then

$$
\mathbf{N} = \begin{bmatrix} N_1 & 0 & \mathbb{L} & \mathbb{L} & 0 \\ 0 & N_2 & 0 & & \mathbb{M} \\ 0 & 0 & N_3 & & \mathbb{M} \\ \mathbb{M} & & & \bigcirc & \mathbb{M} \\ 0 & \mathbb{L} & \mathbb{L} & \mathbb{L} & N_n \end{bmatrix} \xrightarrow{Computation} \begin{bmatrix} n_{00} & 0 & 0 & \mathbb{L} & \mathbb{L} & 0 \\ n_{01} & n_{11} & 0 & & & \mathbb{M} \\ n_{02} & n_{12} & n_{22} & 0 & & \mathbb{M} \\ n_{03} & n_{13} & n_{23} & n_{33} & & \mathbb{M} \\ \mathbb{M} & & & & \bigcirc & \mathbb{M} \\ n_{0n} & n_{1n} & n_{2n} & \mathbb{L} & \mathbb{L} & n_{nn} \end{bmatrix} \quad (10)
$$

where n_{ij} [-] is the number of samples that grow from state i to state j during one year.

Additionally it holds that

$$
\sum_j n_{ij} = N_i. \quad (11)
$$

An estimate for the mean probability for a crack to grow in one years time from state i to state j is then defined by

$$
p_{ij} = \frac{n_{ij}}{N_i}. \quad (12)
$$

Obviously, in this case no samples fall into the last subspace since it represents the failed pipe. In other words, $N_n = N_5 = 0$, $n_{nn} = n_{55} = 0$. The probability p_{55} is necessarily 1 since there are no further degradation states for the crack to grow into.

5.6 Latin hypercube sampling (LHS) method

In the LHS method, the space of possible values for each input variable is divided into equal number of equiprobable subspaces, called stratas. The number of stratas to be used depends on the total number of samples to be taken, or vice versa. For example, 10 samples to be taken in two-dimensional sample space (two variables) requires 10 stratas for both variables.

In this study, each value pair a & c was considered equally probable. If this was not the case, equiprobable stratas should be obtained by first dividing the cumulative probability density function of each variable into aforementioned stratas. Then the boundary values of these stratas could be mapped back into physically meaningful values.

Sampling is then performed so that after each sample, the stratas, within which the sample is located, are rendered as "unavailable" for further sampling. For example, let us perform sampling for two input variables. Let the first sample fall within strata number 1 of variable 1 and within strata number 5 of variable 2. This situation is shown in the upper left frame of Figure 4. This sample renders strata number 1 "unavailable" in further sampling of variable 1. The same applies for strata number 5 in case of variable number 2. This situation is shown in the upper right frame of Figure 4.

Available stratas for the next sample to fall into are stratas number 2, 3, 4···8 for variable number 1 and stratas number 1, 2, 3, 4, 6··· 8 for variable number 2. Lower left frame of Figure 4 shows one possibility for the next sample and the lower right frame the situation after these stratas has been rendered as "unavailable". Each new sample is then taken randomly from the available sample space that is left after the previous takes ones. This is continued until there is only one possible combination of stratas left for the last sample to fall into.
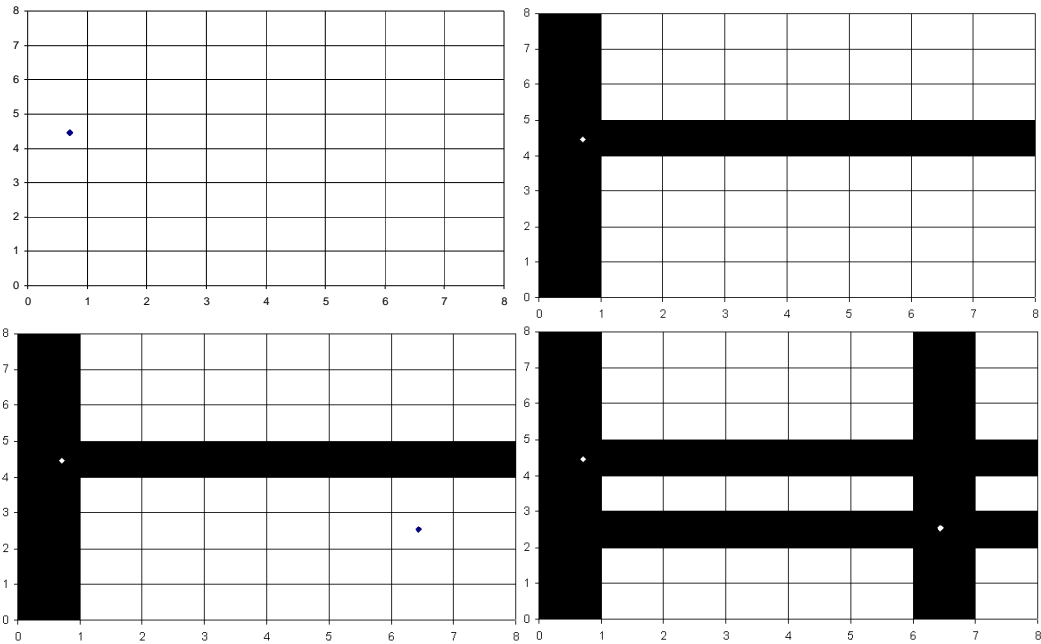
Figure 4.   Upper left: Location of the first sample taken.
Upper right: Rendering of the corresponding stratas as
*   "unavailable" for further sampling.*
Lower left: Location of the second sample taken.
Lower right: Rendering of the corresponding stratas as
*   "unavailable" for further sampling.*

The work by Inman /14/ gives credits for the method to W.J. Conover /15/. However, the first published journal article about LHS was written by McKay, Conover and Beckman /16/. Mathematical formulation of the LHS method can be found for example in the work by Olsson et al. /17/.

Inman et al. /18/ have also proposed a version of the method, which reduces the correlation between the input variables. If desired, as shown for example by Hardyanto /19/, specified amount of correlation between the input variables can also be included into the LHS. This is a useful feature if there exists correlation between, for example, the initial depth of the crack and its initial length.

More detailed discussion about the topic can also be found in the Appendix A of the work by Cronvall et al /6/.

6 Results

Estimated conditional state transition probabilities p_{11}, p_{12}, p_{22}, p_{23}, p_{25}, p_{33} and p_{35} are shown in Figures 5 - 7 as a function of sample size. This sample size is not the sample size falling within any particular crack state but instead the total number of samples that fall into entire range of the crack states. The probabilities computed using different sampling methods are marked with dots of different colour as follows:

- orange for the ones obtained using Monte Carlo sampling,

- dark blue for the ones obtained using the LHS and

- light green for the ones obtained using the LHS within each crack state separately.

Additionally, the estimate for the accurate value is marked in the mentioned figures with the black horizontal line.

Probabilities p_{13}, p_{14} and p_{15} are zero: a crack does not transfer from state 1 into states 3, 4 or 5 within a year under given loading circumstances. Probabilities p_{24} and p_{34} are also absent in the figures. These probabilities are relatively small (0.0009648 for p_{24} and 0.0021797 for p_{34}) and so they could not be estimated with a reasonable accuracy using neither MC simulation nor the LHS method with the given sample sizes.

Each dot in the figures is based on one sample only. Therefore the location of each individual dot in vertical axis is somewhat based on change. The closer the dot is to the black horizontal line, the more accurate is the probability obtained using this particular sample. Since each dot is based on one sample only, goodness of a sampling method cannot be assessed only looking individual points and their distance from the horizontal line. However, if one of the sampling methods were clearly better than the others, it should show at least in the majority of the cases. For example, if the LHS method was much better than the MC simulation, the majority of the blue dots should be located much nearer to the horizontal line than the orange dots.
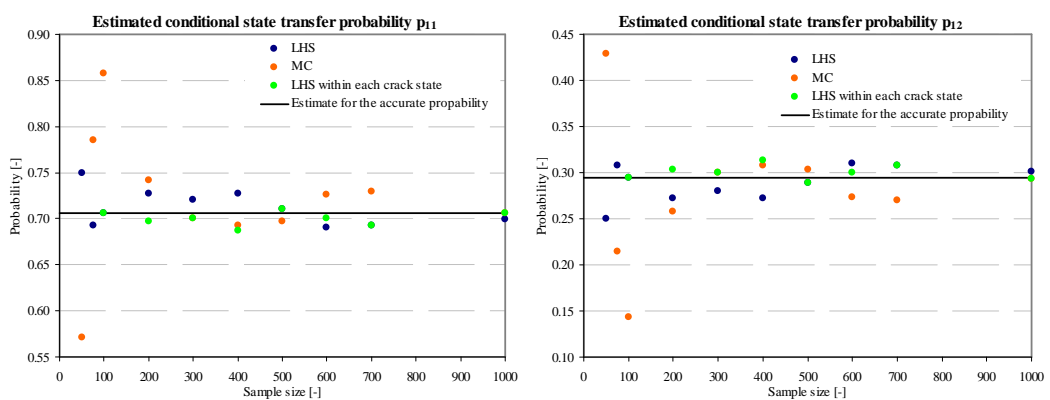

Figure 5.   Estimates for the conditional state transition probabilities p_{11} (left) and p_{12} (right) as a function of the sample size.
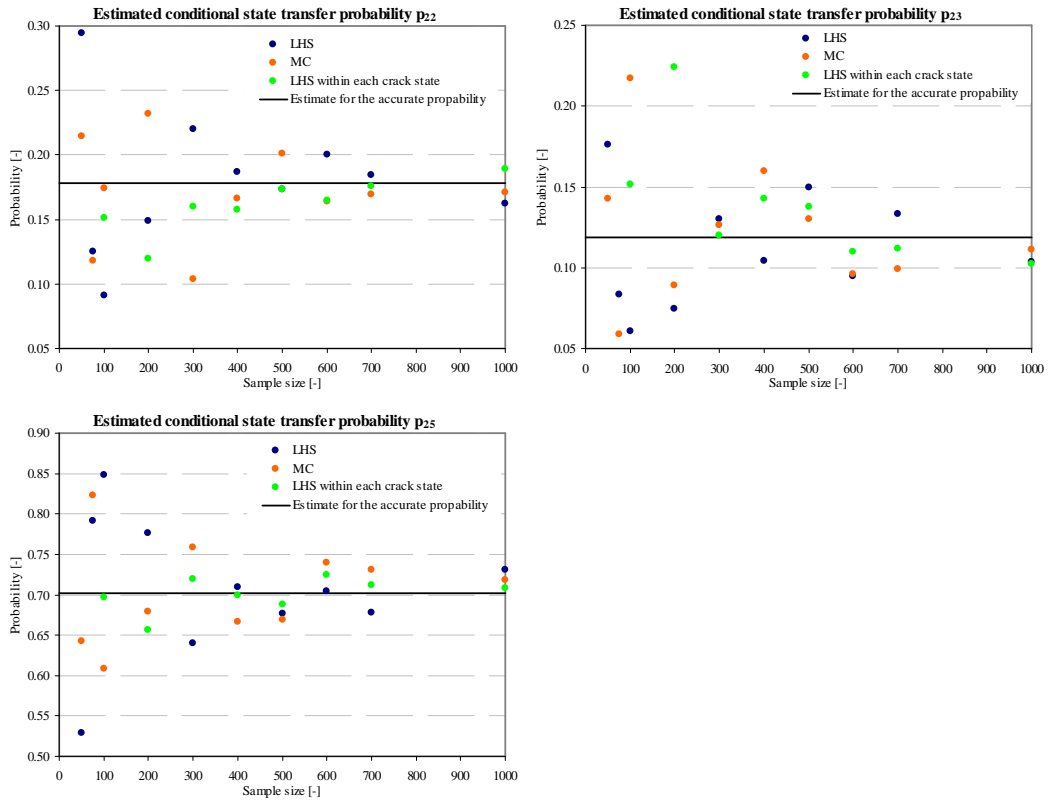
Figure 6.   Estimates for the conditional state transition probabilities p_{22} (upper left), p_{23} (upper right) and p_{25} as a function of the sample size.
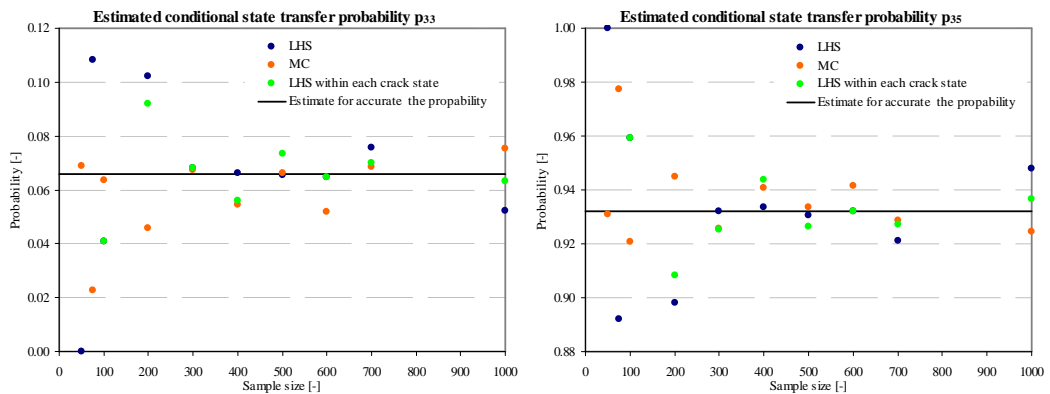

Figure 7.   Estimates for the conditional state transition probabilities p_{33} (left) and p_{35} (right) as a function of the sample size.

In a larger scale, goodness of the methods was assessed by giving each method "correctness" points based on how accurate estimate it yields for each probability p_{ij} with different sample sizes. The one giving the closest estimate gets 3 points, the second one 2 points and the least accurate one 1 point. These individual points are then summed up across the sample sizes ranging from 100 to 1000 and probabilities p_{11}, p_{12}, p_{22}, p_{23}, p_{25}, p_{33} and p_{35}. This yields 8*7=56 "categories" of probabilities in which the points are rewarded. Finally, it should be noted that the probabilities are not totally independent. For example, in this case $p_{11} + p_{12} = 1$.

Distribution of these "correctness" points across the sample size space is shown in Figure 8. Summing up these points gives the following results: LHS method: 96 points, MC simulation: 104 points and LHS method when applied separately for each system degradation state:120 points.
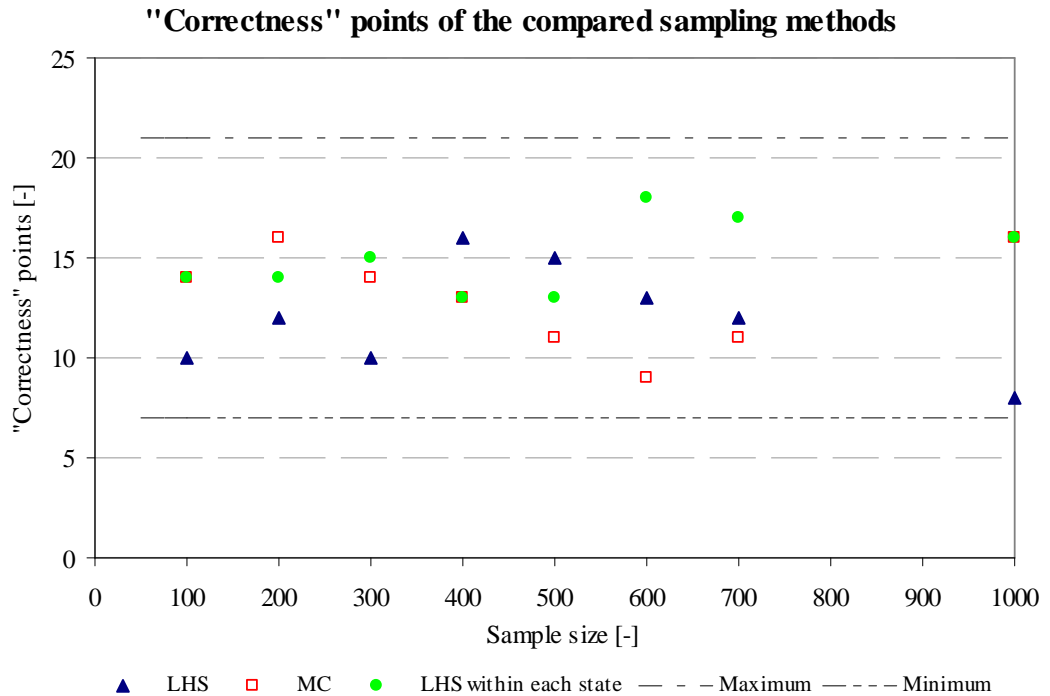

Figure 8.   *Obtained "correctness" points of the compared sampling methods as a function of sample size.*

7   Conclusions

The results of the case study indicate that the LHS method might not increase the efficiency of the sampling when performed in two dimensions and compared to the Monte Carlo simulation. When applied to each system degradation state at a time, some increase in the efficiency can be achieved. Based on the results, it is the most efficient of the three studied sampling methods. Since the number of the studied sample sizes was quite limited, and since sampling was performed only once with each method using each particular sample size, coincidence cannot be totally ruled out as a cause of these results.

Some of the assumptions made in the analyses do not comply exactly with the observations made from the real world. For example, based on the results documented by Brickstad /7/, there exists some correlation between the depth and the length of a crack. However this correlation is quite small and based on relatively small population data. Furthermore, including the correlation between the variables should not influence in the efficiency of the investigated sampling methods in relation to each other.

A more difficult problem is that of assessing small state transition probabilities. None of the studied methods seems well suited for the estimation of very small probabilities of p_{24} and p_{34}. Subset simulation, as proposed by Au and Beck /21/, might provide a solution for this problem and it points out one possible direction for further study.

Estimation of probabilities p_{44} and p_{45}, for which the corresponding areas A_{44} and A_{45} in Figure 3 are small compared to the whole area A ($A_1 + A_2 + A_3 + A_4$), does not succeed if sampling is performed without heavy weighting of this system degradation state number. For example, probability $p_{44} = 0.01529$ requires at least 1000 samples to fall within the crack state 4 in order to be estimated with some confidence. When the width of this system degradation state is 1 % of the total width of all the states, this would require at least 100 000 samples to be taken. Therefore heavy weighting should be applied for this system degradation state if properly accurate probabilities were to be achieved. Another option would be to assume that $p_{44} = 0$ and $p_{44} = 1$.

8 Summary

Assessment of the probability of failure constitutes one part of the risk informed in-service inspection methodology. To make things complicated, this probability is dependent on numerous factors. These factors are in turn determined by the degradation mechanisms to which the investigated component is susceptible to. In addition, they might well correlate with each other.

Since the question is about what can happen in future, most of these factors cannot be treated as deterministic. It can only be estimated how probable it is that these factors will have certain values. It is typical that the probability of failure to be estimated is very low. This raises a question about efficiency of different sampling methods that can be used when assigning values for the variables in governing equations of different degradation mechanisms. The choice of correct sampling method saves considerable amount of analysis time and effort.

The study discussed in this paper was limited to stress corrosion cracking degradation mechanism in welds of nuclear power plant (NPP) piping systems. Since the question is about future possibilities, i.e. probability of failure, probabilistic fracture mechanics (PFM) is the tool used in the study. In PFM analyses, different input variables in governing crack growth equations are assigned values on the basis of their assumed probabilistic distributions. One PFM analysis therefore consists of numerous deterministic analyses performed with different sets of sampled values for the input variables. The main aim of this work was to investigate and compare efficiency of different methods that can be used when sampling input values for PFM analyses.

The study was performed as a case study in which it was assumed that stress corrosion (SC) causes a circumferential, semi-elliptical crack to grow from the inner surface of a NPP pipe at the location of a weld. This choice allowed concentration to two variables while assuming deterministic values for the other variables. Depth of a crack and its length were chosen as parameters to be varied in the analyses. No correlation was assumed to exist between these two variables, which is at least a reasonable assumption.

Taken that the welding material is usually a known alloy of steel, deterministic values were assigned for the associated material parameters. This is justified because the variation of these parameters is quite small compared to variation of the other parameters.

The stress distribution which strives to increase the size of the crack was also taken as a given value. This is at least partly justified by the fact that welding residual stresses, which constitute the majority of the total stress distribution, are generally not known and neither is their probability distribution. Usually these welding residual stresses are taken into account by assigned them relatively conservative value, typically taken from some applicable fitness-for-service procedure. This was the approach followed in this study too.

In a continuum mechanics sense, growth of a crack is continuous. However, in this study it was treated also at more general level of discrete crack growth states, called also as system degradation states. When a crack grows, it eventually experiences transition from one system state into the following state; unless it is detected during inspections and repaired. During one year, a crack can, in theory, experience multiple transitions.

Usage of the crack state concept enables mathematical modelling of possible crack growth using Markov chain methodology. In this context, crack states become system states of the Markov chain. Usage of crack state concept also simplifies estimation of probability of detection of existing cracks in in-service inspections. In this case the probability of detection can be assigned for limited number of crack states instead of some continuously distributed parameter.

Depth of an existing crack is commonly used as a parameter which determines how probable it is to detect the crack in inspections. The probability of detection is simply divided into two categories of very unlikely to detect and possible to detect. The depth of a crack is also used as an indicator when deciding whether the cracked piping component is repaired right away or later and whether there exists a leak at the location of the crack. These were the factors that determine the number of crack states used in this study.

In this study, as a result of the PFM analysis, probability estimates are obtained for a crack, initially in one system state, to experience transition into one of the following system states during one time increment. As the number of samples used in the PFM analyses increases, the obtained estimates generally become increasingly accurate.

Three different ways to perform the sampling were studied. First, both Monte Carlo (MC) simulation and Latin Hypercube Sampling (LHS) method were performed so that the space of possible values for each variable was considered as a whole entity. Additionally, the LHS procedure was also performed separately for each crack state.

It was detected that the LHS method did not consistently increase accuracy of the estimates for the state transition probabilities when compared to the ones obtained using MC method. This was contrary to expectations. However, the LHS method, when performed separately for each system degradation state, did yield somewhat better results. Due to limited number of samples, this can of course be coincidence. For estimation of small state transition probabilities, none of the investigated methods yielded satisfying results when using a reasonably large sample size.

References

1. Gosselin, S.R. EPRI's new in-service inspection program. Nuclear News, November 1997. pp 42-46. ISSN 0029-5574.

2. Simola, K., Holmberg, J-E., Cronvall, O., Männistö, I., Vepsä, A. RI-ISI pilot study of the Shut-down cooling system of the Olkiluoto1/2 NPP units, 2005, VTT Industrial Systems, Espoo, 44 p. + App. 23 p. Research report BTUO72-051318.

3. Cronvall, O., Männistö, I., Holmberg, J-E., Pulkkinen, U. Continuation of the RI-ISI pilot study of the Shut-down cooling system of the Olkiluoto 1/2 NPP units. 2006, VTT Industrial Systems, Espoo, 60 p., Research report TUO72-056667.

4. Cronvall, O., Männistö, I., Simola, K. Development and Testing of VTT approach to risk-informed in-service inspection methodology. Final report of SAFIR INTELI INPUT Project RI-ISI, 2007, VTT, Espoo, 43 p. VTT Tiedotteita – Research Notes: 2382. ISBN 978-951-38-6914-4; ISBN 978-951-38-6915-1. http://www.vtt.fi/inf/pdf/tiedotteet/2007/T2382.pdf

5. Jansson, C., Morin, U. Assessment of Crack Growth Rates in Austenitic Stainless Steels in Operating BWRs. Proc. of Eight International Symposium on Environmental Degradation of Materials in Nuclear Power System – Water Reactors. August 10 – 14, 1997. Amelia Island, Florida. Pp 667-674.

6. Cronvall, O., Simola, K., Vepsä, A., Alhainen, J. RI-ISI pilot study of the Auxiliary feed water system of the Olkiluoto OL1 NPP unit. 2008, VTT, Espoo, 48 p. +App. 13p.

7. Brickstad, B. The Use of Risk Based Methods for Establishing ISI-Priorities for Piping Components at Oskarshamn 1 Nuclear Power Station. SAQFoU-Report 99/05, SAQ Kontroll AB, 1999. 83p.

8. Hazelton, W., S., Koo, W., H. Technical Report on Material Selection and Processing Guidelines for BWR Coolant Pressure Boundary Piping - Final Report. Report NUREG-0313, Rev. 2-F. U.S. Nuclear Regulatory Commission, January 1988. 38+9 p.

9. Irwin, G.R., Analysis of Stresses and Strains near the End of a Crack Traversing a Plate. Journal of Applied Mechanics, Vol. 24, 1957, pp. 361–364.

10. Varfolomeyev, I. et al. BESIF 1.0: Stress Intensity Factors for Surface Cracks under 2D Stress Gradients. IWM-Report T 14/96, Fraunhofer-Institut für Werkstoffmechanik (IWM), July 1996. 42 p.

11. Busch, M. et al. KI-Factors and Polynomial Influence Functions for Axial and Circumferential Surface Cracks in Cylinders. IWM-Report T 18/94, Fraunhofer-Institut für Werkstoffmechanik (IWM), October 1994. 41 p.

12. Busch, M. et al. Polynomial Influence Functions for Surface Cracks in Pressure Vessel Components. IWM-Report Z 11/95, Fraunhofer-Institut für Werkstoffmechanik (IWM), January 1995. 88 p.

13. American Petroleum Institute (API), Recommended practice for fitness-for-service. API 579. Washington DC, American Petroleum Institute, 2000.

14. Inman, R. L. 1999 Latin Hypercube sampling. Encyclopedia of statistical sciences, Update volume 3, Wiley, NY pp. 408-411.

15. Conover, W. J. 1975. On a better method of selecting values of input variables for computer codes. Unpublished manuscript.

16. McKay, M. D., Conover, W. J. and Beckman, R. J. A comparison of three methods for electing values of input variables in the analysis of output from a computer code. Technometrics 1979, Vol. 21. Iss. 2. pp. 239-245.

17. Olsson, A., Sandberg, G. and Dahlblom, O. On Latin hypercube sampling for structural reliability analysis. Structural safety, Elsevier Science, 2003,Vol. 25, pp 47 – 68, ISSN 0167-4730.

18. Inman, R. L. and Conover, W. J. A distribution free approach to inducing rank correlations among input variables. Communications in statistics, Part B – Simulation and computation, 1982, Vol. 11, pp. 311-334.

19. Hardyanto, W., Groundwater Modelling Taking into Account Probabilistic uncertainties. 2003, Freiberg On-Line Geosciences Vol. 10, ISSN 1434-7512. 117 p.

20. ABB ATOM Materialhandboken – Normer och datablad för metalliska material. ABB Atom AB, 1999.

21. Au, Siu-Kui & Beck, James L. Estimation of small failure probabilities in high dimensions by subset simulataion. Probabilistic Engineering Mechanics, 2001, Vol.16, pp. 263-277.