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INHOMOGENEITY SCREENING CRITERION FOR THE ASTM E1921 T_0 ESTIMATE BASED ON THE SINTAP LOWER-TAIL METHODOLOGY

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Abstract

The Master Curve brittle fracture toughness estimation method described in the ASTM E1921-11 test standard is based on a theoretical scatter and size effect assumption and makes use of a maximum likelihood estimation method to determine the fracture toughness transition temperature T_0. The estimation method in E1921-11 is valid only for macroscopically homogeneous steels. If the steel is inhomogeneous, the maximum likelihood method applied in E1921-11 becomes unreliable. Here, a simple screening criterion, based on the SINTAP lower-tail estimation method, is proposed and the efficiency and limitations of the criterion is shown for a variety of different types of inhomogeneity.

Keywords

Master Curve, Inhomogeneity, Screening criteria, SINTAP lower-tail method.

Nomenclature

BM	Bimodal inhomogeneity.
$f(T_{0i})$	Distribution function of T_0 values for multimodal inhomogeneity.
K_0	Normalising fracture toughness corresponding to 63.2 % cumulative probability.
K_{CENS}	Censoring fracture toughness.
K_{JC}	Brittle fracture toughness as defined in ASTM E1921-11.
K_{min}	Lower limiting fracture toughness fixed as 20 MPa√m in ASTM E1921-11.
MC	Master Curve.
MM	Multimodal or random inhomogeneity.
MML	Maximum likelihood method.
MOT	Minimum of three equivalent estimate.
n	Total number of data.
p_a	Portion of more brittle constituent in a bimodal distribution.
P_f	Cumulative probability.
P_{false}	Probability of incorrect screening result.
$P_{false+unconservative}$	Probability of incorrect screening result with significant unconservative error.
r	Number of uncensored data according to ASTM E1921-11.
S	Survival probability $=1-P_f$
T	Test temperature.
T_0	Master Curve transition temperature corresponding to median fracture toughness 100 MPa√m for 25 mm specimen thickness.
T_{0a}	T_0 of constituent a, for multimodal inhomogeneity.
T_{0ave}	Average T_0 value for multimodal inhomogeneity.

T_{0b}	T_0 of constituent b, for multimodal inhomogeneity.
T_{0eff}	Effective T_0 value for inhomogeneous material.
$T_{0eff20\%}$	T_{0eff} corresponding to 20 % fractile.
$T_{0eff5\%}$	T_{0eff} corresponding to 5 % fractile.
T_{0E1921}	T_0 according to standard ASTM E1921-11.
T_{0i}	Individual T_0 value for multimodal inhomogeneity.
$T_{0(max)}$	Maximum T_0 value for single point estimates used in SINTAP step 3.
T_{0ref}	Reference T_0 value for inhomogeneous material.
$T_{0sintap}$	Final T_0 estimate for SINTAP method.
T_{0step1}	T_0 estimate for SINTAP step 1.
T_{0step2}	T_0 estimate for SINTAP step 2.
β	ASTM E1921-11 sample size uncertainty factor.
δ_i	Censoring parameter.
ΔT_0	ASTM E1921-11 margin adjustment for T_0.
ΔT_{0ab}	Temperature difference between bimodal components T_{0a} and T_{0b}.
σ_{exp}	Measure of experimental uncertainties in ASTM E1921-11.
σT_0	Standard deviation of multimodal inhomogeneity.

Introduction

The Master Curve (MC) brittle fracture toughness estimation method described in the ASTM E1921-11 test standard [1] is based on a theoretical scatter and size effect assumption and makes use of a maximum likelihood estimation method to determine the fracture toughness transition temperature T_0. Since the method is described in the E1921-11 standard and has been described numerous times by various authors, the method is not described here in any more detail. The required information is available in the standard or can be read in more detailed form in e.g. [2-4].

The estimation method in E1921-11 is applicable only for macroscopically homogeneous steels. If the steel is inhomogeneous, the maximum likelihood method applied in E1921-11 becomes inaccurate. For such materials inhomogeneous Master Curve analysis methods have been developed [5]. Specifically two different types of inhomogeneities have been addressed: bimodal (BM) and multimodal (MM). The bimodal inhomogeneity analysis method focuses on materials with clearly two separate fracture toughness distributions. Typically, non-heat-treated weld heat affected zones, with a narrow clearly more brittle region, show a bimodal inhomogeneity [5]. It should be noted that a non-heat treated weld has many different microstructures over a small region. Each of these microstructures could have its own characteristic toughness and therefore, the bimodal assumption may not automatically be appropriate. The multimodal (or random) inhomogeneity analysis [5] describes better the possible normal variation of the macroscopic toughness properties of base materials, castings and forgings.

The bimodal Master Curve distribution is defined by Eq. (1)

$$P_f = 1 - p_a \cdot \exp\left\{-\left(\frac{K_{JC} - K_{min}}{K_{0a} - K_{min}}\right)^4\right\} - (1 - p_a) \cdot \exp\left\{-\left(\frac{K_{JC} - K_{min}}{K_{0b} - K_{min}}\right)^4\right\} \tag{1}$$

K_{0a} refers to the more brittle constituent having a probability p_a and K_{0b} refers to the tougher constituent having a probability $1-p_a$. K_{0a} and K_{0b} are connected to their respective transition temperatures T_{0a} and T_{0b} through Eq. (2).

$$K_{0x} = 31 + 77 \cdot \exp\left\{0.019 \cdot (T - T_{0x})\right\} \quad \left[MPa\sqrt{m}, °C\right] \tag{2}$$

The multimodal distribution is basically identical to the standard Master Curve distribution. However, the transition temperature is not a constant, but varies for individual fracture toughness values. The probability distribution function for an individual T_{0i} is in this case defined by Eq. (3).

$$f(T_{0i}) = \frac{e^{-\frac{(T_{0i} - T_{0ave})^2}{2 \cdot \sigma T_0^2}}}{\sigma T_0 \cdot \sqrt{2\pi}} \tag{3}$$

The use of the inhomogeneity analysis methods require, however, a minimum of 20 to 30 test results [5, 6], whereas the standard assessment only requires between 6…9 test results to provide a valid T_0 estimate. This raises the problem of how to decide whether a material is homogeneous or heterogeneous. A simple solution would be to assume that the material is always inhomogeneous and to perform a sufficiently large number of tests. This would require the use of much material and would make the testing clearly more expensive. Most

importantly, it would unduly penalise homogeneous materials. A solution for this problem would be the use of a simple inhomogeneity screening criterion to decide if the material is homogeneous or inhomogeneous.

The screening criterion should be such that the probability of falsely recognising a homogeneous material as inhomogeneous is sufficiently small. The criterion should also be able to recognise materials with a significant inhomogeneity with a high probability. At the same time, the probability that a T_0 value resulting from an inhomogeneous material, falsely recognised as homogeneous, is not significantly un-conservative with respect to a T_0 value that would be descriptive of the material. This raises the question about which T_0 value is descriptive of an inhomogeneous material.

A multimodal material is described by the average T_0 (T_{0ave}) and the standard deviation of T_0 values in the material (σT_0). A reproduction of T_{0ave} would be descriptive of the average behaviour of the materials toughness, but this value is seldom used in a structural integrity assessment. Generally, a fracture toughness estimate corresponding to some cumulative probability level is used. For less critical structures, commonly a so-called "minimum of three equivalent probability level" is used. The use of this is mainly historical and stems from when assessments were based on the lowest fracture toughness value out of three tests. This probability level corresponds approximately to a 20th percentile [7]. More critical structures apply usually a more stringent 5th percentile fracture toughness value. If the descriptive T_{0eff} value for a material is defined as the T_0 value of a homogeneous material that describes the desired fracture toughness of the inhomogeneous material with the same probability level (e.g. 20 % or 5 %), it is possible to develop a consistent description of homogeneous and inhomogeneous materials. Figure 1 show as an example a case where the material has a

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multimodal inhomogeneity with a 20°C standard deviation. Figure 1 was developed by estimating the total failure probability for a specific K_{JC} and T with Eq. (4) [7]. From this, percentiles could be determined as a function of temperature.

$$P_{tot}\left(K_{JC},T\right)=1-\int_{T_{0\min}}^{\infty} f\left(T_{0}\right)\cdot S_{T}\left(K_{JC},T_{0}\right)\cdot dT_{0} \tag{4}$$

The figure shows that if the 20th percentile fracture toughness is described by a homogeneous material, the effective $T_{0eff20\%}$ is 8°C higher than T_{0ave}. For a 5th percentile the effective $T_{0eff5\%}$ is 12°C higher than T_{0ave}. Table 1, which was developed in the same way as Figure 1, shows the relation between T_{0eff} and different inhomogeneities. In this table T_{0ref} identifies the temperature that T_{0eff} is related against. T_{0b} refers thus to the more ductile component in a bimodal distribution and p_a to the portion of the more brittle component in the bimodal distribution. T_{0b} is chosen as reference because it represents the value of the "homogeneous" (more common) material. Also, T_{0a} is overconservative with respect to the effective T_0 and is therefore not well suited as reference. ΔT_{0ab} gives the temperature difference between the bimodal components T_{0a} and T_{0b}. T_{0ave} and σT_0 are the two parameters describing the multimodal distribution. If the difference between T_{0eff} and T_{0ref} is less than 10°C the inhomogeneity is not to be considered significant, since the difference is of the same order as the uncertainty in the T_0 estimate for a homogeneous material. Table 1 shows also the relation between the ASTM E1921-11 standard T_0 value (T_{0E1921}) and T_{0ref}. For bimodal materials the standard T_0 value is always close to the more ductile constituent, when its content is 50 % or more of the combined material. However, the T_0 is always positioned between the two component T_0 values. For a multimodal material, the standard T_0 estimate is un-conservatively biased with respect to T_{0ave}.

The SINTAP Lower-Tail Analysis Method [8]

The SINTAP method is intended for the analysis of small data sets, where the uncertainty related to the data set size becomes an important factor. It is intended to give representative lower bound estimates suitable for structural integrity analysis purposes. Thus, the SINTAP method is not intended to be used e.g. to determine transition temperature shifts or in other cases where the average fracture toughness is of interest. For a homogeneous material, the SINTAP method provides on the average a 10% lower fracture toughness estimate than the standard Master Curve [7]. For inhomogenous data sets, the difference is larger, because the inhomogeneity causes the standard estimate to be biased as seen from Table 1.

The SINTAP lower-tail analysis contains three steps, leading to a T_0 value denoted $T_{0sintap}$. Step 1, which is equivalent to the standard E1921 analysis, gives an estimate of the median value of fracture toughness assuming homogeneous material behaviour. Step 2 performs a lower-tail MML estimation, checking and adjusting for any undue influence of excessive values in the upper-tail of the distribution. Step 3 performs a minimum value estimation to check and make allowance for gross inhomogeneities in the material. In step 3, effectively, an additional safety factor is incorporated for cases where the number of tests is small. It is recommended in SINTAP [8] that all three steps are employed when the number of tests to be analysed is between 3 and 9. With an increasing number of tests, the influence of the additional safety factor for small data sets is gradually reduced. For 10 and more tests, SINTAP requires only steps 1 and 2 to be used. However, step 3 may still be employed for indicative purposes, especially when there is evidence of gross inhomogeneity in the material (e.g. for weld or heat affected zone material). In such cases, it may be judged that the

characteristic value is based upon the step 3 result, or alternatively, such a result may be used as guidance in a sensitivity analysis or used to indicate the need for more experimental data, when appropriate.

The preliminary tasks including data censoring and specimen size adjustment are performed according to the standard MC analysis in accordance with ASTM E 1921-11. Step 1 consists of a standard MC determination of K_0 or T_0 (T_{0step1}), shown in Figure 2. Step 2 performs a lower-tail estimation in the following way, shown in Figure 3:

(a) Censor all data whose toughness K_{JC} exceeds a K_{CENS} value given by Eq. (5) to be equal to the K_{CENS} value given by Eq.(5), setting δ_i for the censored data equal to 0 and δ_i for all other data fulfilling the size criterion equal to 1.

$$K_{CENS}(T_0) = 30 \text{ MPa}\sqrt{m} + 70 \text{ MPa}\sqrt{m} \cdot \exp\left\{0.019/°C \cdot (T - T_0)\right\} \qquad (5)$$

(b) Use this "upper-tail" censored data set to obtain a new estimate of T_0 (T_{0step2}) by performing a standard E1921 analysis.

(c) Compare the T_0 values from steps 1 and 2. If the new T_0 is greater than the previous T_0, repeat the upper-tail censoring, using the new value as a benchmark. Continue the iteration until a constant or maximum value of T_{0step2} is obtained.

If the number of specimens in the data set is less than 10, perform step 3 (minimum value) estimation. This is done as follows (Figure 4):

a) Calculate the maximum value of T_0 (based on a single data point), $T_{0(max)}$, using all non-censored data, i.e. where $\delta_i = 1$, using Eq. (6).

$$T_{0(max)} = \max\left| T_i - \frac{\ln\left\{\dfrac{\left(K_{JC_i} - 20\,MPa\sqrt{m}\right)\cdot\left(\dfrac{n}{\ln 2}\right)^{1/4} - 11\,MPa\sqrt{m}}{77\,MPa\sqrt{m}}\right\}}{0.019} \right| \qquad (\delta_i = 1) \qquad (6)$$

Note that T_i is the test temperature of a specimen of toughness K_{JCi} and n is the total number of test results in the data set. Eq. (6) is simply a size adjusted Master Curve estimate corresponding to median fracture toughness (confidence level 0.5), where the effective size is defined by n. The constant (ln2) simply adjusts for the difference between the median K_{JC} and K_0.

(b) Compare $T_{0(max)}$ and T_0 from step 2. If $T_{0(max)} - 8°C > T_{0(step2)}$, there is indication that the data is inhomogeneous and $T_{0(max)}$ should be taken as the representative value of T_0. The value of 8°C, corresponds approximately to a 10 % difference in K_0, which reflects the bias when using the SINTAP procedure for a homogeneous material.

Figure 5 shows the difference between the standard T_0 value and the effective value describing a 20th percentile fracture toughness, taken from Table 1, as a function of the difference between the SINTAP step 1 and step 2 estimates for various types of inhomogeneities. For bimodal inhomogeneities where the ΔT_{0ab} difference is of the order of 20°C (±10°C) or less and multimodal inhomogeneities where σT_0 is 10°C or less, the difference between the standard T_{0E1921} value and $T_{0eff20\%}$ is less than 10°C. The same is the

case, if the amount of bimodal inhomogeneity (p_a) is of the order of 10 % or less, regardless of ΔT_{0ab}. Again, since a 10°C difference is of the same order as the uncertainty in the T_0 estimate for a homogeneous material, these types of inhomogeneities are thus insignificant for non-critical applications of the standard T_0 value. For larger inhomogeneities, the standard T_0 estimate becomes clearly unconservative. The SINTAP lower tail analysis was developed to safeguard against such situations.

Figure 6 shows as comparison the difference between the SINTAP step 2 T_0 value and $T_{0eff20\%}$, as a function of the difference between the SINTAP step 1 and step 2 estimates for various types of inhomogeneities. In this case, for a multimodal inhomogeneity, the σT_0 must be clearly larger than 30°C before the SINTAP estimate becomes significantly unconservative. For bimodal inhomogeneities the ΔT_{0ab} difference must be 40°C (±20°C) or more and the amount of brittle constituent must be close to 25 %, before the SINTAP estimate becomes significantly unconservative. If the amount of brittle constituent is close to 50 % or more, or clearly less than 25 %, the SINTAP estimate describes well $T_{0eff20\%}$ (less than 10°C unconservatism). It should be emphasized that this is exactly the application that the SINTAP lower tail methodology was developed for. Being a general purpose structural integrity procedure, SINTAP targeted the same probability level as the conventional minimum of three (MOT) estimate. The picture changes a little when the more critical 5th percentile is used for the comparison.

Figure 7 shows the same comparison as in Figure 5, but this time the effective T_0 corresponding to a 5th percentile ($T_{0eff5\%}$) is used. The trend is mainly similar as in Figure 5, but for $T_{0eff5\%}$ even small amounts of bimodal inhomogeneity ($p_a \approx 10$ %) can cause a large error if ΔT_{0ab} is clearly larger than 20°C.

Figure 8 shows the same comparison as in Figure 6, but this time using $T_{0eff5\%}$. Except for clearly inhomogeneous materials ($\sigma T_0 \gg 20°C$ and $\Delta T_{0ab} > 40°C$ with $p_a < 25$ %), the SINTAP step 2 estimate provides a satisfactory description of $T_{0eff5\%}$. Satisfactory means that the error is not larger than 10°C. However, in the case of severe inhomogeneities with a small p_a, even the SINTAP estimate may be severely unconservative. Thus, a simple use of the SINTAP lower tail estimation methodology does not, per se, guarantee a sufficiently reliable T_0 estimate. The SINTAP estimate needs to be combined with an inhomogeneity screening criterion. This is described next.

Inhomogeneity Screening Criterion

The inhomogeneity screening criterion is based on a comparison of the difference between the SINTAP step 2 T_0 and the standard ASTM E1921-11 T_0 (or SINTAP step 1).

ASTM E1921-11 contains an expression for margin adjustment of T_0 accounting for the uncertainty in T_0 that is associated with the use of only a few specimens to establish T_0. The margin expression for an 85 % two-tail confidence has the form of Eq. (7) [1].

$$\Delta T_0 = \sigma\left(Z_{85}\right) = 1.44 \cdot \sqrt{\frac{\beta^2}{r} + \sigma_{exp}^2} \tag{7}$$

β is the sample size uncertainty factor defined in ASTM E1921-11 and σ_{exp} is the contribution of experimental uncertainties. For the use as a screening criterion, where the same data set is undergoing different analyses, σ_{exp} can be disregarded.

The screening criterion becomes thus simply as Eq. (8).

$$T_{0step2} - T_{0step1} \leq 1.44 \cdot \sqrt{\frac{\beta^2}{r}} \Rightarrow \text{homogenous}$$

$$T_{0step2} - T_{0step1} > 1.44 \cdot \sqrt{\frac{\beta^2}{r}} \Rightarrow \text{inhomogenous}$$

(8)

It should be emphasized that r refers to the number of uncensored data in the SINTAP step 1 analysis, not what is left after step 2 censoring.

The screening criterion was tested on different types of inhomogeneities, using Monte Carlo simulation. This consisted of defining different distributions with varying amounts of inhomogeneity and randomly generating virtual fracture toughness values from them. Nine evenly spaced temperatures covering $\pm$ 40°C from T_{0ave} or from $(T_{0a}+T_{0b})/2$ were used. Two different realistic data set sizes were examined, n = 9 and n = 18, so that the smaller set had one value per temperature and the larges set had two. The smaller data set was selected because it has a realistic size and is the largest data set, still making use of step 3 in the SINTAP method. The larger data set represents a size that is realistic, if some inhomogeneity in the material is expected. The generation of one fracture toughness value consisted of two random number generations. First, the fracture toughness specific T_0 value was generated assuming either bimodal or multimodal behaviour. Second, using this T_0 value a corresponding K_{JC} value was generated using the Master Curve.

Figure 9 shows the probability of a false screening result. P_{false} was determined by the number of simulations failing Eq. (8) divided by the number of simulations in each case. P_{false} for the

homogeneous case gives the probability that homogeneous material is falsely judged to be inhomogeneous and vice versa. The probability that a homogeneous material is erroneously assumed to be inhomogeneous is only approximately 5 %, regardless of data set size. For significant inhomogeneities or larger data sets, the probability of assuming an inhomogeneous data set to be homogeneous is close to 10 % or less. An inhomogeneity of σT_0 or ΔT_{0ab} ~ 20°C or greater is difficult to catch with a small data set. Also, with decreasing amount of brittle constituent in a bimodal inhomogeneity case the screening becomes unreliable. The main reason for this is that the probability that the inhomogeneity is present in the data set becomes small. In the case that $p_a = 25$ % and $n = 9$, there is a 7.5 % chance that the data set does not contain a single value corresponding to the more brittle constituent. For a $p_a = 10$ % the chance is as high as 39 % making any sensible screening impossible. In this case, the inhomogeneity behaves basically as an outlier and regardless of magnitude the inhomogeneity cannot be recognised from small data sets.

Even more important than the efficiency of the screening criterion to recognise an inhomogeneous data set correctly, is the probability of obtaining an unconservative T_0 estimate in the case of a false screening result. This was examined by determining the proportion of estimates, being both false and having an unconservative error of more than 10°C, in relation to $T_{0eff5\%}$. $P_{false-unconservative}$ represents thus the number of simulations failing Eq. (8) and having an unconservative error of more than 10°C with respect to T_0, divided by the number of simulations in each case. The T_0 estimate is estimated according to SINTAP step 2 or step 3 (for $n \leq 9$). Figure 10 shows a compilation of the different inhomogeneities. For all multimodal and bimodal distributions with p_a ~ 50 %, the likelihood of a false result with a significantly unconservative T_0 value is on the average only of the order of 5 %, regardless of the magnitude of the inhomogeneity. For smaller amounts of brittle constituent

in the bimodal inhomogeneity case the success becomes more unreliable. Since the analysis has been made with respect to the error in relation to $T_{0eff5\%}$, it is independent of the choice of reference temperature (T_{0a} or T_{0b}).

Discussion

Some level of material variations are generally unavoidable in industrially produced material quantities. Often, the magnitude of the variations is not structurally significant. The variations are also often deterministic in nature. Such deterministic variations are e.g. the toughness gradient as a function of thickness location in thick section plates and forgings or the centreline segregation typical for thinner plates. These kinds of material variations are easily handled by proper test specimen sampling, so that the sample is limited to a specific material region. Sometimes, it is not possible to limit the sample to a specific material location so that all specimens represent identical material. The assessment of deterministic material variation is discussed in more detail in [5]. A more difficult type of material variability is connected to local random inhomogeneities that cannot be avoided by specimen sampling. To safeguard against this type of material variability, some kind of screening criterion is needed for ASTM E1921-11.

One problem is connected to the MML fitting used in ASTM E1921-11. Even though it produces excellent results for a homogeneous material, it has the tendency to strongly emphasize the tougher material for an inhomogeneous material. One possibility would be to use a simple mean estimate of T_0 in line with Eq. (9). Reference [5] gives an example of the use of Eq. (9) in the assessment of a multimodal data set. The problem here is, that Eq. (8)

does not allow for censoring, i.e. all test results must fulfil the validity criterion in the standard.

$$\bar{T}_0 = \frac{\sum_{i=1}^{n}\left\{T_i - \frac{\ln\left(\frac{K_{JC_i} - 30MPa\sqrt{m}}{70MPa\sqrt{m}}\right)}{0.019°C^{-1}}\right\}}{n} \tag{9}$$

When applicable, Eq. (9) can be used as an additional screening aid, since it represents an estimate of the mean T_0 value. If the result of Eq. (9) differs significantly from the standard T_0 estimate this is an additional indication that the material is inhomogeneous.

The SINTAP lower-tail method, proposed here, does penalise a homogeneous material slightly. To study this, the cumulative distribution of $T_{0sintap}$ estimates from the Monte Carlo simulation was compared with $T_{0eff5\%}$ (For a homogeneous material $T_{0eff5\%}$ is equal to T_0). The cumulative distributions are shown in Figures 11 and 12 for the small and large data sets. In Figure 11, $T_{0sintap}$ is the product of all three SINTAP steps, whereas in Figure 12, $T_{0sintap}$ is the product of SINTAP steps 1 and 2. For small data sets (e.g. n = 9), SINTAP introduces, for a homogeneous material, an average bias of about 11°C (corresponding to 50 % cumulative probability), compared to the standard expression bias of 2…4°C (Figure 11). For larger data sets (e.g. n = 18), the same bias decreases to 2°C (Figure 12). The strength of the SINTAP lower-tail method lies in that it rewords increased testing, but does not unduly penalise the use of smaller data sets. The most likely type of non-deterministic material variability connected to welds, plates and forgings is the multimodal in nature. This type of inhomogeneity is well handled by the SINTAP lower-tail method inhomogeneity screening criterion. Most problematic are bimodal inhomogeneities, containing only a small portion of brittle

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constituent. This type of materials can never be reliably assessed with small data sets, since the inhomogeneities act as outliers.

Summary and Conclusions

The SINTAP lower-tail methodology has been used to develop a simple inhomogeneity screening criterion for the Master Curve T_0 transition temperature.

The screening criterion can be expressed simply in the form of Eq. (8), where T_{0step1} and T_{0step1} refer to the step 1 and step 2 T_0 estimates in the SINTAP procedure, β is the sample size uncertainty factor defined in ASTM E1921-11 and r is the number of uncensored data in the SINTAP step 1 analysis.

$$T_{0step2} - T_{0step1} \leq 1.44 \cdot \sqrt{\frac{\beta^2}{r}} \Rightarrow \text{homogenous}$$

$$T_{0step2} - T_{0step1} > 1.44 \cdot \sqrt{\frac{\beta^2}{r}} \Rightarrow \text{inhomogenous}$$

(8)

The screening criterion works well for multimodal inhomogeneities and bimodal inhomogeneities with close to equal amounts of ductile and brittle constituents. When combined with the SINTAP T_0 estimate, the probability of falsely judging an inhomogeneous material as homogenous and making more than a 10°C error in the descriptive T_0 value is only approximately 5 %. The probability of falsely judging a homogeneous material as being inhomogeneous is also only approximately 5 %. Bimodal inhomogeneities, containing only a small portion of brittle constituent can never be reliably assessed with small data sets, since the inhomogeneities act as outliers. For such materials the screening criterion is ineffective.

Acknowledgements

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Table 1. Relation between T_{0eff} for homogenous material and different percentiles of inhomogeneous materials. The standard T_0 value is denoted here T_{01921} for clarity.

Type	ΔT_{0ab} °C	p_a	σT_0 °C	T_{0ref}	$T_{01921}-T_{0ref}$ °C	$T_{0eff20\%}-T_{0ref}$ °C	$T_{0eff5\%}-T_{0ref}$ °C
Homog.	0	0	0	T_0	0-1	0	0
BM	20	0.5		T_{0b}	7	12	13
BM	20	0.25		T_{0b}	3	7	8
BM	20	0.1		T_{0b}	1	3	3
BM	40	0.5		T_{0b}	9	29	30
BM	40	0.25		T_{0b}	3	18	19
BM	40	0.1		T_{0b}	1	7	11
BM	60	0.5		T_{0b}	11	47	49
BM	60	0.25		T_{0b}	5	32	38
BM	60	0.1		T_{0b}	1	9	24
MM			10	T_{0ave}	-4	4	5
MM			20	T_{0ave}	-15	8	12
MM			30	T_{0ave}	-25	14	20
MM			40	T_{0ave}	-37	21	30
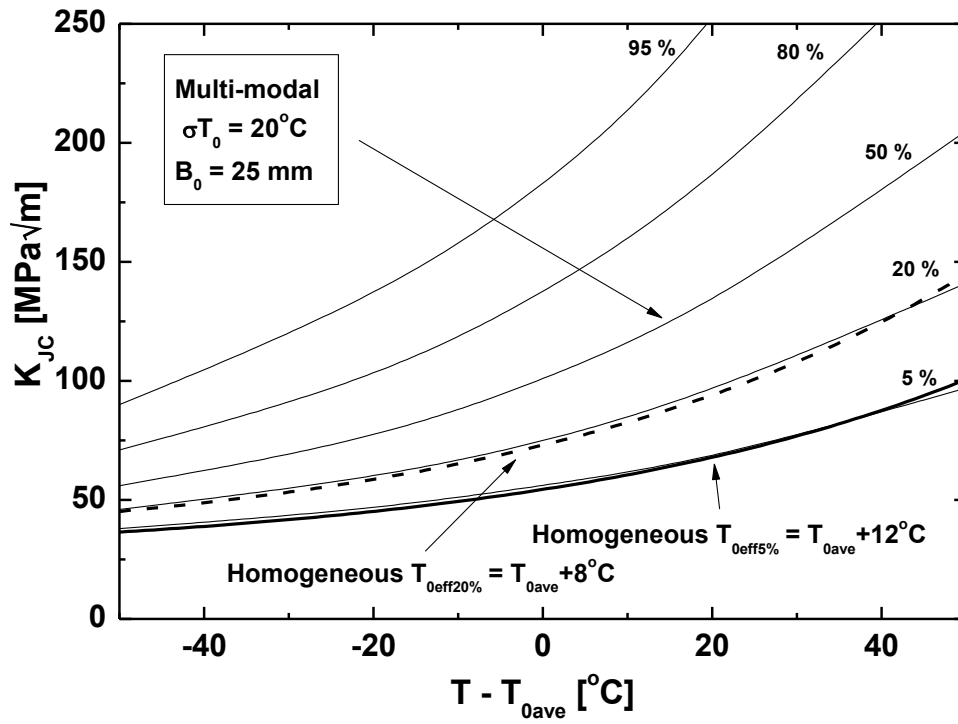
Figure 1. Definition of the effective descriptive homogeneous T_{0eff} values with respect to 5th and 20th percentiles for a multimodal inhomogeneous material.
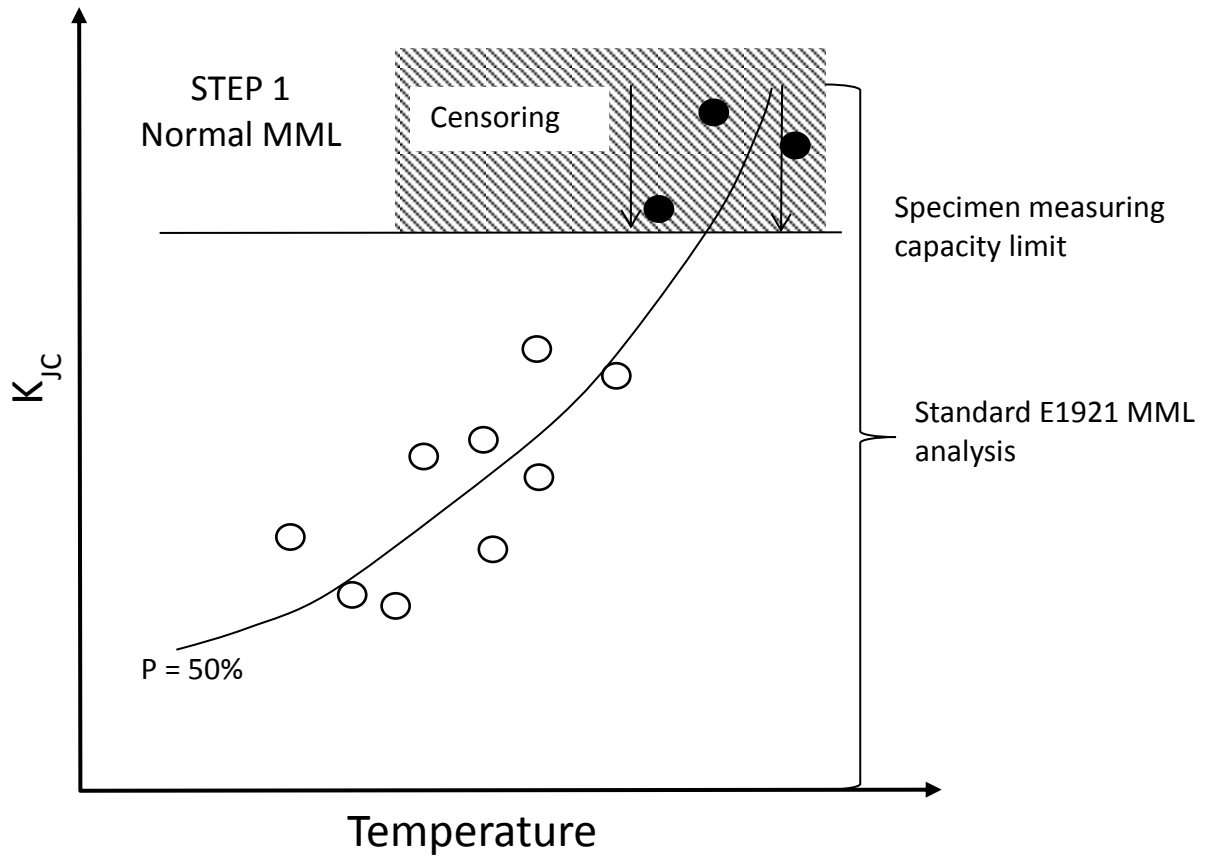
Figure 2. Principle of the SINTAP step 1 analysis (standard Master Curve estimation).
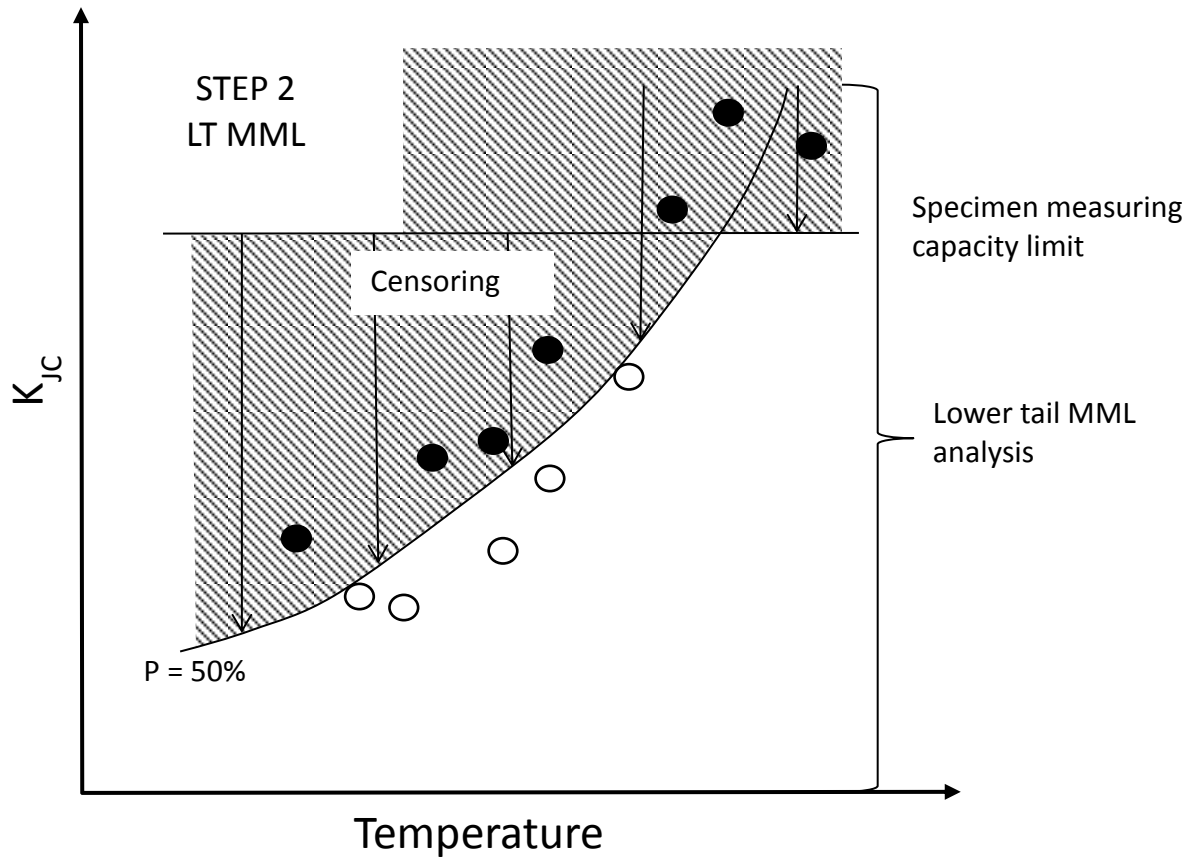
Figure 3. Principle of the SINTAP step 2 analysis (lower-tail estimation).
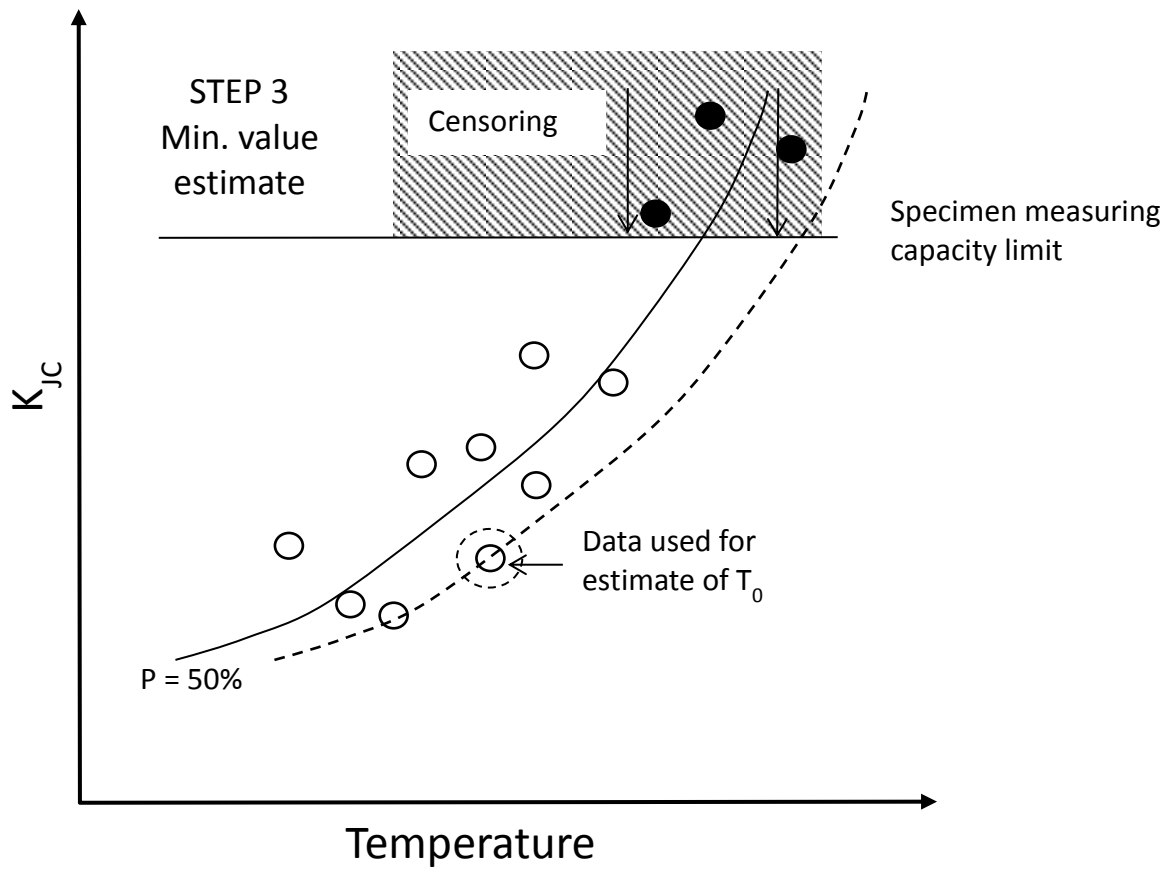
Figure 4. Principle of the SINTAP step 3 analysis (minimum value estimation).
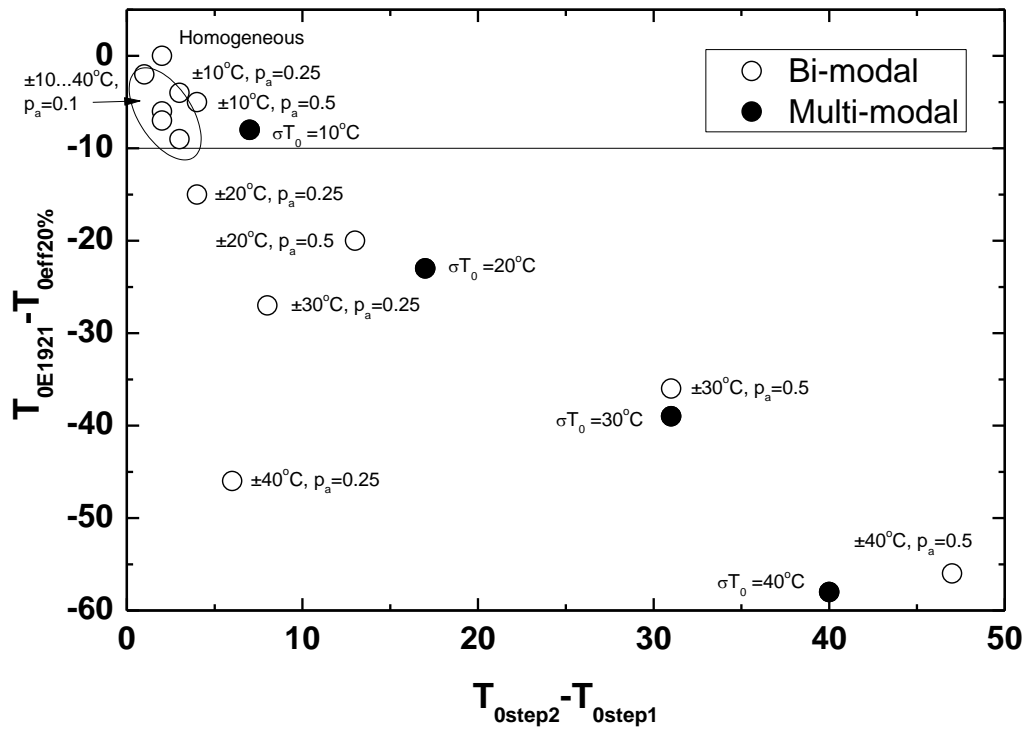
Figure 5. Difference between standard T_{0E1921} and effective value describing 20th percentile toughness, $T_{0eff20\%}$, for different types of inhomogeneities.
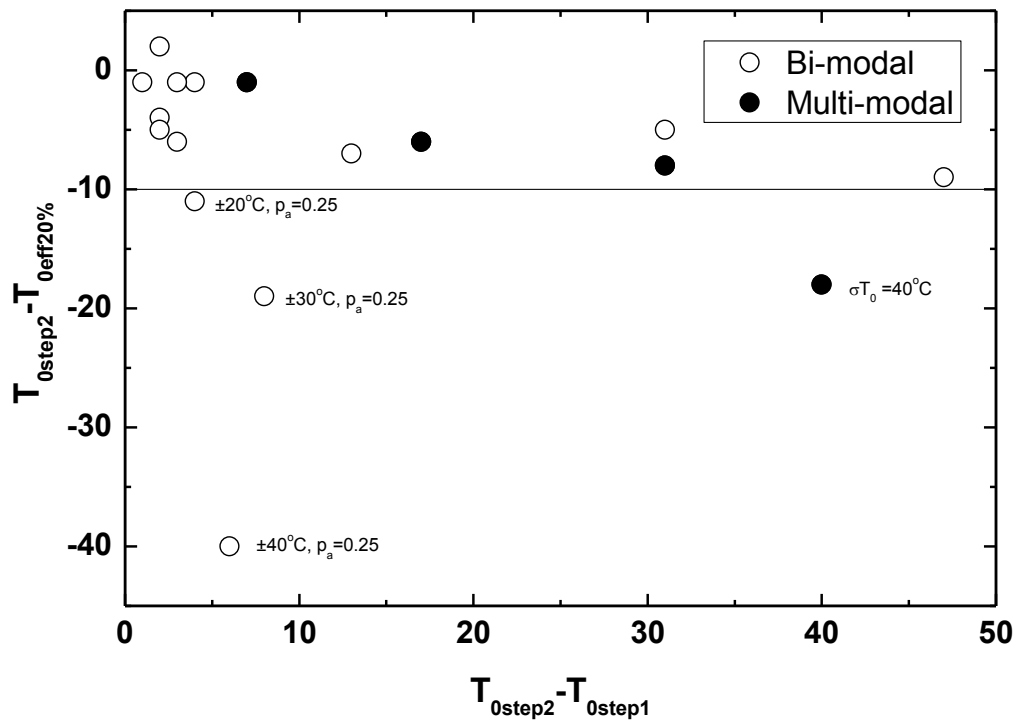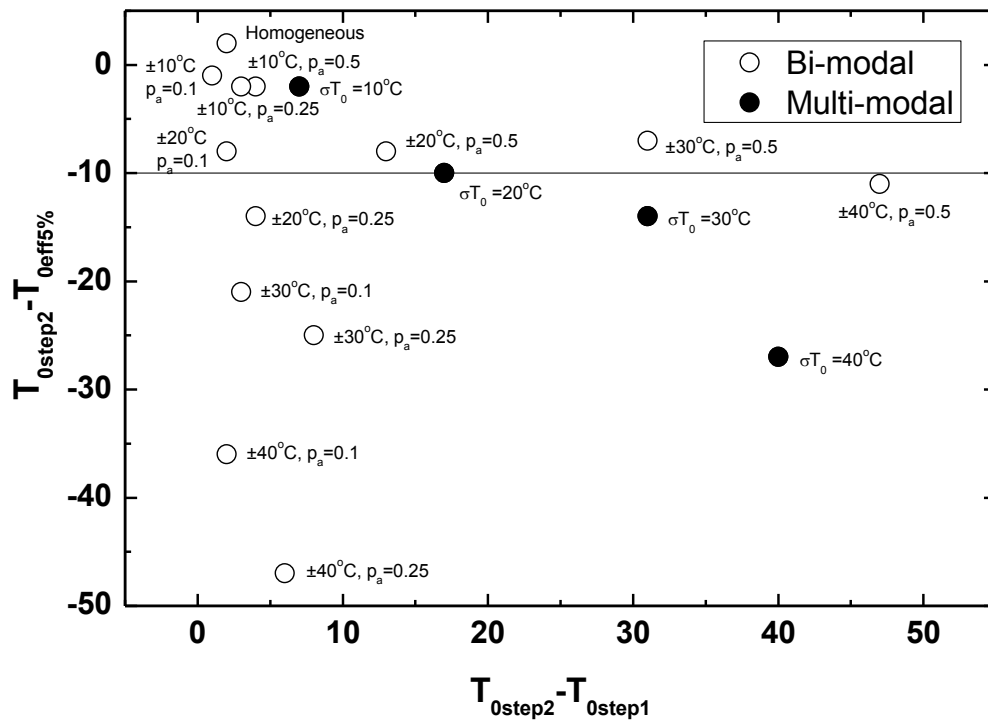
Figure 6. Difference between SINTAP step 2 T_{0step2} and effective value describing 20th percentile toughness, $T_{0eff20\%}$, for different types of inhomogeneities.
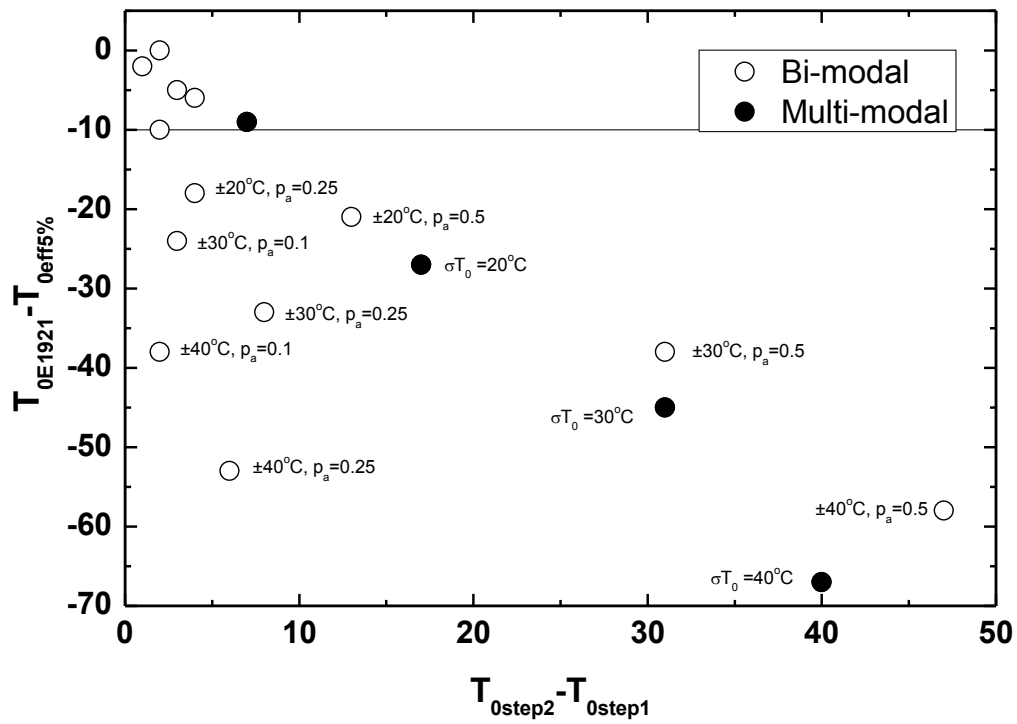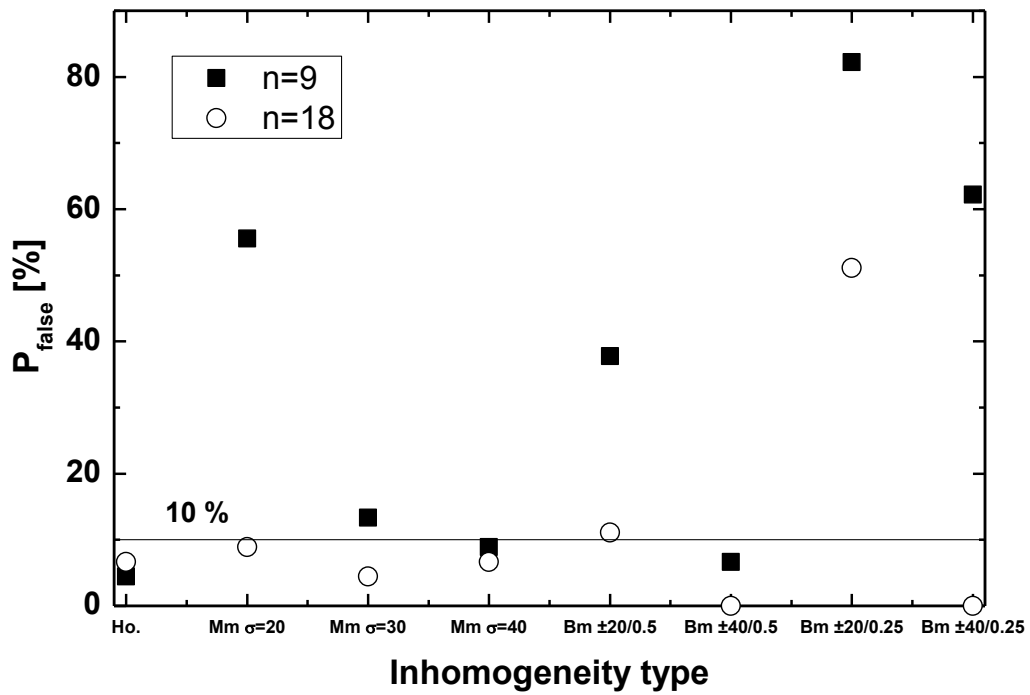
Figure 7. Difference between standard T_{0E1921} and effective value describing 5th percentile toughness, $T_{0eff5\%}$, for different types of inhomogeneities.

Figure 8. Difference between SINTAP step 2 T_{0step2} and effective value describing 5th

percentile toughness, $T_{0eff5\%}$, for different types of inhomogeneities.

Figure 9. Probability of a false screening of a homogeneous or inhomogeneous material for two different data set sizes.
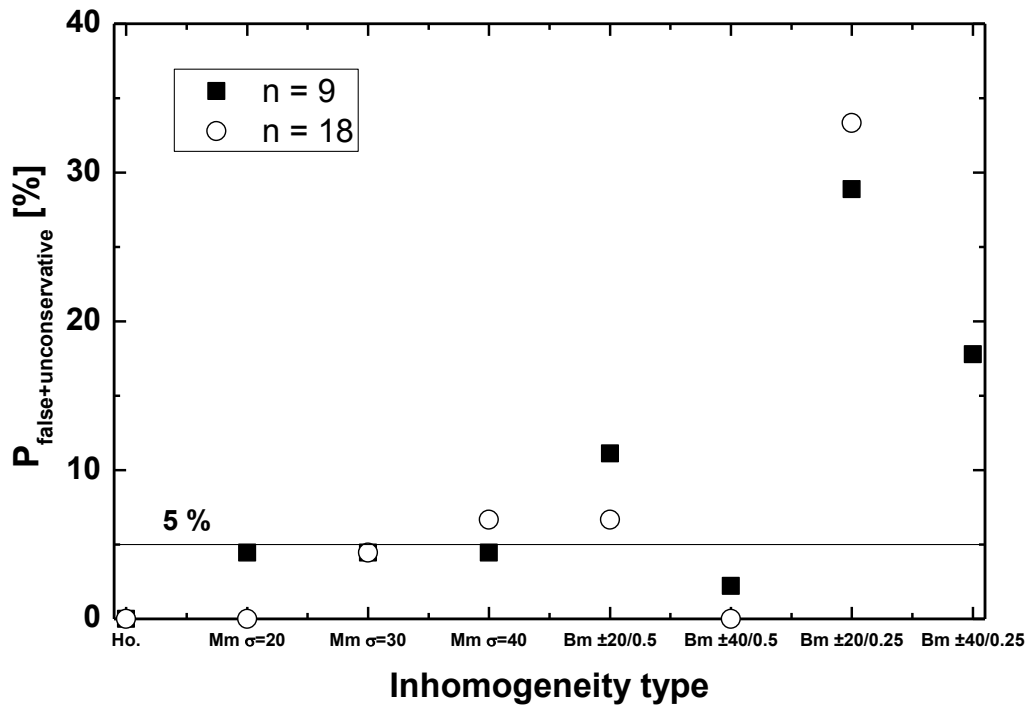
Figure 10. Probability of a false screening and a significantly non-conservative T_0 estimate of a homogeneous or inhomogeneous material for two different data set sizes.
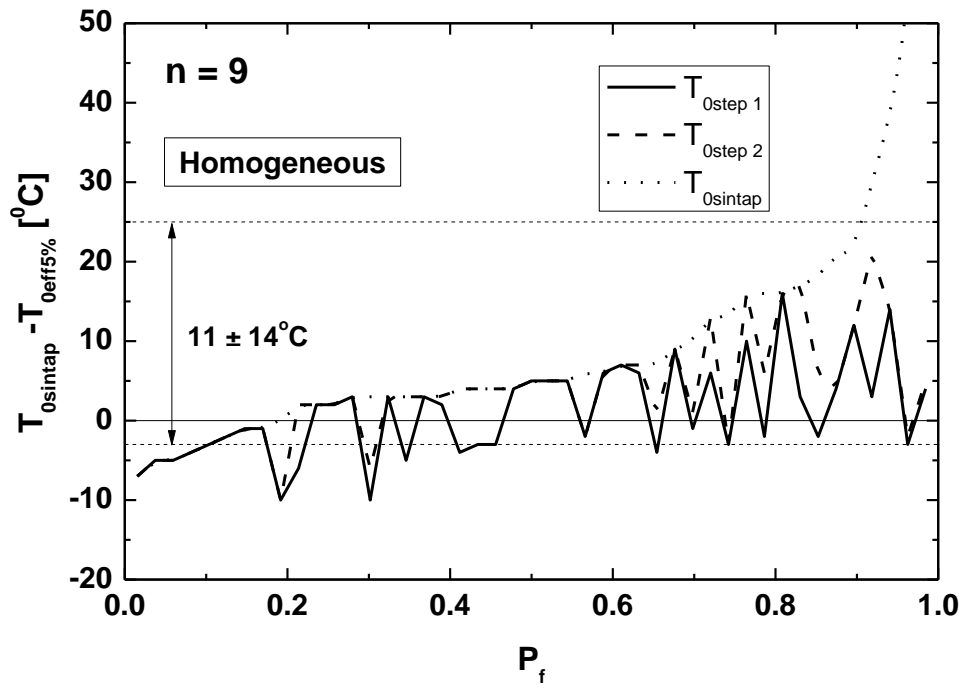
Figure 11. Bias on T_0, introduced by using SINTAP lower-tail assessment method for a data set size that includes step 3.
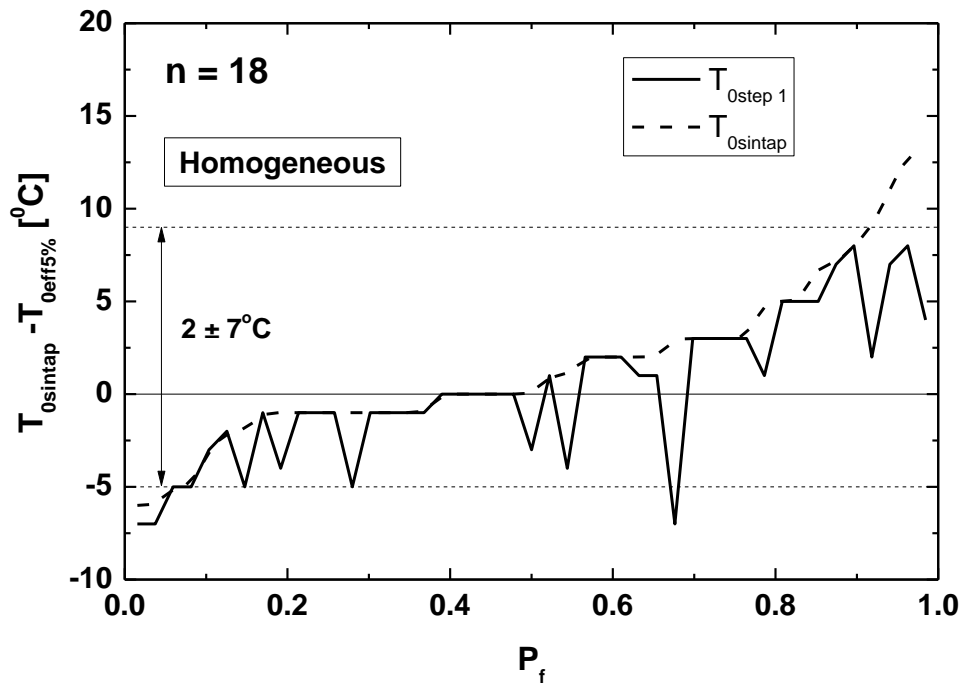
Figure 12. Bias on T_0, introduced by using SINTAP lower-tail assessment method for a data set size that excludes step 3.