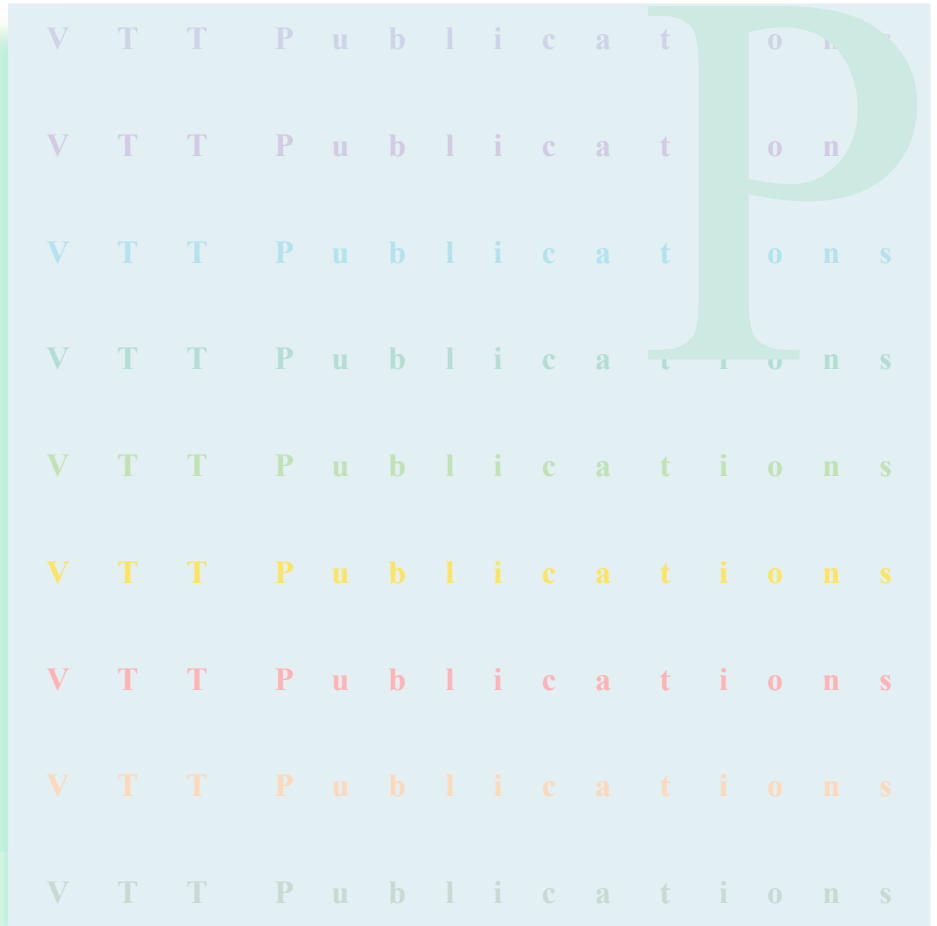Jouko Pakanen & Sami Karjalainen

An ARMAX-model approach for estimating static heat flows in buildings

A method for computerised energy allocation systems

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Abstract

An energy cost allocation system records the energy consumption of a building and divides the overall energy costs between the flats. Because the indoor temperatures of rooms are usually not equal, heat flows between flats cannot be avoided. Hence, in order to ensure fair energy costs per flat the system should be able to determine the static heat flow rates, preferably without utilising design data or *in-situ* measurements. This paper presents a new method for estimating static heat flows between neighbouring rooms. It also outlines the instrumentation and technical properties of the energy allocation system (EAS), needed in implementation. The approach is strictly technical, focusing only on heat transfer issues. Energy cost allocation is not considered.

The approach is based on the proposed control techniques and the ARMAX-model describing dynamic behaviour of heating power. The model is created for each room/flat of a building. Parameter values are identified using real-time measurements collected from each room/flat and its environment. Only ordinary instrumentation is required. The tuning of parameters takes a few days using a fifteen-minute sampling time. A prerequisite for successful estimation is the overall control and the precise adjustment of the room temperatures at specified level. This is accomplished by the suggested set point control. All test runs are performed in a simulated office hotel using the TRNSYS simulation program. Realistic inner and outer heating loads, and their daily schedules and variations are included. The results are encouraging, but further research is needed, especially in a real building environment.

Preface

This paper presents an ARMAX model approach for estimating static heat flows through walls and structures. The objective was to develop a feasible, technical solution to be applied in energy allocation systems.

The authors, Jouko Pakanen and Sami Karjalainen, are both researchers of VTT Building and Transport. Apart from working at VTT Jouko Pakanen is also a docent at Tampere University of Technology.

Contents

List of symbols

Notations

a	Model parameter
$A(z^{-1})$	Polynomial
b	Model parameter
$B_k(z^{-1})$	Polynomial
C	Constant parameter
$\dot{C}_f$	Capacitive flow of filtration air
$\dot{C}_e$	Capacitive flow of exhaust air
C_R	Room thermal capacity
d	Model parameter
$D(z^{-1})$	Polynomial
$e(t)$	White noise
$E(t)$	Convective load caused by electric power, i.e. lighting, machines, etc.
h	Lumping constant
h_{si}	Convective heat transfer coefficient
K_p	Gain of P-type control
$M[\,]$	Model definition, order of polynomials specified in the parenthesis
$O(t)$	Convective heat load caused by occupation
$q(t)$	Heating power
$q_C(t)$	Combined convective heat loads
$Q(t)$	Measured heating power
Q_{DC}	DC-value of heating power
Q_S	Heat flow rate (heat transferred per unit time)
$\overline{Q_S}$	Mean value of Q_S

$S(t)$	Convective heat load caused by solar radiation
t	Time variable
$u_k(t)$	Input signal
$u_o(t)$	Outdoor temperature
$u_R(t)$	Room temperature
$u_{SP}(t)$	Set point temperature
$\bar{u}_R(t)$	Mean value of room temperature
$U_i(t)$	Measured process variable
$U_{i,DC}$	DC-value of measured process variable
$v(t)$	Ventilation air rate
$V(\Theta)$	Loss function
z^{-1}	Backshift operator
Δ	Increment
$\Theta(t)$	Parameter vector
$\Psi(t)$	Data vector
$\delta(t)$	Difference between set point and controlled signal
$\varepsilon(t)$	Prediction error
$\bar{\varepsilon}(t)$	Residual (posterior prediction error)
$\gamma(t)$	Gain sequence

Abbreviations

ARMA	Autoregressive moving average
ARMAX	Autoregressive moving average with exogenous variables
ARX	Autoregressive with exogenous variables
BAS	Building automation system
EAS	Energy allocation system

ECAS	Energy cost allocation system
EMS	Energy management system
HAS	Home automation system
MA	Moving average
VAV	Variable air volume

1. Introduction

1.1 Individual metering and energy cost allocation systems

Individual metering in buildings consists of measurements made in a room or a flat concerning the consumption of heating or cooling energy, electricity and/or water. Other measurements may also be included. The principle is that the measurements determine how the inhabitants share the costs of the consumptions. Sometimes only the amount of consumed energy is essential. Such systems are called energy cost allocation systems.

In some countries individual metering of energy consumption is enforced by law. It is also recommended by the European Commission and every member state is urged to apply this principle. Such regulations have created a number of technical solutions and commercial metering products (Kimari 1994, Műgge 1997). Apart from differences in technical implementations, they have different ways of dealing with the energy flows between flats and rooms, and further energy cost allocation. Some systems use special cofactors to correct the monthly energy costs according to the location of the flat in the building, while others use design data for static heat flow calculations to estimate the energy transfer between flats. Nevertheless, all currently available methods only give rough approximate results. Consumers are aware of the problem, and this is probably one reason why energy cost allocation systems are not widely applied in Finland.

It is evident that a metering system that only collects consumption data, and possibly makes one or two additional measurements, cannot determine actual energy flows between rooms. The available information is not sufficient. For example, structures, materials, furniture, occupation and internal heat gains vary from room to room. Ventilation is different in each flat due to the structures, duct network, air leakages and location of the flat. Thus, allocation of energy costs using insufficient information will lead to large errors.

The intent of the present paper is not to dictate how energy costs should be shared. The view is strictly technical and concentrates on the problem of how to estimate static heat flow through walls and structures between rooms and flats. A

technical system that produces such information will be here referred to as an energy allocation system (EAS).

1.2 Selected approach

Conventional energy allocation systems collect only consumption data. Such systems are usually not even designed to determine static heat flows in buildings. If they do this, calculations are merely based on data, which cannot take into account the changing conditions in the building envelope, structures, loads or furniture. Hence, the starting point for a more realistic energy allocation system should be an assumption that a good approximation of heat flow through walls and structures requires information of all the variables affecting the dynamic heating power of the room or flat. By increasing the number of measurements and adopting a proper control and data processing strategy better estimates can probably be made. Moreover, if the method further leads to a technical solution, it should be feasible and commercially viable.

The presented procedure for heat flow estimation is based on the proposed set point control techniques and the dynamic model of heating power. Model derivation starts from the basic heat balance of an occupied space. The dynamic thermal model is modified into an ARMAX-model, which is a typical black-box model. In its final form the model contains only a few parameters. Measured signals from the building environment are used as input variables while identifying the parameters. A central and unique idea of the solution is to utilise identified dc-levels of heating power in the estimation algorithm. A comparison of the dc-levels of two ARMAX-models results in an approximation of static heat flow rate through a wall or a slab. Obviously, it is difficult to determine the accurate numerical value of actual heat flow. Therefore, the derived method gives only an estimate.

Generally, the estimation method should be verified in a real building, but such a procedure will evidently be a tedious process. That is why experimental validation was undertaken using the TRNSYS computer program, which simulates the dynamical thermal behaviour of an office building. The simulation environment enables the user to create realistic heating loads, including daily schedules and variations, to compute heat transfer through structures, take care

of environmental disturbances, etc. Naturally, the thermal behaviour generated by the simulation program is only an approximation, but it is a good starting point in the early stage development of the method.

1.3 Earlier results

A number of authors have applied ARMAX models or related models (ARMA, ARX etc.) in building environments. Energy analysis and fault detection are typical applications. The following authors, for example have published papers on these issues: Yoshida et al. (1996), Pakanen (1992), MacArthur et al. (1989), Rabl (1988) and Crawford & Woods (1985).

In the recent years, similar models have attracted growing attention in the identification of building components (Bloem 1993, 1996). The objective of these studies is to achieve more information from *in-situ* measurements by means of identification techniques. The ARMAX-model was one of the applied approaches. Although the ARMAX-model has been widely applied even in building research, the method developed in this paper for estimating static heat flows through walls and structures is believed to be new.

2. Static heat flow estimation; the approach

2.1 Room control phases

The estimation of heat flow is made in several phases. It requires both data recordings and special control of indoor temperatures, as well as tuning of parameters and some final calculations. Below the procedure is described using a simple configuration of two adjacent rooms.

Consider a one-storey building consisting of two rooms *A* and *B*, with one common wall (figure 1). Actually, each room consists of a small flat, with a separate heating system combined with indoor temperature control. The physical properties, such as furniture, infiltration, ventilation, inner and outer heating loads, etc., are also unique to each room. The set point of the indoor temperature is supervised by an energy allocation system. The EAS also collects data from the room and its environment using appropriate instrumentation. The rooms and their indoor temperatures are controlled in the following way.
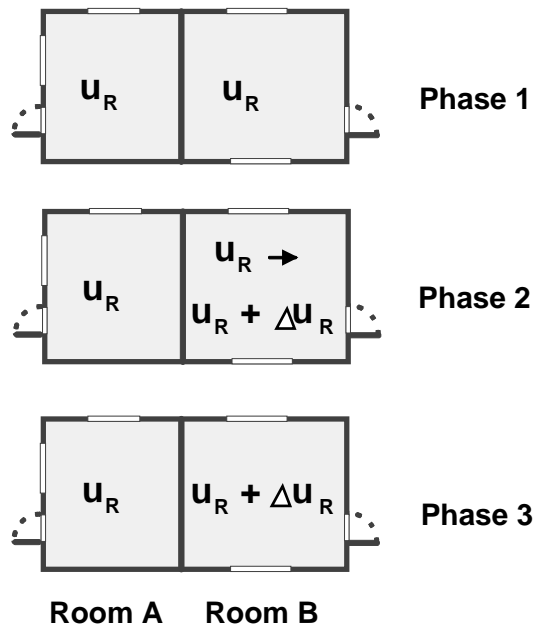
Figure 1. The three phases of the heat flow estimation procedure.

Phase 1. Assume that both rooms have the same indoor temperature u_R, which is kept stable for a predefined period, later referred to as the identification period. During that time, the heating power $Q(t)$ and the other available process variables are recorded from room A and its environment.

Phase 2. This phase is needed for raising (or lowering) the internal temperature of room B and for waiting that the building structures achieve their stationary temperatures. The time constants of thermal processes may be several hours. That is why phase 2 takes at least one day. No data recordings or identification are performed.

Phase 3. The indoor temperature in room B is now different ($u_R + \Delta u_R$). Both indoor temperatures are kept stable, as in phase 1. Again the process variables are recorded for a period equal in length to phase 1. In this case (figure 1), heat flows from room B to room A and decreases the need for heating power in room A.

2.2 Temperature control

The estimation of static heat transfer requires special set point control. During the identification period, the mean value of the indoor temperature in the room/flat is kept at the predefined level by adjusting the set point of the controller. Usually, short-term overshoots or undershoots of indoor temperature due to inner or outer disturbances are acceptable and have no effect on the set point. Over- or undershoots may be caused by the opening of a window or the daily solar radiation, etc. Now, if such a temporal temperature change occurs, the set point must be recalculated for the rest of the time period. The whole control procedure can be defined as follows:

1) The indoor temperature of the room is monitored continuously at discrete time instants, starting at the beginning of each identification period. In practice the time step might be a few minutes. At each time step the control system computes recursively the current mean value $\bar{u}_R(t)$ of the temperature $u_R(t)$.

$$\bar{u}_R(t) \quad = \quad \bar{u}_R(t-1) + \frac{1}{t}\left[u_R(t) - \bar{u}_R(t-1)\right]. \tag{1}$$

2) Due to possible changes in the room temperature a new set point $u_{SP}(t)$ is calculated at every time instant. If n is the total number of time instants during the whole identification period, and $u_{SP}(0)$ is the targeted set point temperature, then, at time instant t the new set point for the rest of the identification period is

$$u_{SP}(t) \quad = \quad \frac{n}{n-t} u_{SP}(0) + \frac{t}{n-t} \bar{u}_R(t) \quad . \tag{2}$$

3) The control system must also account for unusual situations. The set point $u_{SP}(t)$ should be regularly checked against the lower and upper alarm limits. Possible crossing leads to abnormal indoor conditions and may be a good reason for interrupting the identification.

2.3 A few remarks

Observe that in the case presented in figure 1, the data gathered from room B cannot be directly used in static heat flow computations. The reason is that a raise in the indoor temperature of room B in phase 2 causes more static heat flow through all surfaces, not only through the common wall. Obviously, this will lead to incorrect results.

The room arrangement of figure 1 is very simple and designed only for illustrating the estimation procedure. In a real building the situation may be more complicated. Especially, in the case of an apartment building the estimation procedure covering all rooms must be designed carefully. A complicated structure also means a longer time to go through all the rooms/flats.

The above expressions were derived by assuming that indoor temperatures remain stable during the identification. This needs some explanation. Actually, only the indoor temperature in room A should be kept the same level during the phases 1 and 3, and to be more specific, only their mean values should be equal. Hence, slight temporal variations may occur in the room temperature, but the targeted mean value should be achieved by the end of the phase. For room B, the level and stability of the temperature are not so critical. During phase 3, the set point temperature of room B can be chosen freely, and during phase 1 it need not

be exactly the same as in room *A*. Only the temperature difference between phases 1 and 3 is meaningful for room *B*.

2.4 Basic procedure

The aim of the phases 1, 2, and 3 is to create appropriate circumstances for heat flow estimation. Basically, the indoor conditions in room *A* have not changed during phases 1 and 3. The only difference is the static heat flowing from room *B* during phase 3. Hence, an easy way to determine the static heat flow would be to compare the mean valued heating powers of room *A* recorded during the phases 1 and 3. So, assume first that Q_1 and Q_3 represent heating powers of the room *A* in phases 1 and 3. Then, the approximate heat flow rate between the rooms is simply

$$Q_S \quad = \quad \overline{Q}_1 - \overline{Q}_3 \; . \tag{3}$$

Evidently, due to inner and outer disturbances the expression (3) is only a rough approximate. But computing the difference (3) *m* times and averaging their results will give a better numerical result for static heat flow. This can be done by

$$\overline{Q}_S \quad = \quad \frac{1}{m} \sum_{n=0}^{m-1} \left[\overline{Q}_{4n+1} - \overline{Q}_{4n+3} \right], \tag{4}$$

where the subscripts refer to consecutive phases, starting from phase 1 according to figure 1. Now, each phase of the sequence 1,5,9,… etc. is similar to phase 1, and each phase of the sequence 3,7,11,… etc. corresponds to phase 3 in figure 1. All even numbers refer to phase 2.

The main drawback of this approach is that *m* becomes relatively large and the procedure takes a lot of time, even several months in practice. Therefore, it is feasible to find a solution, which gives a reasonable result in a shorter time period.

2.5 Chosen approach

One alternative would be to find a mathematical expression to relate the heating power of the room to the inner and outer disturbances. When the influence of the disturbances is better known their effect on the final result can be reduced. This means that besides the heating power $Q(t)$ also other available process variables must be recorded from room A and its environment, such as room temperatures $U_R(t)$, outdoor temperature $U_o(t)$, solar radiation $S(t)$, electric power $E(t)$, etc.

Usually, a dynamic thermal model of a building contains only the main features of the process, since it is not even feasible to measure all effective variables and/or noise components. In addition, an exact mathematical relationship between the heating power and the input variables is not known. Thus, one obviously has to be satisfied with an approximate expression. Consequently, if a solution for the heat flow is found it is still an approximation. This means that in order to achieve relevant numerical values for the heat flow between rooms, one has to resort to averaging according to equation (4).

The following pages present a solution, which is based on the dynamic thermal model of the heating power. The solution applies the above control procedure and utilizes the phases 1, 2 and 3 or multiple of them. It will be shown that the proposed method gives better results than the basic approach of chapter 2.4, which focuses only on the recorded heating powers. As a result, applying the new method for heat flow estimation the number of phases 1 and 3 (m) will be small.

3. Modelling and identification

3.1 Dynamic thermal model

The dynamic thermal behaviour of a building is complicated. If electrical analogy is applied the building consists of a complex network of thermal resistances and capacitances, where heat transfer is based on conduction, convection and radiation. The network is not even linear or time invariant. Due to these difficulties a number of simplifications must be made in modelling.

The following thermal model focuses only on dynamics and physical quantities, which have a direct influence on heating power. Other aspects, such as predicting peak heat demands or annual energy costs or sizing HVAC equipment, etc., typical of building design or energy simulation analysis, are not included. The presented solution is actually a single-path model implemented in an EAS, which, in turn, is interfaced to a building, its rooms, and the heating process. Consequently, the EAS provides the necessary real time measurements for model identification. Together, they enable the user to predict power fluctuation, which is the basis for static heat transfer calculations between rooms. The final model is developed in two phases. First, the model structure is derived analytically by producing a mathematical expression. This leads to a parametric model, which can be identified utilising experimental data.

The dynamic thermal model is restricted to the occupied space of the building. The occupied space here refers to a flat or a room in a multi-storey building or a townhouse. All the three terms: occupied space, room and flat are used interchangeably throughout the paper.

The convective heat balance of occupied space can be expressed analytically as

$$C_R \frac{du_R(t)}{dt} = \sum_i h_{si} A_i \left[u_i(t) - u_R(t) \right] + \dot{C}_f u_o(t) - \dot{C}_e u_R(t) + q_C(t), \qquad (5)$$

where the summing goes through the surfaces of the space and $q_C(t)$ refers to convective heat loads, such as heating, solar radiation, lighting, machines and occupation:

$$q_C(t) \;=\; q(t) + S(t) + E(t) + O(t). \tag{6}$$

Analytical solutions of (5) are rare and call for substantial simplifications in the equation. Therefore, heat balance calculations are usually performed numerically. This is best accomplished by discretizing the heat balance equation. For example, the time derivative of the indoor temperature could be replaced by its approximate solution

$$\frac{du_R(t)}{dt} \;\cong\; \frac{\left[u_R(t) - u_R(t-h)\right]}{h}. \tag{7}$$

As a result the heat balance is computed at equal time intervals.

The internal temperature of an occupied space is kept stable by means of a temperature controller. A P-type controller is typical in buildings. An output increment of the controller can be written as

$$dq(t) \;\approx\; \Delta q \;=\; K_p\left[\delta(t) - \delta(t-1)\right], \tag{8}$$

where $\delta(t)$ means the difference between the set point temperature $u_{SP}(t)$ and the indoor temperature $u_R(t)$. The notation $\delta(t - 1)$ refers to the earlier temperature difference (Takahashi et al 1970). If the set point is kept stable, both sides are divided by Δt, then approximately

$$\frac{dq(t)}{dt} \;\approx\; \frac{\Delta q}{\Delta t} \;=\; -\frac{K_P}{\Delta t}\left[u_R(t) - u_R(t-1)\right]. \tag{9}$$

So, when a P-controller is used, the derivative of heating power $q(t)$ is proportional to the difference between the current and previous indoor temperatures.

By comparing equations 5, 7 and 9 its easy to conclude that $q'(t)$ in equation 9 is approximately proportional to the time derivative of indoor temperature. This is supported by the fact that the sampling time of the temperature controller is short compared to the time step usually applied in heat balance calculations.

Consequently, if $u'_R(t)$ in equation 5 is replaced by an expression containing $q'(t)$, a new expression for the heating power is produced. It applies to the power of an occupied space or a flat, where indoor temperature is kept stable by a P-controller. A general, discretized version of this new expression can be written as

$$q(t) \quad = \quad \sum_{i=1}^{m} a_i q(t-i) + \sum_{j=1}^{n} b_j u_1(t-j) + ... + \sum_{p=1}^{r} b_p u_k(t-p), \qquad (10)$$

where the input variables u_i, $i = 1,2,3,...$ represents the room, surface and outdoor temperatures, and the effects of ventilation, radiation, occupation, lighting, machines, etc. The parameters a_i, b_j, b_p are constants, which depend on the structures, the environment and the heating process. Some, but not all of the parameter values could be solved from the design data. All parameter values are therefore identified.

The mathematical model of equation 10 is linked to only a few measurable process variables. In addition to them several unmeasurable process variables, including disturbances acting on the system, have an influence on the heating power. Due to economical and/or technical reasons it is not feasible to measure all effecting physical quantities. However, some of their information content, especially periodic trends and daily variations can be included, if the expression is amended by a noise model $e(t)$. After introducing these modifications, the dynamic thermal model of the heating process can be written as a linear stochastic difference equation with constant parameters

$$q(t) = \sum_{i=1}^{m} a_i q(t-i) + \sum_{j=1}^{n} b_j u_1(t-j) + ... + \sum_{p=1}^{r} b_p u_k(t-p) + \sum_{s=1}^{v} d_s e(t-s),$$

$$(11)$$

where $e(t)$ represents a zero mean white noise of variance λ^2. A more perfect notation would require the addition of a discrete dead time constant at each time instant of the input variables. Because their exact values are unknown and they are difficult to find out, discrete dead times are neglected.

Equation 11 is actually an ARMAX-model of $q(t)$. It is convenient to write the model as

$$A(z^{-1})q(t) \quad = \quad B_1(z^{-1})u_1(t) + \cdots + B_k(z^{-1})u_k(t) + D(z^{-1})e(t), \qquad (12)$$

where z^{-1} denotes a back shift operator and the polynomials $A(z^{-1})$, …, $D(z^{-1})$ are defined as

$$
\begin{aligned}
A(z^{-1}) \quad &= \quad 1 + a_1 z^{-1} + \cdots + a_m z^{-m} \\
B_1(z^{-1}) \quad &= \quad b_{11} z^{-1} + \cdots + b_{1n} z^{-n} \\
&\vdots \\
B_k(z^{-1}) \quad &= \quad b_{k1} z^{-1} + \cdots + b_{kn} z^{-r} \\
D(z^{-1}) \quad &= \quad 1 + d_1 z^{-1} + \cdots + d_v z^{-v}
\end{aligned}
\qquad (13)
$$

An ARMAX model represents a linear, finite-order system with stationary disturbances. Several identification methods can be applied to determine the model parameters from experimental data.

3.2 DC-level of the the model

An essential aspect of energy allocation is the need to estimate static heat flow between adjacent rooms or flats with different internal temperatures. This is accomplished by comparing parameters related to the dc-level of heating power. Before proceeding to this point, the parameter must be extracted from the heating power signal.

Variation of the heating power $q(t)$ and the input variables $u_i(t), i=1,\cdots,k$ of (11) are tied to the measured process variables $Q(t)$ and $U_i(t), i=1,\cdots,k$ and their static dc-values by the expressions

$$q(t) \quad = \quad Q(t) - Q_{DC}, \qquad (14)$$

$$u_i(t) \quad = \quad U_i(t) - U_{i,DC}, i = 1,\cdots,k. \qquad (15)$$

Several methods are available for determining the dc-values (Isermann 1982, Söderström & Stoica 1989). The applied method is based on Isermann's paper. First, $U_{i,DC}$ values are estimated by averaging the measurements of input

variables. Then, Q_{DC} is evaluated from the identified ARMAX model in the following way. First, equation 11 or 12 is presented as

$$q(t) \;=\; \mathbf{\Psi}_0^T(t)\mathbf{\Theta} + e(t), \tag{16}$$

where $\mathbf{\Psi}_0$ and $\mathbf{\Theta}$ denote the data and parameter vectors. Usually, when it is not necessary to evaluate dc-values the vectors contain only the output, input and disturbance data in addition to the tuned parameters. But now the parameter vector is amended by a constant parameter C and the corresponding component in $\mathbf{\Psi}_0$ is set to one

$$\mathbf{\Psi}_0^{\,\cdot}(t) = \left[-q(t-1),\ldots,-q(t-m); u_1(t-1),\ldots,u_1(t-n); \ldots ; \right.$$
$$\left. u_k(t-1),\ldots,u_k(t-r); e(t-1),\ldots,e(t-v); 1 \right] \tag{17}$$

$$\mathbf{\Theta} \;=\; \left[a_1,\ldots,a_m; b_{11},\ldots,b_{1n}; \ldots; b_{k1},\ldots,b_{kr}; d_1,\ldots,d_v; C \right]^T \tag{18}$$

So, the constant parameter C will get a specific value during identification similarly to the other parameters. The meaning of the constant C becomes apparent when the identified model is written out using (14) and (15). First

$$Q(t) = -\sum_{i=1}^{m} a_i Q(t-i) + \sum_{j=1}^{n} b_j U_1(t-j) + \ldots + \sum_{p=1}^{r} b_p U_k(t-p) +$$
$$\sum_{s=1}^{v} d_s e(t-s) + C \tag{19}$$

If now $t \to \infty$ all variables approach their static values, the zero mean noise variables disappear and the constant C can be solved as

$$C \;=\; (1 + a_1 + \ldots + a_m)Q_{DC} - (b_{11} + \ldots + b_{1n})U_{1,DC} - \ldots$$
$$- (b_{1k} + \ldots + b_{1r})U_{k,DC} \tag{20}$$

According to (19) constant C is a basic level of the heating power. All fluctuation is built up on the constant C.

3.3 Heat flow estimation using the model

Static heat flow estimation follows the procedure outlined in chapter 2. Assume first an ideal case, where the indoor and outdoor conditions are kept unchanged during the phases 1 and 3. Obviously, the model (19) for phase 1 and its parameters is also suitable for phase 3, with the exception of the parameter C. An equation corresponding to (20) can be written, first for the model of phase 1:

$$Q_{DC1} \quad = \quad \frac{C_1}{1 + a_1 + \cdots + a_m} + \frac{b_1 + b_2 + \cdots + b_n}{1 + a_1 + \cdots + a_m} U_{DC1}, \tag{21}$$

where for clarity only one input variable is included. The corresponding equation for phase 3 is

$$Q_{DC3} \quad = \quad \frac{C_3}{1 + a_1 + \cdots + a_m} + \frac{b_1 + b_2 + \cdots + b_n}{1 + a_1 + \cdots + a_m} U_{DC1}, \tag{22}$$

Because in (22) the only changed variables are C_3 and Q_{DC3}, it is clear that

$$Q_{DC3} = Q_{DC1} - Q_S, \tag{23}$$

where Q_S represents the static heat flow from the adjacent room. Now, inserting (23) into (22) and subtracting (22) from (21) leads to a solution for Q_S

$$Q_S = \frac{C_1 - C_3}{1 + a_1 + \ldots + a_m}. \tag{24}$$

In practice, the solution (24) holds only for an ideal case. Subtraction of the constant parameters of any two arbitrary models of the form (20) will probably not lead to the intended solution. However, if the inner and outer environmental conditions during the phases 1 and 3 are similar, although not identical, then the model parameters a and b in phases 1 and 3 could be kept the same. A practical indicator that these conditions are fulfilled, could be a successful execution of the identification algorithm in phase 3 using the parameters of phase 1.

As a result, corresponding to (20), the following equations can be written for the phases 1 and 3

$$Q_{DC1} = \frac{C_1}{1+a_1+\cdots+a_m} + \frac{b_1+b_2+\cdots+b_n}{1+a_1+\cdots+a_m} U_{DC1}$$

$$Q_{DC3} = \frac{C_3}{1+a_1+\cdots+a_m} + \frac{b_1+b_2+\cdots+b_n}{1+a_1+\cdots+a_m} U_{DC3}. \qquad (25)$$

Now, if the dc-value of the input variable U_{DC1} approaches U_{DC3} the situation comes closer to the ideal case (24). At any rate, if the dc-characteristics of the input variables are close to each others, equations (25) will lead to an approximation of Q_S

$$Q_S \approx \frac{C_1-C_3}{1+a_1+...+a_m} = Q_{DC1} - Q_{DC3} - \frac{b_1+b_2+...+b_n}{1+a_1+...+a_m}(U_{DC1}-U_{DC3}). \quad (26)$$

Thus the solution is similar to equation (24). In practice, the result can be achieved in the following way. First, model (19) and its parameters are identified during phase 1. Consequently, a constant C_1 is produced. Then, during phase 3, identification continues but concentrating only on the constant C parameter. All the other parameter values are kept unchanged after phase 1. The second identification generates a constant C_3 and Q_S can be calculated.

The theoretical result presented above does not guarantee that the heat flow estimation will be successful. Several requirements must be met in order to achieve feasible results. Apart from familiarity with the problem and the applied techniques the following issues must be kept in mind:

1) Identification during the phases 1 and 3 must be synchronized, i.e., they must start and stop at exactly the same time of the day and preferably on the same, consecutive weekdays.

2) Distance between the modelled and real heating power must be the same in phases 1 and 3.

3) The model structure must be right and proper for both identification periods. An inappropriate model easily leads to unstable operation, especially in critical conditions.

4) The environmental circumstances must be suitable for identification. For example, a long period of warm weather, during which no heating is required, may ruin the model identification or overemphasize some features. Even short exceptional periods may have an effect on the results.

3.4 Identification

Minimization algorithm

The dynamic thermal behaviour of a heating process can be described by an ARMAX-model as shown in equations 12 and 13. Unfortunately, the model parameters are not known and it is difficult to find out their equivalents in real building structures. Luckily, the measured data collected from the building and its environment makes it possible to identify the parameters.

The following identification algorithm applies pseudolinear regression, which is also known as the extended least squares method. Pseudolinear regression is a special case of the prediction error method (Söderström & Stoica 1989, Ljung & Söderström 1983). The algorithm is based on the minimization of a criterion function $V(\Theta)$, derived from equation 16. In this case $V(\Theta)$ is considered a linear regression model, although the MA-part of (16) refers to a more complicated model. In anyway, by solving $e(t)$ of N measurements $V(\Theta)$ can be written as

$$V(\mathbf{\Theta}) \quad = \quad \sum_{k=1}^{N} \left[q(t) - \mathbf{\Psi}_0^T(t) \mathbf{\Theta}(t) \right]. \tag{27}$$

By following the linear regression procedure one can easily find out that the unmeasurable noise variables $e(t - 1)$, ... , $e(t - v)$ in (17) must be replaced with some measurable estimates. Otherwise the original procedure cannot be applied. A natural choice for $e(t)$ is a residual $\bar{\varepsilon}(t)$. It is computed using the parameter estimates

$$\hat{\mathbf{\Theta}} = \left[\hat{a}_1(t), \cdots, \hat{a}_m(t); \hat{b}_{11}(t), \cdots, \hat{b}_{1n}(t); \cdots; \hat{b}_{k1}(t), \cdots \hat{b}_{kr}(t); \hat{d}_1(t), \cdots, \hat{d}_v(t); \hat{C}(t) \right]^T$$

$$\tag{28}$$

with the following expression

$$\bar{\varepsilon}(t) \;=\; q(t) - \mathbf{\Psi}^T(t)\,\hat{\mathbf{\Theta}}(t), \tag{29}$$

where $\mathbf{\Psi}^T(t)$ is written as

$$\mathbf{\Psi}^T(t) = \big[-q(t-1),...,-q(t-m)\,;u_1(t-1),...,u_1(t-n)\,; ... \,;$$
$$u_k(t-1),...,u_k(t-r)\,;\bar{\varepsilon}(t-1),...,\bar{\varepsilon}(t-v)\,;1\big]. \tag{30}$$

Finally, after some mathematical operations the pseudolinear regression algorithm can be written as

$$\varepsilon(t) \;=\; q(t) - \mathbf{\Psi}^T(t)\,\hat{\mathbf{\Theta}}(t-1),$$

$$R(t) \;=\; R(t-1) + \gamma(t)\big[\mathbf{\Psi}(t)\,\mathbf{\Psi}^T(t) - R(t-1)\big], \tag{31}$$

$$\hat{\mathbf{\Theta}}(t) \;=\; \hat{\mathbf{\Theta}}(t-1) + \gamma(t)\,R^{-1}(t)\,\mathbf{\Psi}(t)\,\varepsilon(t).$$

The above equations need several brief comments. First, note that instead of the residual, $\bar{\varepsilon}(t)$ in equation (31) is written using the prediction error $\varepsilon(t)$, available at time t–1. Secondly, the expressions (31) are presented in a recursive form. This is not obligatory, but an algorithm of this kind was utilized, when the results of the following chapters were computed. Recursive algorithms may be useful if a number of identifications are computed at the same time in one computer or when the computation is executed by a small microprocessor based control system. Also note that the last expression of (31) is written in a simplified form. Usually, the matrix inversion R^{-1} is substituted by an inversion of a scalar.

Preparatory treatment of data

The process data usually requires some prefiltering or other preparatory treatment before it can be applied to a minimization algorithm. For example, low pass filtering reduces the unnecessary high-frequency noise components of the measured signals. The situation is somewhat different when the data comes from a simulated process. In that case, the measurements do not contain similar disturbances, or even any process noise. All the data presented in this paper have been generated in this way. That is why a white noise component is added to all measurement signals. This is illustrated in table 1, which presents the process variables of the ARMAX-model and the added percentage white noise in the measurement. The white noise is created by a noise-generating algorithm (Press et al. 1992), which is one procedure in the identification program modules. Yet, process noise, which consists of unmeasurable disturbances exciting the process, is not included in the data.

Table 1. Structure of the ARMAX-model and added white noise related to each measurement.

Variable or constant	Order of the polynomial	Added white noise [%]
$q(t)$	2	2.0
$U_o(t) - u_R(t)$	3	1.0
$v(t)$	3	9.0
$S(t)$	3	4.0
$E(t)$	3	2.0
$O(t)$	3	3.0
1 [*]	1	-
$e(t)$	3	-

[*] refers to the constant parameter C.

Model validation

In addition to the model parametrization and identification, one has to determine the proper number of parameters for polynomials in (13). Both under- and over-parametrization have negative effects on the results. Thorough analysis of parametrization becomes rather complicated. Therefore, model validation was performed by testing the prediction error $\varepsilon(t)$ and by visually comparing the model and the measured process output. A perfect model generates a white noise prediction error during minimization. Similarly, $\varepsilon(t)$ should have a symmetric distribution, i.e., in the sequence of $\varepsilon(1), \varepsilon(2), ..., \varepsilon(N)$ the sign should change, at an average every second time step and there should be an equal number of positive and negative values of $\varepsilon(t)$ (Söderström & Stoica 1989). By varying the number of parameters in each polynomial (13) and by performing the above two tests, the model structure of table 1 was produced: *M[2,3,3,3,3,3,1,3]*. The order of the polynomials in the input parameters seems rather high, but time delays of heat transfer in buildings are rather long. The applied time step was 0.25 *h*.

On the following pages specific ARMAX-models are quoted using the above notation: *M[2,3,3,3,3,3,1,3],* where the first and last numbers in the parenthesis refer to the order of polynomials $A(z^{-1})$ and $D(z^{-1})$ of (13). All the other numbers refer to the orders of the polynomials of the applied input variables, i.e., in this case $B_1(z^{-1}), ..., B_5(z^{-1})$, and the constant parameter *C*.

Observe that the model *M[2,3,3,3,3,3,1,3]* is a result of a general, but not very profound model validation procedure. However, the suggested model structure is somewhat too complicated for heat flow estimation. This will be shown later.

4. Issues concerning technical implementation of an EAS

So far, the analysis has discussed only the theory of heat flow estimation and ignored the technical requirements of an EAS that would allow the method to be applied. If the presented theory is valid for the estimation of static heat flow rates through walls, is there available a technology for developing an advanced, low-cost, technically and commercially feasible EAS?

The energy allocation system applying the method should be equipped with instrumentation that gathers useful information, from the building and its environment at low cost. The system should be able to record at least the following measurements:

1) From each room/flat: indoor air temperature, heating power, ventilation rate, electric power, occupation

2) From each facade: outdoor air temperature, solar radiation

3) From the whole building: wind velocity

Most of the above measurements, including wind velocity, solar radiation, temperature, or power measurements are conventional and can be accomplished by means of ordinary building automation instrumentation. Appropriate commercial sensors are available, which give relevant information of these physical quantities at low or moderate cost. Their technical implementation will therefore not be discussed here. However, some of the sensors are not ordinarily available in every building. Wind velocity sensor is one instrument of this kind. It is a known fact that wind pressure on a building facade increases air infiltration and hence indirectly also energy consumption. It would thus be reasonable to include wind velocity as one of the process variables in the dynamic model. An even better and a more economical alternative might be to use the sensor to inform the EAS of inappropriate wind conditions for identification. This is due the fact that air infiltration caused by wind pressure varies as a function of wind direction and one sensor cannot give a comprehensive view of the effect on the whole building. Hence the EAS should

be equipped with a number of wind velocity sensors, installed all over the building. This is not an economical solution, however.

Similar problems are also implicit in other measurements. Several solar radiation sensors are needed in each facade to compensate for shadows caused by other buildings, trees, etc. Fortunately, proper solar sensors are not expensive. Moreover, indoor air should be measured from each room of the flat. In order to get a good view of the average air temperature the sensor should be located in the middle of the room, which is usually not possible. This is an example of the cases, in which one has to accept a compromise between the costs and the optimum performance of instrumentation.

One of the rarely measured process variables is the ventilation rate of the room/flat. Nowadays, commercial products are available which are able to provide information of ventilation rates in each room of a residential or apartment building and even control the ventilation rate. Obviously, such techniques will be more common in the future and ventilation rate might become a standard, measurable signal. Such techniques would be appropriate for buildings equipped with Variable Air Volume (VAV) systems, especially if the technique is available at low cost.

Unfortunately, VAV is rarely used in residential and apartment buildings. A more common arrangement is centralized air exhaust, in which one powerful fan runs at two different speeds according to a daily schedule. Several rooms or even flats are connected to the same fan through a duct network. One may assume that the ventilation rate remains stable in each apartment unless strong wind pressure, the opening of a window or a door or a similar event changes the air pressure balance in the duct network. The centralized air exhaust system can only offer rough information of the actual ventilation rate. This is accomplished by measuring the fan power stage, rather than the actual exhaust air rate. This is probably enough for the identification and estimation of static heat flows, provided that air pressures in the ducts remain stable and air flows in balance. It is not necessary to measure the exact air ventilation rate in each flat for promoting energy allocation because the energy consumption caused by air ventilation is included in the energy costs of the flat. Thus, if these conditions are acceptable, a rough and low-cost procedure of exhaust air rate measurement is easy to arrange directly by means of the control relays of the fan.

Occupation is another rarely applied measurement, but it is becoming more popular due to the low-cost infra-red sensors widely applied in alarm systems. A similar technique is applicable to an EAS. It allows an easy way to determine the number of persons entering a flat or a room. The information is necessary for a good estimate of inner heating loads.

It is clear that extra instrumentation increases the investment costs of the system. This is not in line with the requirement for an economical solution. But the instrumentation needed in an EAS is mostly similar to that applied in home and building automation systems (HAS, BAS) and/or energy management systems (EMS). They are conventional systems of the kind usually installed in every building. Thus, an economical solution obviously involves an EAS with its individual metering procedures combined with a HAS, BAS or EMS and without any parallel instrumentation. The combined system utilizes data processing capabilities and standard interfacing techniques typical of a HAS, BAS or EMS and extra instrumentation provided by an EAS in control, energy management and individual metering procedures. Consequently, by increasing instrumentation and partly sharing resources with the other technical systems of a building, it is possible to design a feasible and probably also commercially viable energy allocation system.

5. Simulated building

5.1 TRNSYS simulation program

The test building, its structures, operation and environmental conditions were created using TRNSYS, a modular building simulation program (Klein et al. 1996). TRNSYS is a collection of system components, with each component representing a model of a functional part of a building. The user can define the parameters and inputs for each model. He may also apply the model output as an input to the next model and thus create a combination of interconnected components, which operate as one large simulation system. TRNSYS is implemented using the FORTRAN programming language. Each system component is described by a subroutine.

TRNSYS contains two different building models, Type 19 and Type 56. The former is suitable for single-zone and the latter for multi-zone simulations. TRNSYS is able to compute the thermal behaviour of a building with up to 25 thermal zones. In this work Type 56 was applied to twelve zones.

The following brief introduction clarifies the elements and modelling of a zone used in TRNSYS. A more comprehensive description can be found in Klein et al. (1996). The building model assumes one real air node per zone. The convective heat balance of the zone consists of the following gains (figure 2): ventilation $Q_{vent,i}$, infiltration $Q_{infl,i}$ (air flow from outside), internal convective gains $Q_{int,c,i}$ (by people, equipment, lighting etc.), convective heat flow from all inside surfaces $Q_{surf,i}$, gains due to convective air flow from other zones $Q_{cplg,i}$ (coupling), and convective heating $Q_{heat,c,i}$ (radiators etc.). The thermal capacity of the air node consists the capacities of the zone air and the furniture.

Furthermore, TRNSYS utilizes radiative heat flows in each zone (figure 3). The following gains are included: internal radiative gains $Q_{int,r,i}$ (by people, equipment, lightning etc.) received by the wall, solar gains through windows $Q_{sol,i}$ received by the wall, long-wave radiation exchange $Q_{long,i}$ between the wall and all the other surfaces, and the radiative part of heating $Q_{heat,r,i}$ (radiators etc.) received by the wall.
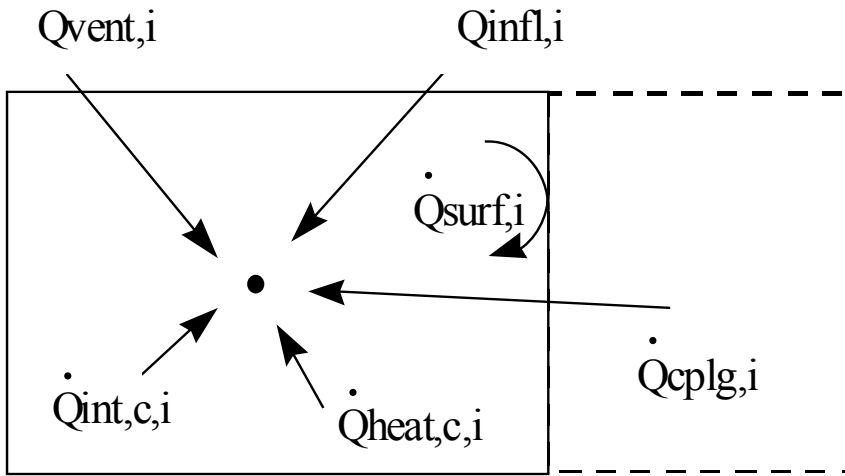
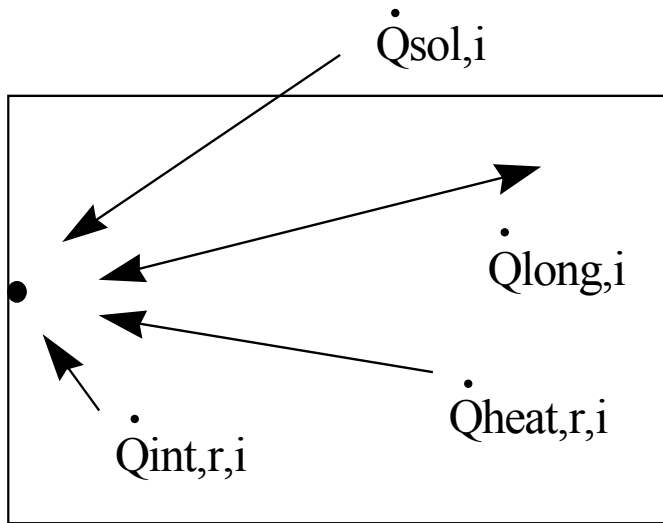*Figure 2. Heat balance on the zone air node consisting of convective gains.*

Figure 3. Radiative energy flows on one wall.

Besides the zone air temperature, TRNSYS applies an artificial air node, denoted as star temperature. By means of this artificial node, the net radiative and convective heat fluxes from the inside surfaces can be computed.

The walls, windows and slabs are essential structures of the TRNSYS zone model. Energy balances at the surfaces give combined convective and radiative heat fluxes. Conductive heat transfer through a wall, slab or window is computed using the Transfer Function Method, which is based on response factors, i.e., the thermal history of the structure.

All of the above features are included when the energy balances are written for each zone. A mathematical description of the energy balances leads to a linear set of equations resulting in air temperatures for each zone at each time step.

Although the zone and building model of TRNSYS is versatile and applicable to numerous cases, simulation programs can only approximate the dynamic thermal behaviour of a real building. When such programs are designed several simplifications must be made. The typical assumptions of TRNSYS and other corresponding simulation programs are:

1) Air temperature is uniform all over the room.

2) Zone surfaces are isothermal, i.e., only one surface node per wall is needed.

3) Heat transfer coefficients are constant and uniform over each surface.

4) Radiative heat transfer from a surface to the air and to other surfaces is proportional the corresponding temperature differences.

5) Heat transfer through a wall, slab or window is one-dimensional.

6) Solar gains through windows are distributed to all walls.

Building simulation programs have restrictions that have an inevitable effect on the predicted dynamic thermal behaviour of a zone or building (Lomas et al 1997). This must be kept in mind when the results of the following chapters are considered.

5.2 Description of the test building

The simulated building is an office hotel. It consists of twelve rooms (zones), located in two storeys, with five office rooms and a corridor in both storeys. The total floor area of the building is 239 m^2. The area of the smallest room is 12 m^2. The other floor areas vary from 18 to 25 m^2. Figure 4 illustrates the layout of the building. The building is assumed to be located in the central part of Finland. Weather data from Jyväskylä was utilised. The following tables show some characteristic features of the building. Table 2 shows the U-values of the structures and table 3 the air flows. The ordinary ventilation rate for the whole building is 1.05 1/h. It is assumed that supply air has the temperature and humidity of the ambient air. Ventilation is operated from Monday to Friday between 7 am and 5 pm. Infiltration changes the air content of the rooms all the time.

1. floor

15 (12.4 m2)

14 (24.9 m2)

16 (28.5 m2)

11 (18.0 m2)

12 (17.6 m2)

13 (18.0 m2)

2. floor

25 (18.7 m2)

24 (18.7 m2)

26 (28.5 m2)

21 (18.0 m2)

22 (17.6 m2)

23 (18.0 m2)
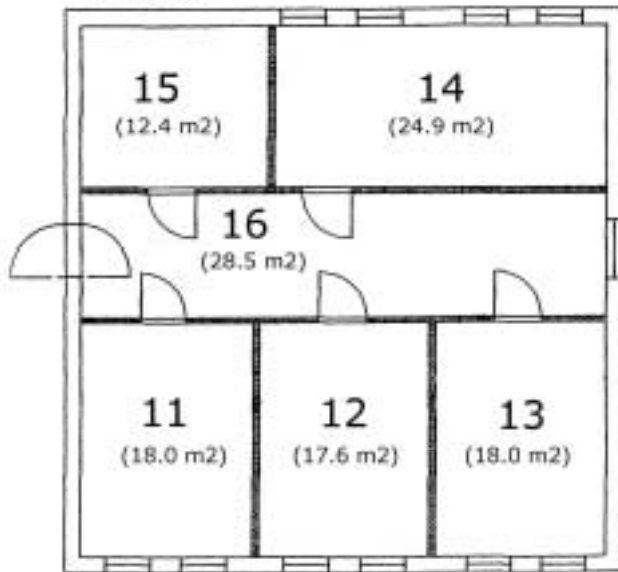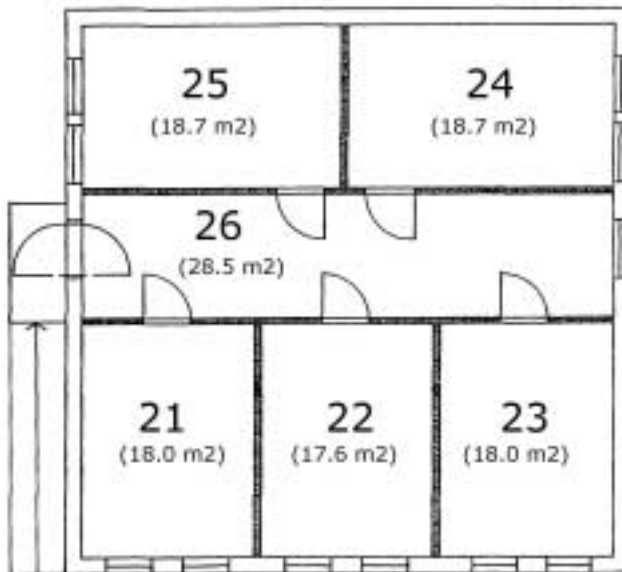
Figure 4. Layout of the test building.

Table 2. U-value and capacity of the structures. U-values include convective heat transfer coefficients, which are 3.1 W/m²-K for inside surfaces and 17.8 W/m²-K for outside surfaces.

Structure	U-value [W/m²-K]	Capacity [kJ/m²-K]
External wall, case 1	0.28	401
Internal wall (wall between zones), case 1	1.34	282
Internal wall, case 2	0.80	20
Internal wall, case 3	0.34	22
Ground floor	0.29	204
Floor between storeys	0.79	384
Roof	0.20	422
External door	0.54	13
Internal door	1.06	6
Window	1.33	-
Window in external door	1.80	-

Table 3. Air flows in a building, reference case

Room number	Ventilation air flow (dm3/s)	Infiltration air flow (dm3/s)	Coupling air flow (dm3/s)	Exhaust air flow (dm3/s)
11	18.0	1.5	3.1	22.6
12	17.6	0.9	3.0	21.5
13	18.0	1.5	3.1	22.6
14	24.9	1.9	4.3	31.1
15	12.4	0.7	2.1	15.3
16	14.3	1.4	-15.7	0.0
21	18.0	1.5	3.1	22.6
22	17.6	0.9	3.0	21.5
23	18.0	1.5	3.1	22.6
24	18.7	1.6	3.2	23.5
25	18.7	1.6	3.2	23.5
26	14.3	1.4	-15.7	0.0

The following internal gains are included in the simulated building. One person is working in each office room from Monday to Friday from 8 am to 4 pm. The rooms 14, 24 and 25 are different in that they have two occupants. Every office worker has a lunch hour during the working day, and they go out for lunch. Lighting is on from Monday to Friday between 8 am and 4 pm. Lighting heat gain is 13 W/m^2 in the office rooms and 5 W/m^2 in the corridors. Furthermore, all the other office rooms, except the rooms 12 and 23, have one computer each.

It is assumed that all the room temperatures of the building are controlled according to the procedure presented in chapter 2. A heater has been installed, but no cooling is available. Thus, on a warm day, the indoor temperature may temporarily rise over the set point temperature. 60 % of the heating load is convective and 40 % radiative.

The starting time of simulation is the beginning of the year, and the length of simulation is 16 weeks. Sampling time is 15 minutes.

6. Analysis of the ARMAX-model in the simulated environment

6.1 Approach

Before applying the ARMAX-model in heat flow computations, it is reasonable to analyze the model itself. How does the model fit in the simulation environment and what are the expected results in ideal cases? Such an examination gives more insight into the performance of the model and helps to make fair judgements about the approach in a real building environment. The following analysis is possible because behaviour of the simulated building is precisely known, and the introduction of modifications in the building envelope, structures, inner and outer disturbances is easy and all test runs can be repeated many times if needed.

6.2 Parameter values as a function of disturbances

The following two tables show how the parameters of an ARMAX-model vary due to changes in the indoor temperatures of neighbouring rooms. The presented parameter values are the results of two identifications of (12), concentrating on the heating power of the same room. First, the indoor temperature of both the monitored room and the surrounding rooms are kept stable at 20°C. Table 4 presents the data in this case. Then, the indoor temperatures of the surrounding rooms are raised to 23°C. The identification is performed again, generating the parameters of table 5. Both cases are computed using exactly the same weather and disturbance data. Without knowing the behaviour of an ARMAX-model one would assume that an obvious change would occur in parameter *C*, which accumulates the dc-values of the inputs and the output. However, slight variation in parameter values can be seen in all parameter values. The tables clearly indicate that changes do not focus only on one or two parameters but almost all of them. Thus, extracting the effect caused by a modification introduced into a room or its environment is difficult by direct comparison of the parameter values. The result is evident, of course and typical of black box models, but gives a more detailed view of the model structure and an insight into its parameter values.

Table 4. Parameter values of the reference case. Indoor temperatures are set to 20 °C. The applied model is M[3,2,2,2,2,2,1,3].

Par. n:r	a	b_1	b_2	b_3	b_4	b_5	c	C
1	-0.9213	-0.0105	-0.0646	-0.0002	0.1424	0.0063	0.7357	0.0086
2	-0.4115	0.0095	-0.1201	-0.0003	0.4884	0.0349	-0.2230	-
3	0.3735	-	-	-	-	-	-0.0083	-

Table 5. Parameter values of the case where the indoor temperatures of the neighbouring rooms are set to 23 °C, while the other settings are the same as in the reference case. The applied model is M[3,2,2,2,2,2,1,3].

Par. n:r	a	b_1	b_2	b_3	b_4	b_5	c	C
1	-0.8827	-0.0108	-0.0212	-0.0002	0.1041	0.0033	0.7095	-0.0060
2	-0.4518	0.0097	-0.1579	-0.0003	0.5085	0.0375	-0.1902	-
3	0.3779	-	-	-	-	-	0.0030	-

Figures 5 and 6 show a sample of the predicted and real heating power. They illustrate the cases presented in tables 4 and 5. The overall agreement of the curves in each figure seems to be good. In figure 5 all indoor temperatures are equal, and in figure 6 the indoor temperatures of the neighbouring rooms are higher.

The figures clearly show the central idea of the heat flow estimation. If the lowest levels of real heating power in both figures are compared, the effect of the room temperature difference becomes visible. In figure 5 the lowest point is about 350 to 400 W and in figure 6 between 0 and 50 W. Hence, their difference, which is the static heat flow between rooms is approximately 300 to 400 Watts (exactly 337 W). Unfortunately, the illustrated case is not real, since both curves are computed using the same weather and disturbance data. This is not possible in practice and makes the problem more complicated and demanding. Therefore the proposed method is based on comparing the dc-levels of heating power. The

purpose of the ARMAX-model is to reduce the disturbing effects of weather and other process variables, so that the final comparison could be made between pure static heat flows.

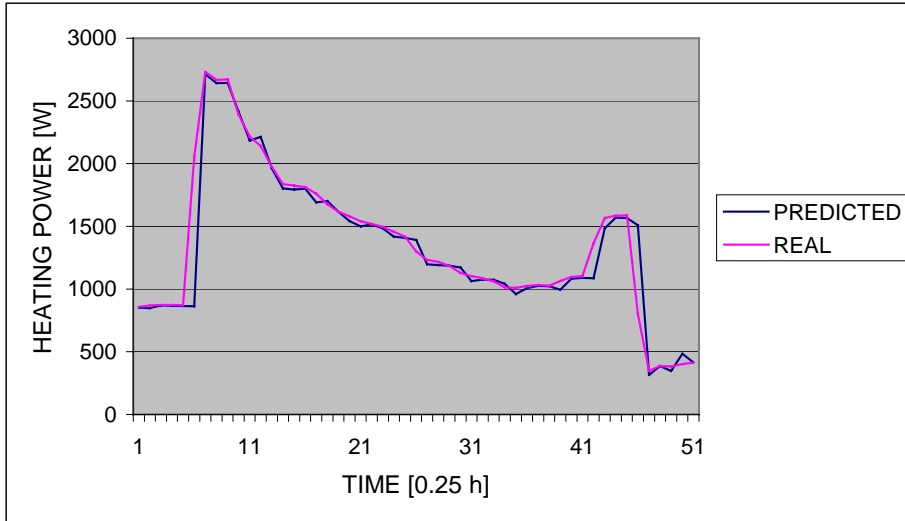*Figure 5. A sample of real and predicted heating power of room 21. All room have equal indoor temperature (20 C). In this case $Q_{DC} = 0.979$ kW.*
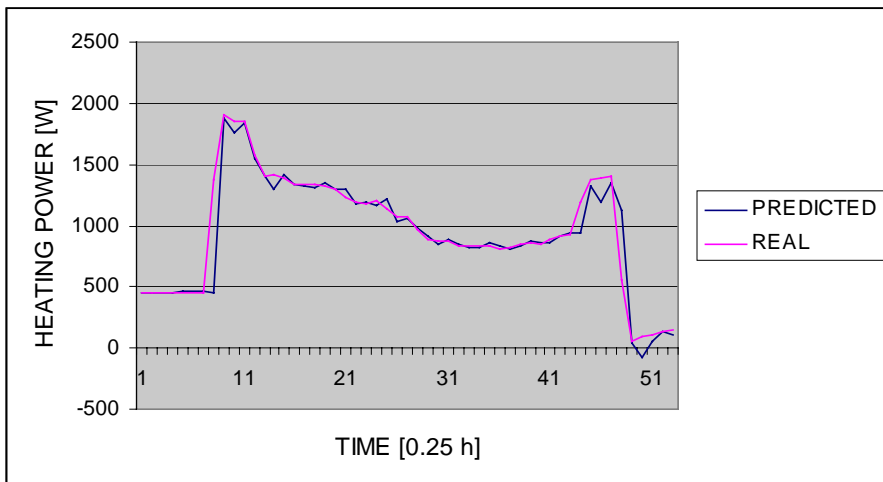

Figure 6. A sample of real and predicted heating power of room 21 when the indoor temperature of the neighbouring rooms is 23 C. Due to the static heat flow from neighbouring rooms, the dc-level of heating power is lower than in figure 4. In this case $Q_{DC} = 0.642$ kW.

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6.3 Static heat flow in an ideal case

Static heat flow estimation according to chapter 2 is done in three different phases. Model identification is performed in the phases 1 and 3. Phase 2 is needed for steering room temperatures to new values. The final, estimated heat flow is a mean value, computed from the sampled data of several test runs (equation 24). Now, assume that the weather data in addition to the data of inner disturbances is exactly identical in the phases 1 and 3, which is not possible in real buildings. Obviously, such an arrangement will be seen in the results as smaller deviations from the mean valued heat flow. Therefore, the properties of the estimation procedure can be examined in a case that is close to ideal.

Tables 6, 7, and 8 present the results of identified static heat flow between room 21 (24) and the neighbouring rooms. Estimation is performed in an ideal case as presented above. The results are obtained using an identification time of five days. The applied ARMAX-model is *M[0,2,2,2,2,2,1,3]*.

Case 1. Tables 6 and 7 present a case where all room temperatures are first set to 20 °C (phase 1) and the temperatures in the neighbouring room are then raised to 23 °C (phase 3). In this way the net heat flow between room 21 (24) and the neighbouring rooms is achieved. By changing the thermal properties of the inner walls, several cases are computed.

Case 2. The test runs of table 8 differ from the above. First, (phase 1) all room temperatures are kept at the same level (20 °C), then the indoor temperature of room 21 (24) is raised to 23 °C (phase 3). As a result, the net heat flow between room 21 (24) and any of the surrounding rooms can be estimated.

The results in the tables set a minimum level for the errors. They can be later compared with the estimation results of more realistic test runs.

Table 6. Actual and estimated static heat flow rate between room 21 and all the neighbouring rooms through inner walls and slabs when the U-value of the inner walls is gradually changed (case 1). The results are mean values of a few estimates.

U-value of the inner walls/slabs/door [W/m2-K]	Actual static heat flow rate through the inner walls [W]	Estimated static heat flow rate through the inner walls [W]	Resulting error [%]
0.34/ 0.79/ 1.06	116	122	4.9
0.80/ 0.79/ 1.06	181	187	3.3
1.34/ 0.79/ 1.06	325	327	0.6

Table 7. Actual and estimated static heat flow rate between room 24 and all the neighbouring rooms through the inner walls and slabs when the U-value of inner walls is gradually changed (case 1). The results are mean values of a few estimates.

U-value of the inner walls/slabs/door [W/m2-K]	Actual static heat flow rate through the inner walls [W]	Estimated static heat flow rate through the inner walls [W]	Resulting error [%]
0.34/ 0.79/ 1.06	120	118	4.3
0.80/ 0.79/ 1.06	187	192	2.8
1.34/ 0.79/ 1.06	338	339	0.4

Table 8. Actual and estimated static heat flow rate between room 21 and the neighbouring room(s) when the U-value of the inner walls is 1.34 [W/m2-K] (case 2). The results are mean values of a few estimates.

Static heat flow between rooms	Actual static heat flow rate through the inner wall [W]	Computed static heat flow rate through the inner walls [W]	Resulting error [%]
21 and 22	142	141	0.5
21 and 11	61	62	1.4
21 and 26	101	103	2.2

6.4 Number of input variables

Obviously, errors increase when one or more input signals are dropped out of the model, but what is the error if all inputs are removed? Table 9 shows the influence of a few input signals. One input is not meaningful, but if all inputs are removed, the total error is about thirty percent compared the real static heat flows. The error could be greater but the MA-part of the ARMAX-model is able to adopt some of the cyclical variations of the heating power and compensate for the dropped input signal.

Table 9. Comparison of identified static heat flows through the inner walls in room 24 by gradually reducing the model structure.

U-value of the inner walls [W/m2-K]	Actual static heat flow rate [W]	Error, when air ventilation not included [%]	Error, when only $u_o(t) - u_R(t)$ included [%]	Error, when no inputs included [%]
0.34	120	2.5	14.2	27.5
0.80	187	3.7	14.4	27.8
1.34	340	4.7	16.2	36.2

6.5 Effect of ARMAX-model structure

The model validation in chapter 3 was based on the testing of prediction errors and visual inspection of the model behaviour. No further validation was applied, although the order of the polynomials $A(z^{-1}),\dots,C(z^{-1})$ seemed to be somewhat high. In order to prevent overparametrization and to further validate the model structure, another method was applied.

Now, a few promising models are compared in a test procedure, where the criterion for a good model is a successful static heat flow estimate (Q_S). First, the ARMAX-model is identified as usual. Each identification takes five days from Monday to Friday. Then, an estimate is computed. This is repeated ten times. Furthermore, an ideal case is assumed (the same measured data is used for the phases 1 and 3). Finally, a mean value of the ten estimated Q_S is calculated and denoted as $\overline{Q}_S$.

Table 10 presents the results. The bottom line presents the estimated Q_S, which should be 338 W. Observe that the best models contain no past values of $q(t)$, although such an expression was suggested by the validation procedure of chapter 2. In addition, the order of the other polynomials is lower than that proposed in table 1. A factor that may have contributed to the lower order of the polynomial $A(z^{-1})$ can be seen from equation 17. When Q_S is calculated, polynomial $A(z^{-1})$ is the divider. Variation in the numerical values of the polynomial easily causes larger deviations in Q_S. Also note that the results of table 10 only apply in an ideal case. The situation and the needed model may be different when the circumstances for estimation become more difficult.

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Table 10. Comparison of ARMAX-models used to estimate static heat flow rate Q_S in an ideal case. Columns on the right present order of the polynomial for each variable. The last row presents the estimated Q_S.

Variable or constant	Model 1	Model 2	Model 3	Model 4	Model 5
$q(t)$	0	0	1	2	0
$u_i(t) - u_R(t)$	1	2	2	2	3
$v(t)$	1	2	2	2	3
$S(t)$	1	2	2	2	3
$E(t)$	1	2	2	2	2
$O(t)$	1	2	2	2	2
1 [*]	1	1	1	1	1
$e(t)$	3	3	3	3	3
	339 W	338 W	361 W	348 W	339 W

[*] refers to the constant parameter C.

6.6 Time steps needed for identification

Tables 11 and 12 present the effect of identification time on Q_S values. The idea is to find out a proper number of time steps for model identification. The applied model is presented in tables 4 and 5. According to tables 11 and 12, a few days are enough. A longer identification period does not give crucially better results. Five days still seems to be a rather long period if the identification must be performed several times for heat flow determination. However, a shorter period can be achieved by special arrangements. One method is to set the initial parameter values according to a previous test run and to shorten the total time noticeably. Another solution would be to start the identification several days before the phase (1 or 3). When the phase starts, the minimization algorithm already has processed the parameters closer to their final values. In addition, modification of the minimization algorithm may have an influence on the convergence rate. For example, Ljung & Söderström (1983) have shown that the gain sequence $\gamma(t)$ in (31) has an effect on the convergation rate. When these arrangements are used, one week may be a minimum for all the three phases 1, 2 and 3.

Table 11. Comparison of identified static heat flows through the inner walls in room 21 using different numbers of time steps for identification.

Actual static heat flow rate [W]	Error using 480 time steps (5 days) [%]	Error using 960 time steps (10 days) [%]	Error using 4800 time steps (50 days) [%]
112	2.5	2.5	3.5
179	2.7	5.9	0.0
329	4.1	4.7	2.7

Table 12. Comparison of identified static heat flows through the inner walls in room 24 using different numbers of time steps for identification.

Actual static heat flow rate [W]	Error using 480 time steps (5 days) [%]	Error using 960 time steps (10 days) [%]	Error using 4800 time steps (50 days) [%]
120	1.7	0.8	1.7
187	0.5	1.6	2.7
340	0.9	2.6	3.5

7. Results of static heat flow estimation

7.1 Approach

All the following results are based on the procedure and expressions presented in chapters 2 and 3. The applied building is the simulated office hotel of chapter 4. Collecting and processing the measured data is done in a similar manner as would be done in a real building environment. This means that each identification is performed using unique weather and disturbance data. Each identification is also followed by a phase 2, during which the neighbouring indoor temperature is steered to a new value. It is assumed that the time period of phase 2 is long enough for the structures to achieve their new stationary temperatures. The procedures are applied in the rooms 14, 21, 24 and their neighbouring rooms. The applied temperature differences between adjacent rooms are 3.0°C and 1.5 °C. Observe that the former value is so high as to be probably difficult to achieve in real apartment building.

7.2 Static heat flow through walls and slabs

The following simulated test is directly based on the procedure described in chapter 2. First, the indoor temperatures of room 21 (24) and the neighbouring rooms are kept stable at 20°C (phase 1), and then the temperatures in the neighbouring rooms are adjusted to and kept at 23°C (phase 3). Identification of the phases 1 and 3 takes five days from Monday to Friday, producing a static heat flow Q_S, solved using equation 26. Phase 2 consists of the weekend, i.e., Saturday and Sunday, between the phases 1 and 3. The situation is equal to the case 1 in chapter 4.3, but unique weather and disturbance data are now applied to the phases 1 and 3. The applied ARMAX-model was *M[0,2,2,2,2,2,1,3]*.

The table presents three cases, where U-value of the inner walls differ (see table 2). In each case, five estimations have been made. U-values of other structures do not change. So, the total heat flow consists of summed heat flow from the neighbouring rooms through walls and slabs to room 21 (24). This is computed by TRNSYS-program. Table 13 contains results received from room 21 and its environment and table 14 illustrates corresponding results for room 24.

Table 13. Actual and estimated static heat flow rate from neighbouring rooms to room 21 through inner walls and slabs when the U-value of the inner walls is gradually changed.

U-value of the inner walls [W/m2-K]	Total, actual static heat flow rate [W]	Estimated static heat flow rate (Q_S) [W]	Resulting error [%]
0.34	116	139	+ 19.8
		96	- 17.2
		140	+ 20.6
		139	+ 19.8
		104	- 10.3
0.80	181	226	+ 24.9
		174	- 11.6
		205	+ 13.0
		160	- 11.5
		167	- 7.6
1.34	325	296	- 8.9
		349	+ 7.0
		367	+12.9
		307	- 5.5
		361	+ 11.1

Table 14. Actual and estimated static heat flow rate from neighbouring rooms to room 24 through inner walls and slabs when the U-value of the inner walls is gradually changed.

U-value of the inner walls [W/m2-K]	Total, actual static heat flow rate [W]	Estimated static heat flow rate (Q_S) [W]	Resulting error [%]
0.34	120	149	+ 24,0
		160	+ 33.3
		92	- 23.1
		99	- 17.3
		150	+ 25.0
0.80	187	215	+ 14.9
		151	- 19.2
		200	+ 7.2
		226	+ 20.8
		165	- 11.7
1.34	338	363	+ 7.6
		291	- 13.8
		349	+ 3.5
		372	+ 10.1
		314	- 7.2

7.3 Static heat flow through a wall

The procedure of chapter 2 is followed again. First, the indoor temperatures of room 21 and the neighbouring rooms are kept stable at 20°C (phase 1). Then, only the indoor temperature of room 21 is steered to and kept at 23°C, while the others remain at 20°C (phase 3). Both phases take five days from Monday to Friday, producing a static heat flow Q_S, using equation 26. Phase 2 consists of the weekend, as earlier. The procedure is repeated five times, similarly to tables 13 and 14.

The results are presented in table 15. The numbers on the last row illustrate the heat flow estimation between room 21 and all the adjacent rooms.

Table 15. Actual and identified static heat flow rate between room 21 and the neighbouring rooms when the U-value of inner walls is 1.34 [W/m2-K].

Static heat flow between rooms	Actual static heat flow rate through the inner wall [W]	Estimated static heat flow rate through the inner wall (Q_S) [W]	Resulting error [%]
21 and 11	62	71	+ 14.7
		31	- 50.3
		96	+54.8
		33	- 46.2
		99	+ 60.4
21 and 26	101	146	+ 44.9
		110	+9.4
		69	- 31.7
		47	- 53.1
		42	- 58.6
21 and 22	142	165	+ 15.9
		96	- 32.6
		174	+ 22.5
		128	- 10.1
		186	+ 31.2
21 and 11,22,26	331	384	+ 16.2
		359	+ 8.6
		300	- 9.4
		324	-2.2
		366	+ 10.5

7.4 Mean valued heating power vs. the new method

As noted in chapter 2, a simple method for heat flow estimation is to apply the mean valued heating power, computed during the phases 1 and 3. Their difference is an approximate solution for the static heat flow. Figure 7 presents a comparison of the mean valued method and the new ARMAX-model –method. The target value of the static heat flow is 325 W, and it is presented as the

straight line in the figure. The largest variations result from the mean valued method. The curve generated using the new method follows rather close to the target line. Probably only a few points are needed for a reasonable heat flow estimate.
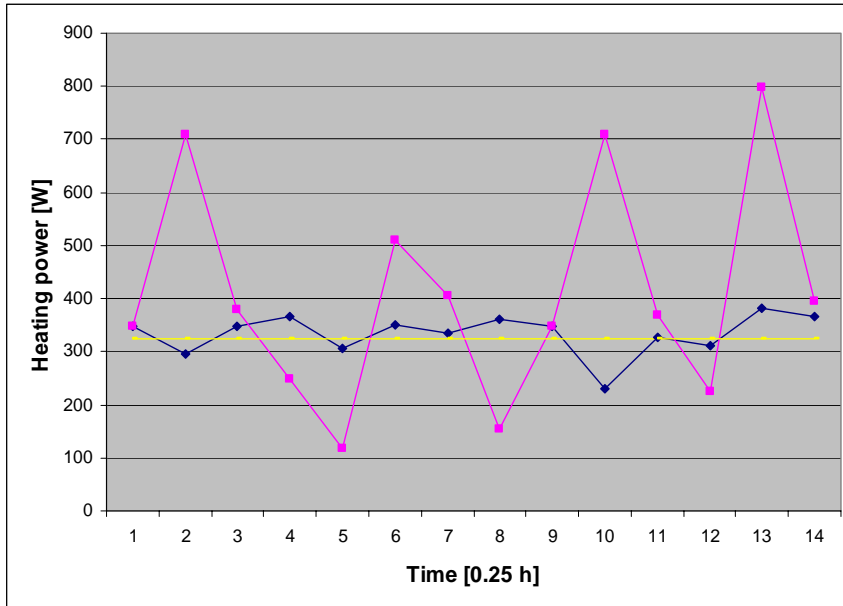

Figure 7. Comparison of static heat flow estimates generated with the aid of mean valued heating power (the curve with the largest fluctuation) and the method based on the ARMAX-model (the curve with smaller fluctuation). The straight line represents the targeted, static heat power flowing into the room 21.

7.5 Error as a function of static heat flow

The previous results clearly indicate that the error increases when the heat flow value decreases. Based on the collective results, the error as a function of heat flow can be outlined. Figure 8 presents such a curve. The data for the curves are gathered from the rooms 14, 21, 24, and their neighbouring rooms. The figure shows that the estimation error is less than 10 % if heat flow values are large enough. Observe that the curve illustrates the expected error for a single estimation only. By performing the estimation procedure a few times and by calculating a mean value of the results, a smaller percentage error is obtained.
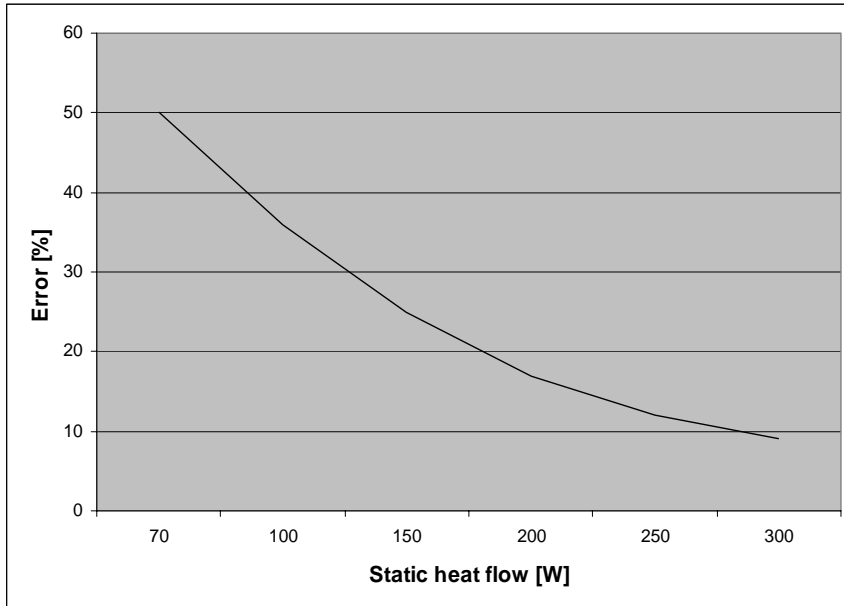
Figure 8. Approximate resulting error as a function of static heat flow value.

7.6 Remarks from the test runs

The heat flow estimates between the rooms 21 and 26 seemed to be more difficult than the others. Many of the trials failed. This is assumed to be due to the high coupling air rate of room 26 (table 2). The coupling air rate is not included in the ARMAX-model. If this is true, similar results can be expected when one tries to estimate the heat flows between the adjacent rooms of a flat. If a door between the rooms remains open, the large coupling airflow may ruin the heat flow estimation. One possibility to avoid the problem is to identify the whole flat in one model.

Although not shown by the tables, the results also contained some outliers, i.e. erroneous points, which seemed to be caused by poor modelling, the influence of noise, abnormal exciting signals, etc. and resulted in unstable operation during identification. Outliers differ more or less from the typical estimate values. However, many outliers can be detected by close monitoring of the identification algorithm using a proper diagnostic procedure. The applied estimation procedure

did not contain any diagnostic features. Therefore, some outliers were removed from the data set manually. The problem is that some deficiencies in estimation had no or only a slight effect on the result. One objective in the future will be to set criteria for a successful estimation and to design a diagnostic algorithm capable of detecting unsuitable estimates.

The applied ARMAX-model was *M[0,2,2,2,2,2,1,3]*. The method is clearly sensitive to the applied model. A slight modification in the model structure may cause considerable changes in the results. The test runs also indicated that one ARMAX-model is not enough for all cases if the optimal performance is targeted. So, a good model is a prerequisite for successful estimation.

The errors presented in the above tables only concerns the simulated cases. Evidently, the errors will be larger in a real building. One source of error that has not yet been discussed is caused by inaccuracies in indoor temperature control. The estimation procedure assumes that the mean valued indoor temperature can be kept at the same level during the phases 1 and 3. In practice, it may be difficult to achieve and maintain these temperatures. Consequently, offsets in indoor temperatures will increase errors in the estimated static heat flows.

8. Discussion

The foregoing pages raise several questions, such as: What is the error level of the estimation method in a real environment? Is the proposed method stable and robust enough to be applied in practice? The method needs a special control procedure. Is it applicable, and implementable in real HVAC systems using the available instrumentation? If the method turns out to be practical, how heat flow estimation will be made for an apartment building and how long will such a procedure take? What are the necessary actions to be made to resolve its usefulness and perhaps to further develop the method?

Perhaps the most crucial question concerns error levels of the method in a real environment. The above results were achieved by simulating the dynamic thermal behaviour of a building. Although several comparisons have shown that the behaviour of a simulated building can be made consistent and close to a real building, a number of limitations remain. Besides the deficiencies mentioned in chapter 4, the indoor heat loads of the simulated test building were relatively uncomplicated. They contained many abrupt changes, but no arbitrary fluctuation, nor any ordinary noise. So, the variations were not close to natural. Furthermore, disturbances caused by process noise, which are typical of real systems, were not included. Such features may oversimplify the ARMAX-model and make the results unrealistic.

Modelling techniques are valuable tools in the fields of signal processing and control, but also suitable to be used in buildings. Yet, the estimation problem is not trivial. Proper skills, experience and familiarity with the problem are essential for successful results. This is also true of the estimation problem discussed in this paper. Designing estimation software for an EAS, which is sufficiently stable, robust and capable of distinguishing inappropriate initial and/or environmental conditions, will be a demanding task.

The estimation of static heat flows requires the special control technique of chapter 2. The control system must also be able to supervise all room temperatures of the building. Such a control philosophy is technically implementable but not using conventional solutions. However, this does not mean that one has to resort to any exceptional HVAC systems or special instrumentation. Even now, appropriate commercial energy allocation systems

are available that could be easily modified to perform the required control procedure. Nevertheless, inaccuracies in indoor temperature control are clearly one source of error, which will impair heat flow estimations in real buildings. Additional research efforts are necessary to further develop the method and the control procedure.

A fundamental issue, not discussed above is related to estimating of heat flows in an apartment or an office building consisting of several storeys. The estimation is easier in a one-story building, such as a town house, where the flats are separated by one uniform wall. In an apartment building, the estimation must be designed more carefully. An essential question is how to minimize the identification time, when a large number of flats are involved. Moreover, one has to decide how to cope with the rooms inside flats. A conceivable strategy is not to model each room separately but to combine all rooms into one model. Otherwise, coupling air from one room to another will cause problems in identification, as noted above. Still, one has to find out how to combine the rooms with their unique features and disturbances into one ARMAX-model. In an office building, the situation may be different. If the doors are kept closed coupling airflow between rooms is probably not significant.

The estimation method should be useful in cases, where the temperature difference between rooms is smaller than $3\,°C$, i.e., one to two degrees. Consequently, indoor temperature variations become negligible, causing no inconvenience to the occupant. If this is true of the estimation method, estimation can even be designed to be a continuous procedure. Then, an EAS supervises all room temperatures of the building and re-estimates heat flows regularly, provided that the environmental conditions are suitable. This approach takes care of possible changes in furniture, occupation or disturbances. A long-term data recording increases accuracy of the results. By repeating the estimation several times, it is also possible to confirm the previous calculations, i.e., the results will be more reliable.

The above open questions clearly show that the estimation method needs further research before its technical feasibility or commercial viability can be guaranteed. This will not be a straightforward procedure. As shown, simulated environments provide only incomplete circumstances for testing. It would be best to be able to make the verification and further development in a real

building. However, such an approach needs a lot of arrangements, both in measurements and control. In addition, comprehensive *in-situ* measurement of static heat flows between rooms and slabs is not a simple task. Point-wise measurement cannot represent the mean valued heat flow through a wall. A lot of instrumentation is needed, which may even disturb or cause unexpected errors in the results. In anyway, the ordinary inaccuracies of *in-situ* heat flow measurements are typically 5 to 10 %. Perhaps the best solution would be to run a simulated and a real building in parallel, which means that the dynamic thermal behaviour of the real test building is also simulated using TRNSYS or a similar computer program. If both approaches give consistent results, the experimental validation of the estimation method becomes more reliable. Such an arrangement might be a good platform for further testing and developing the estimation method.

9. Summary

Estimation of static heat flow between adjacent rooms based only on on-line measurements and special control techniques turns out to be a difficult problem. One solution is to model the heating power of a room with an ARMAX-model, and to utilize the identified dc-level in estimating the static heat flow. However, not all physical variables affecting the dynamics of heating power can be included or modelled. The same is also true of some disturbances involved in the heating process. These features have a negative influence on the accuracy of resulting heat flow values. Still, the presented method gives moderate, encouraging results in the test building.

All the test runs were made in a simulated environment. The applied simulation program makes it possible to vary the structural and environmental properties of the building. It is a good platform for the early stage of research and development. However, the applied test environment contains many deficiencies compared to a real building. Therefore, no final decision about the technical feasibility or the commercial viability of the method can be made. Also, many questions concerning details and practices of the method are still awaiting answers and need further study, preferably by applying data from a real building. That is why the method must be verified in a more realistic test environment before proceeding to details or making any final judgements.

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Title

An ARMAX-model approach for estimating static heat flows in buildings
A method for computerised energy allocation systems

Abstract

An energy cost allocation system records the energy consumption of a building and divides the overall energy costs between the flats. Because the indoor temperatures of rooms are usually not equal, heat flows between flats cannot be avoided. Hence, in order to ensure fair energy costs per flat the system should be able to determine the static heat flow rates, preferably without utilising design data or *in-situ* measurements. This paper presents a new method for estimating static heat flows between neighbouring rooms. It also outlines the instrumentation and technical properties of the energy allocation system (EAS), needed in implementation. The approach is strictly technical, focusing only on heat transfer issues. Energy cost allocation is not considered.

The approach is based on the proposed control techniques and the ARMAX-model describing dynamic behaviour of heating power. The model is created for each room/flat of a building. Parameter values are identified using real-time measurements collected from each room/flat and its environment. Only ordinary instrumentation is required. The tuning of parameters takes a few days using a fifteen-minute sampling time. A prerequisite for successful estimation is the overall control and the precise adjustment of the room temperatures at specified level. This is accomplished by the suggested set point control. All test runs are performed in a simulated office hotel using the TRNSYS simulation program. Realistic inner and outer heating loads, and their daily schedules and variations are included. The results are encouraging, but further research is needed, especially in a real building environment.

Keywords

buildings, heat flows, energy consumption, room temperatures, energy economy, costs, heat transfer, heating, models, measurement, thermal load, simulation, indoor air