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Selection and Identification of a Shape Function for Modeling Degradation as a Gamma Process

O. SAARELA, B. H. NYSTAD, A. TAIPALE and O. VENTÄ

ABSTRACT

This paper proposes an approach for estimation of Remaining Useful Life (RUL), in which data from earlier lifetimes is utilized in the identification of the functional shape of degradation. A set of candidate shape functions are identified, and one of them is selected for RUL estimation, and consequently for inference, based on information theory. The feasibility of the approach is studied using functional forms of degradation acquired through air filter loading experiments and air quality measurements representing degrading factors. The study indicates the usefulness of utilizing data from previous lifetimes when reliable enough a priori information of the degradation phenomena is not available. The observed improvement in accuracy was considerable at long prediction horizons, which is beneficial especially in Condition-Based Maintenance applications (not limited to air filters) with infrequent maintenance shutdowns.

STRUCTURAL HEALTH MODELING AND THE GAMMA PROCESS

Estimation of Remaining Useful Life (RUL) is a key element, and also a major challenge in cost-effective Condition-Based Maintenance (CBM). Traditional lifetime models are based on time to failure data. However, failure data is scarce in reality and non-existent for new equipment designs whereas condition and performance data is more abundant. These data describe underlying degradation processes facilitating their identification as state space models. In this approach estimation of RUL becomes estimation of the degradation process to reach a predefined threshold level. If the functional form of the condition and performance degradation is modeled properly, one can then in principle estimate the RUL without the need of failure data. The predefined threshold can be either deterministic or stochastic. It is often decided based on engineering experience, analysis of historical failure data, or applicable standards.

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Monotonic degradation processes evolving in only one direction can be modeled as gamma processes. This model structure is an attractive choice since it describes monotonic evolution over time in tiny random increments representing the effects of randomly varying deteriorating factor such as stress on a structure [1]. Reported successful applications range from individual components [2] to large structures [3].

A gamma process $\{X(t), t \geq 0\}$ with shape function $v(t)$ and scale parameter u is a stochastic process with Gamma-distributed increments and has the following properties:

(1) $X(0) = 0$ and $X(\infty) = \infty$ with probability one,
(2) $X(\tau) - X(t) \sim \mathrm{Gamma}(v(\tau) - v(t), u), \forall\, \tau > t \geq 0$, and
(3) $X(t)$ has independent increments.

The probability density function $f_{X(t)}$ of $X(t)$ is consequently $\mathrm{Gamma}(v(t), u)$, with expectation value and variance given by

$$\mathrm{E}\big(X(t)\big) = \frac{v(t)}{u}, \ \mathrm{Var}\big(X(t)\big) = \frac{v(t)}{u^2} \tag{1}$$

A fault is defined as the up-crossing of the degradation process $X(t)$ of a possibly stochastic threshold level Y. Let the time at which the fault occurs be denoted by the lifetime T_Y. Due to the monotonic behavior of the gamma process, the lifetime distribution can then be written as

$$F(t) = \Pr(T_Y \leq t) = \Pr(X(t) \geq Y) = \int_{x=0}^{\infty} \int_{y=0}^{x} f_{X(t)}(x) f_Y(y)\, dy\, dx \tag{2}$$

where Y has probability density function f_Y. The threshold can also be modeled as a deterministic quantity y. [3]

Selection of the shape function $v(t)$ representing the functional form of degradation is discussed below. This function must be monotonic with $v(0) = 0$ and $v(\infty) = \infty$. Given a data set consisting of inspection times t_i, $i = 1, \dots, n$ and corresponding degradation observations x_i ($x_0 = 0$ at time $t_0 = 0$), parameters of $v(t)$ can be identified from degradation increments $\delta_i = x_i - x_{i-1}$ by, e.g., maximizing the likelihood function

$$\mathcal{L}(\delta) = \prod_{i=1}^{n} f_{X(t_i) - X(t_{i-1})}(\delta_i) \tag{3}$$

SELECTION OF THE SHAPE FUNCTION

In published work, the shape function for gamma process modeling is commonly selected based on degradation assumedly being proportional to a power law

$$v_p(t) = ct^b \tag{4}$$

which can represent expected degradation phenomena in a range of applications including, e.g., degradation of concrete due to corrosion of reinforcement (b = 1), sulphate attack (b = 2), and creep (b = $^1/_8$) [3]. However, it cannot accurately represent degradation that is faster at both ends of a lifetime (wear-in and wear-out) than in between. For modeling such phenomena, the following shape function is proposed:

$$v_s(t) = c(\sinh a(t-b) + \sinh ab) \quad (5)$$

Introducing other candidates for the shape function obviously implies that either a choice between them has to be made or a mechanism for combining the results computed using each has to be selected. In the absence of reliable enough a priori information the approach has to be selected based on accumulated data. In this work the choice between candidate shape functions is based on measurement data from previous lifetimes. In other words, measurement data from previous replacement intervals is utilized in learning the functional form of degradation and thus to increase the accuracy of RUL estimates with long prediction horizons. Hence the likelihood function for model identification becomes

$$\mathcal{L}(\delta) = \prod_{k=1}^{o} \prod_{j=1}^{m_k} \prod_{i=1}^{n_{k,j}} f_{X(t_{k,j,i})-X(t_{k,j,i-1})}(\delta_{k,j,i}) \quad (6)$$

where m is the number of previous lifetimes considered and o is the number of inspected corresponding items. To reflect the current rate of degradation the scale parameter u is identified from the most recent data from the lifetime whose RUL is being estimated. In the experimental part of this study, values $o = 1$ and $m = 1, \ldots, 4$ were used.

A systematic means for comparing candidate models is available through Akaike's Information Criterion (AIC). It facilitates estimating which of the candidate models loses the least information (in the sense of Kullback–Leibler divergence) from the inadequately known process that generated the data. AICc, a modified version of the criterion including a correction term for short data lengths, is defined for model υ as

$$\mathrm{AICc}(\upsilon) = -2\ln(\mathcal{L}(\upsilon)) + 2k_\upsilon + \frac{2k_\upsilon(k_\upsilon+1)}{n-k_\upsilon-1} \quad (7)$$

where k_υ is the number of parameters identified from n data points. Selecting a model based on the likelihood function alone would guide selection towards a model with most adjustable parameters, allowing the model to describe the available data set accurately. However, identifying many parameters from a limited data set tends to increase the variances of the parameter estimates, making results calculated from the model more unreliable. Models that minimize AICc provide practical compromises between the accuracies and reliabilities of the respective candidate models. [4]

REMAINING USEFUL LIFETIME OF AIR FILTERS

Fibrous filters are widely used for air cleaning purposes due to their reliable operation and relatively low prize. Normally the lifetime of a filter is determined by reaching a certain pressure drop over the filter. Exceeding the designed pressure drop level causes unnecessary power losses in the system and increases the risk of mechanical failure of the filter.

Sometimes filters are changed based on predefined schedules, leading to remarkable parts of filter lifetime being lost and increasing filtration cost unnecessarily. In many cases, however, the pressure drop values are monitored. RUL is not estimated, though, as prediction of pressure drop development is very difficult with current systems. Estimation of RUL of air filters, like many other CBM targets, is complicated by strong variations in operating conditions. In addition to environmental conditions like humidity and temperature, the properties of loading aerosols have significant effects on filter pressure drop development. The most important properties in dust cake formation are particle concentration, size distribution, particle surface properties, and stickiness. [5]

Estimating remaining filter lifetime would enable considerable savings for the operator. One application field is gas turbine inlet air filtration [6], where vast flow of air must be cleaned in widely varying environmental conditions. It is economically essential to maintain power generation during episodes of worst filtration conditions including, e.g., sandstorms, high humidity, and icing as energy price is high during such periods. Reliable RUL information about air filters would provide a significant advantage for power plants when planning their service shutdowns.

EXPERIMENTAL SETUP

The applicability of the proposed approach was studied utilizing functional forms of decreasing RUL acquired from filter loading experiments combined with available air quality data. No information specific to this application was used in RUL estimation below.

In the loading experiments air filters of different designs were exposed to aerosols of different characteristics in altogether 24 combinations. The functional forms of decreasing RUL were acquired by measuring pressure drop, particle concentration and air flow for each combination. Functional forms of RUL degradation were computed as pressure differences (measured in Pa) as functions of cumulative exposures (mg/m^3*h). Figure 1 shows the measured pressure drops from five filter loading experiments. The experimental arrangement is presented in more detail in [7].

RUL-decreasing factors in this study were represented by air quality measurements (Figure 1) from multiple geographical locations, made available by the Finnish Meteorological Institute [8]. These air quality time series provided real-world data characteristics (i.e., non-compliance with theoretical background), especially a variety of autocorrelatedness, non-stationarity, and distribution functions. To preserve these data characteristics, time series of RUL indicators (ΔP) representing multiple lifetimes were generated simply by integrating the measurement data of the particle concentration ($\mu g/m^3$) over time and converting this filter exposure (mg/m^3*h) to ΔP (Pa) with the experimental functional forms. RUL indicator signals for all combinations of 24 functional forms and 121 exposure time series were generated, each indicator signal representing multiple consecutive filter lives. Altogether the data set included 448868 computational filter lives. The prognostics terminology (RUL, End-of-Life, …) used in the next section refer to this data set.

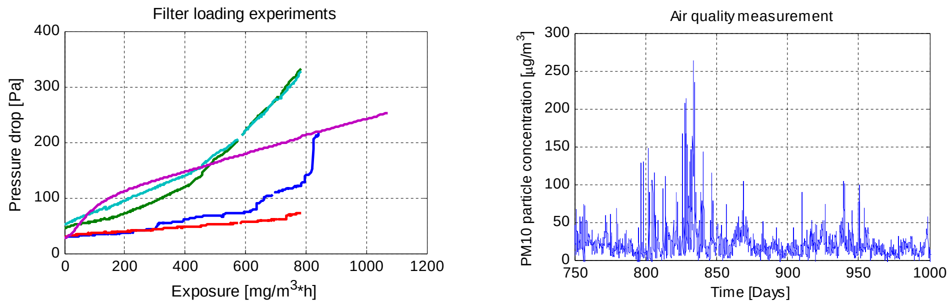

Figure 1. Examples of measurement data used to generate time series representing filter lifetimes. Functional forms of degradation are shown on the left and a time series of a degrading factor on the right.

PERFORMANCE STATISTICS

In order to gain understanding about the feasibility of the proposed approach, RUL estimates were computed from the indicator signals (ΔP) described in the previous section. Test cases were generated by selecting pseudo-random combinations of

- functional form of degradation (from air filter loading experiments),
- time series representing degrading factors (geographical location of air quality measurement),
- the computational lifetime whose RUL to estimate (the "current" life),
- how many (1-4) previous lives to use to learn the functional form of degradation, and
- time instant for computing the estimate (5-99% of current life).

For each of 140000 test cases generated, RUL was estimated using both v_p and v_s as the shape function of the gamma process model. Figure 2 shows examples of RUL estimation with these shape functions. In this data set, based on AICc, v_s was selected over the corresponding v_p in 75.5% of cases. The lengths of the lifetimes where predictions were being made ranged from 0.11 to 7.2 times the lengths of the corresponding previous lifetimes utilized in identifying the shape functions.

For reference, predictions were also computed with a more traditional approach where v_p is identified from the data of the lifetime whose RUL is being predicted. The functional forms of degradation acquired from filter loading experiments were not utilized in RUL estimation.

Figure 3 summarizes the computed RUL predictions. The average values and the 0.5% and 99.5% percentiles are shown as functions of prediction horizon, measured as the "true" RUL value known only a posteriori. As can be seen from the convergence of the percentiles, learning functional forms of degradation from previous lifetimes brings most significant accuracy improvements to long-term predictions, where the functional form cannot yet be inferred from data from the current lifetime. Also, as can be seen, the additional improvement from using more than one previous lifetime for learning is minor or non-existent in this data set. RUL estimates computed with v_s have a narrower distribution than those computed with v_p, which is consistent with AICc-indicated preference of v_s in majority of the cases.

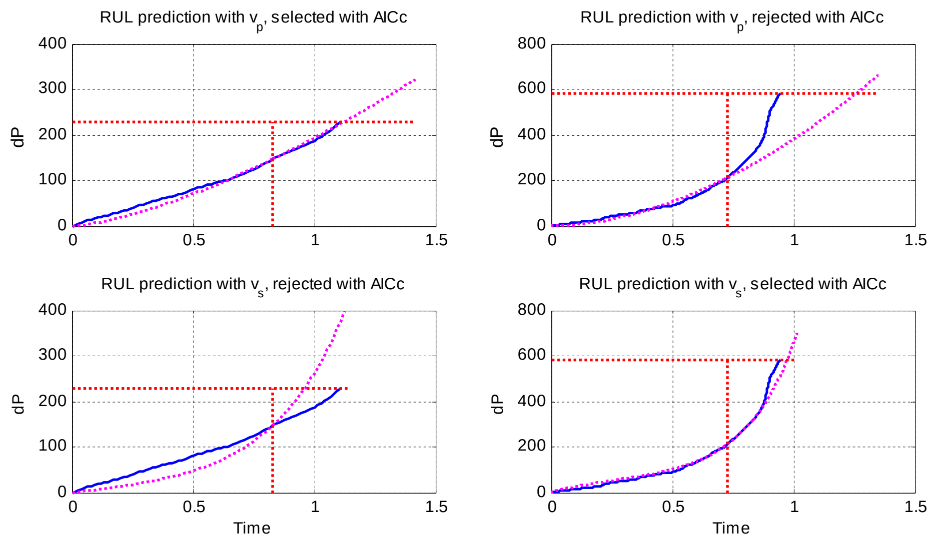

Figure 2. Examples of RUL predictions with two different functional forms of degradation (left and right) and with two different shape functions (top and bottom). Dotted vertical lines show the times the predictions are made and dotted horizontal lines show the degradation levels being predicted. Dotted curves represent the predicted and solid curves the “actual” degradation. The shape functions have been identified and selected with AIC from four previous lifetimes. On time axis value 1 represents the average length of the four previous lifetimes.

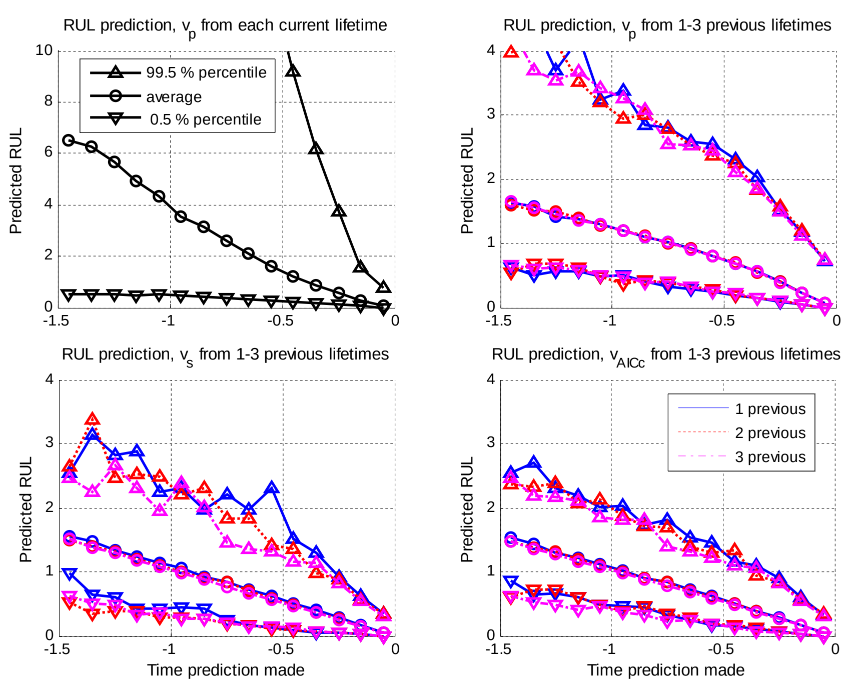

Figure 3. Experimental mean values and 99% confidence limits for RUL estimates. On top left, the shape function v_p is identified from each lifetime being predicted; in other graphs the mutually almost superimposed curves are computed with shape functions identified from 1-3 previous lifetimes. Axes are scaled so that value 1 represents the average duration of all filter lifetimes. Value 0 indicates End-of-Life.

An apparently straightforward way to estimate confidence limits for the RUL estimates would be through the statistical properties of the gamma process, Equation 2. However, this approach is complicated by the mutual dependencies of the parameters of the shape function *v* and the scale *u* parameter as their values are estimated from measurement data. Even though Equation 2 can be easily written to explicitly accommodate the joint PDF of the identified parameters, estimation of such joint PDFs from limited data sets can be challenging.

The large number of cases in this data set facilitates statistical characterization of the computed RUL estimates. A number of metrics have been used to evaluate the performance of prognostic algorithms ranging from basic statistical characterization to application-specific assessment involving cost factors for too early and too late estimates [9]. In this study the estimation accuracy was evaluated in terms of relative prediction error,

$$\Delta_i = \frac{\hat{r}_i - r_i}{r_i} * 100\% \qquad (8)$$

where $\hat{r}_i$ is the RUL estimate for the *i*th test case and choice of shape function. r_i is the corresponding a posteriori value.

Table 1 shows the mean values and the 95% confidence limits (2.5% and 97.5% percentiles) of this relative error for various value ranges of $\hat{r}_i$. For a single test case the RUL estimates computed with different shape functions can fall into different value ranges, so a single test case can contribute to multiple lines of the table. The results suggest that learning the functional form of degradation from previous lifetimes improves the accuracy of RUL estimates, especially at the early stages of filter life. Towards the end of filter life the accuracy improvement becomes less notable. Utilizing data from both previous and current lives for shape function identification is a topic for further study.

TABLE 1. STATISTICS FOR RUL ESTIMATION ERROR.

			Δ			Δ			Δ			Δ		
			v_p from current life			v_p from previous lives			v_s from previous lives			v_{AICc} from previous lives		
RUL estimate in range (1 = average filter life)			**$p_{2.5}$ (%)**	**mean (%)**	**$p_{97.5}$ (%)**	**$p_{2.5}$ (%)**	**mean (%)**	**$p_{97.5}$ (%)**	**$p_{2.5}$ (%)**	**mean (%)**	**$p_{97.5}$ (%)**	**$p_{2.5}$ (%)**	**mean (%)**	**$p_{97.5}$ (%)**
1.8	-	2.0	17	214	618	4	135	461	-7	44	178	-9	36	129
1.6	-	1.8	10	200	628	-3	112	404	-12	38	144	-14	30	119
1.4	-	1.6	-2	183	606	-13	96	409	-14	30	156	-15	26	119
1.2	-	1.4	-16	169	681	-19	84	454	-17	24	124	-18	22	114
1.0	-	1.2	-25	157	726	-23	82	511	-21	20	117	-22	19	108
0.8	-	1.0	-30	136	707	-28	78	556	-26	17	125	-26	17	115
0.6	-	0.8	-38	129	819	-32	82	627	-34	15	133	-31	15	124
0.4	-	0.6	-37	116	692	-35	83	592	-44	17	177	-39	17	172
0.2	-	0.4	-33	79	375	-38	58	357	-52	23	243	-46	25	239
0.1	-	0.2	-35	54	275	-39	34	246	-57	10	169	-49	14	170
0.05	-	0.1	-43	46	273	-47	30	241	-65	0	139	-58	5	144
0.01	-	0.05	-53	24	199	-57	18	192	-76	-14	116	-69	-8	130

DISCUSSION

This paper has proposed an approach to RUL estimation with gamma process modeling when reliable enough a priori information of the degradation phenomena is not available. The computational results indicate that the reliability of RUL estimates can be improved when the functional form of degradation is identified from data from earlier lifetimes. Also, the results support the feasibility of using AICc for selecting a model for RUL prediction from a set of candidate models. For the data set studied, the accuracy of the RUL improved considerably when data from one previous lifetime was utilized; longer data records didn't provide significant further improvement.

The achieved improvement in accuracy was considerable at long prediction horizons. This is beneficial especially in CBM applications (not limited to air filters) in environments with infrequent maintenance shutdowns. For example, planning of an annual maintenance shutdown of a nuclear power plant is aided if RUL estimates indicate that some maintenance actions can be postponed one year further. For short prediction horizons towards the end of component life the use of data from both previous and current lifetimes should be studied.

A key factor for industrial acceptance of the technique is the reliability of the RUL estimates computed. This issue will be further studied, considering e.g. the effects of measurement uncertainties and inspection intervals.

REFERENCES

1. Abdel-Hameed, M. 1975. "A gamma wear process," *IEEE Transactions on Reliability, 24(2)*, 152-153.
2. Nystad, B., G. Gola and J. Hulsund. 2010. "Technical condition assessment and remaining useful life estimation of choke valves subject to erosion," *Annual Conference of the Prognostics and Health Management Society*, Portland, October 10-14 2010.
3. van Noortwijk, J. M. 2009. "A Survey of the Application of Gamma Processes in Maintenance," *Reliability Engineering and System Safety*, 94:2-21.
4. Burnham, K. P. and D. R. Anderson. 2002. Model Selection and Multimodel Inference. New York: Springer-Verlag.
5. Gustavsson, J. (ed), A. Ginestet, P. Tronville, and M. Hyttinen. 2011. Air Filtration in HVAC systems. REHVA Guidebook 11. Brussels: REHVA.
6. Wilcox, M., R. Baldwin, A. Garcia-Hernandez, and K. Brun. 2010. "Guideline for Gas Turbine Inlet Air Filtration Systems", Dallas: Gas Machinery Research Council.
7. Valmari, T., M. Lehtimäki, and A. Taipale. 2006. "Filter Clogging by Bimodal Aerosol," *Aerosol Science and Technology*, 40:255-260.
8. Air Quality in Finland. 2013. http://www.airquality.fi/, accessed 20.1.2013.
9. Saxena, A., J. Celaya, E. Balaban, K. Goebel, B. Saha, S. Saha and M. Schwabacher. 2008. "Metrics for Evaluating Performance of Prognostic Techniques," *International Conference on Prognostics and Health Management PHM 2008*, Denver, October 6-9 2008.