VTT TIEDOTTEITA - MEDDELANDEN - RESEARCH NOTES 1959

Prediction of smoke production and heat release by convolution model

NORDTEST Technical report 1297-96

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VTT Building Technology

TECHNICAL RESEARCH CENTRE OF FINLAND
ESPOO 1999

ISBN 951–38–5440–X (soft back ed.)
ISSN 1235–0605 (soft back ed.)

ISBN 951–38–5441–8 (URL: http://www.inf.vtt.fi/pdf/)
ISSN 1455–0865 (URL: http://www.inf.vtt.fi/pdf/)

JULKAISIJA – UTGIVARE – PUBLISHER

Valtion teknillinen tutkimuskeskus (VTT), Vuorimiehentie 5, PL 2000, 02044 VTT
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Technical editing Leena Ukskoski

Libella Painopalvelu Oy, Espoo 1999

Myllymäki, Jukka & Baroudi, Djebar. Prediction of smoke production and heat release by convolution model. NORDTEST Technical report 1297-96. Espoo 1999, Technical Research Centre of Finland, VTT Tiedotteita – Meddelanden – Research Notes 1959. 29 p.

Keywords fires, fire protection, fire tests, smoke, combustion products, heat release, computation

Abstract

Using information obtained from bench-scale tests, a simple convolution integral model is applied for predicting heat release rate and smoke production rate of large-scale test fires of surface materials. The model is based on a convolution product of time derivative of effective area with the corresponding Cone Calorimeter data per unit area.

In the convolution model of Wickström and Göransson the model is applied only for the prediction of the heat release rate. The analytical expressions for the effective areas were developed from the visual observations of the burning area of the material in the large-scale test. In this report a trial was made to extend the convolution model also to the prediction of the smoke production rate. Because determination of effective areas cannot in this case be based on the observations, they have to be solved by numerical inverse methods.

For the inverse solution of the apriori unknown effective areas, several techniques were developed. Mathematical method and a computer program that uses regularized Singular Value Decomposition was written. Fast Fourier transform technique and simple average solution method were successfully applied as alternative methods. The smoke production rate and heat release rate in the Full Scale Room Corner Test ISO 9705 and in the Cone Calorimeter Test (ISO 9705) were analysed for 9 materials of the EUREFIC set of building products. Effective areas both for the prediction of heat release rate and smoke production rate were calculated.

A correlation was found between the areas of heat release and smoke production but no general area functions for the smoke production rate could be found. Therefore, a smoke prediction computer program could not be written based on the model. Determination of general area functions would need more comprehensive studies and a larger set of test data to be analysed. For future projects, the tools developed in this study give a good starting point.

Foreword

The authors wish to thank NORDTEST and VTT Building Technology for funding this study under the NORDTEST project 1297-96.

Jukka Myllymäki Djebar Baroudi

Contents

List of symbols

$A_q(t)$	burning area as a function of time (m^2)
$A_s(t)$	smoke producing area as a function of time (m^2)
$\mathbf{A}$	coefficient matrix
α	regularization parameter
$H(t-t_c)$	Heaviside function
I	intensity of transmitted light
L	path length of transmitted light (m)
$\dot{m}$	mass loss rate of fuel (MLR) (kg/s)
N_r	basis functions used in interpolation of areas
$\hat{\dot{q}}''(\omega)$	Fourier transform of heat release rate per unit area in Cone Calorimeter(kJ/ m^2)
$\dot{q}''(t)$	heat release rate per unit area in Cone Calorimeter (kW/ m^2)
$\bar{\dot{q}}_{t_c}''$	time average of heat release rate per unit area during time $0 \le t \le t_c$ (kW/ m^2)
$\hat{\dot{Q}}(\omega)$	Fourier transform of heat release rate in Room Corner Test (kJ)
$\dot{Q}(t)$	heat release rate in Room Corner Test (kW)
$\hat{\dot{s}}''(\omega)$	Fourier transform of smoke production rate per unit area in Cone Calorimeter(dB)
$\dot{s}''(t)$	smoke production rate per unit area in Cone Calorimeter (dB /s)
$\bar{\dot{s}}_{t_c}''$	time average of heat release rate per unit area during $0 \le t \le t_c$
$\dot{S}(t)$	smoke production rate in Room Corner Test (dB m^2/s)
$\hat{\dot{S}}(\omega)$	Fourier transform of smoke production rate in Room Corner Test (dB m^2)

Abbreviations

FFT	Fast Fourier Transform
HRR	heat release rate in Room Corner Test (kW)
hrr	heat release rate per unit area in Cone Calorimeter (kW/ m^2)
MLR	mass loss rate of fuel (MLR) (kg/s)
SEA	specific extinction area (m^2/kg)
SPR	smoke production rate in Room Corner Test (dB m^2/s)
spr	smoke production rate per unit area in Cone Calorimeter (dB /s)
SVD	Singular Value Decomposition
THR	total heat release in Room Corner Test (kJ)
thr	total heat release per unit area in Cone Calorimeter Test (kJ/ m^2)
TSP	total smoke production in Room Corner Test (dB m^2)
tsp	total smoke production per unit area in Cone Calorimeter Test (dB)

1. Introduction

The early fire behaviour of building products is important for many aspects of fire safety. The heat release rate is a fundamental variable of fire with which almost all other emission properties are highly correlated. For prediction of emissions of other quantities, the best strategy is to make use of modelling results obtained for the heat release rate.

Another important descriptor of a fire is the smoke production rate. Smoke is produced in almost all fires and presents a major hazard to life. The production of smoke and its optical properties are often measured simultaneously with other fire properties such as heat release and flame spread. The ability of small-scale tests in a cone calorimeter to predict full-scale behaviour is of major interest.

A model for full-scale heat release rate prediction based on convolution integral formulation was developed by Wickström and Göransson (1987, 1992). In the model the large-scale heat release rate of the product was calculated as a convolution product of the time derivative of the burning area and the heat release rate of the product measured on a bench scale. Wickström and Göransson have shown that the results from the Cone Calorimeter (ISO 5660, NT FIRE 048) can be used to predict Room Corner Tests (ISO 9705 or NT FIRE 025). They have used the data obtained in the EUREFIC-program. When the room fire heat release rate is calculated, the only input needed comes from the Cone Calorimeter in terms of the time to ignition and the heat release rate per unit area as a function of time. The results reported by them are based on single tests at an irradiance of 25 kW/m^2. In the Wickström/Göransson model, an analytical formulation for the burning area was used.

A similar convolution integral model has been developed in the CBUF (Combustion Behaviour of Upholstered Furniture) study by the authors (Myllymäki & Baroudi 1996, Babrauskas et al. 1997) for the prediction of full-scale heat release rate of pieces of upholstered furniture. In the model the burning area was determinated using Output Least Squares (OLS) method. An analytical expression for the burning area function was developed.

Correlation of smoke production in the full-scale Room Corner Test and in the Cone Calorimeter has been analysed by Östman et al. (Östman and Tsantaridis 1993, Östman et al. 1992). Several smoke parameters in the Cone Calorimeter and the Room Corner Test were analysed. They used three sets of building products tested in EUREFIC, Nordic round-robin programmes and Scandinavian test results tested at the Swedish National Testing Institute, comprising a total of 28 products. Only data at an irradiance of 50 kW/m^2 were chosen, since all products of the set did ignite at 50 kW/m^2, but not at 25 nor 35 kW/m^2.

The aim of this work has been to try to develop a methodology for correlation of the heat release rate and smoke production rate measurements in the Cone Calorimeter and in the Room Corner Tests. A convolution model has been used both for heat release rate and smoke production rate prediction. A stable integral inversion method based on the classical Tikhonov-regularized method is used to calculate the burning and smoke producing areas.

2. Direct problem: Convolution model

A mathematical model (Direct Problem) to predict the heat release rate (HRR) $\dot{Q}(t)$ on full-scale consists of a convolution product of the time derivative of an effective burning area $\dot{A}_q$ with the corresponding cone calorimeter heat release rate (hrr) $\dot{q}''(t)$ per unit area,

$$\dot{Q}(t) = \int_0^t \dot{q}''(t-\tau)\dot{A}_q(\tau)d\tau \, . \tag{1}$$

A similar convolution model has been used to predict the smoke production rate. It has been assumed that smoke production rate $\dot{S}(t)$ in full-scale tests can be calculated as a convolution product of the effective smoke producing area $A_s(t)$ and smoke production rate $\dot{s}''$ per unit area in the Cone Calorimeter from equation

$$\dot{S}(t) = \int_0^t \dot{s}''(t-\tau)\dot{A}_s(\tau)d\tau \, . \tag{2}$$

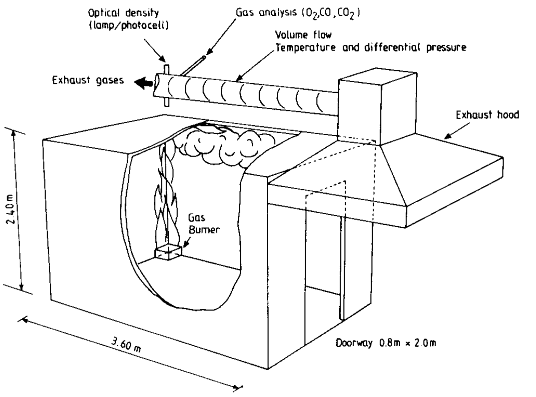
The heat release rate and smoke production from the burning area depend on the irradiance history on the surface, which is caused by the radiative heat transfer between the surfaces and flame and the conditions in the burning room. To simplify the calculation it is assumed that the heat release and smoke production from the surfaces are the same as in the cone calorimeter tests with an irradiance of 50 kW/m^2. In other words, all parts of the tested product are assumed to burn and produce smoke in the same way on the large scale as on the small scale. This is, of course, a great simplification. Only cone data at an irradiance of 50 kW/m^2 were chosen, since all products ignite at 50 kW/m^2, but not at 25 nor 35 kW/m^2. This is also practical because comparisons with correlations obtained by Östman and Tsantaridis can be made, since they have used cone data with the same irradiance level.

The reasons mentioned above mean that the burning area $A_q(t)1$ and the smoke producing area in equations (1) and (2) are dependent on the geometry and structure of the product and conditions in the large scale test. They are *effective areas* and more like correlation parameters of the model, which implicitly take into account the point that many portions of the burning item will be seeing a flux other than 50 kW/m^2 and all the phenomena after the simple flame spread phase. That is why the area $A_s(t)$ is not necessarily the same as $A_q(t)$ calculated from the same test data.

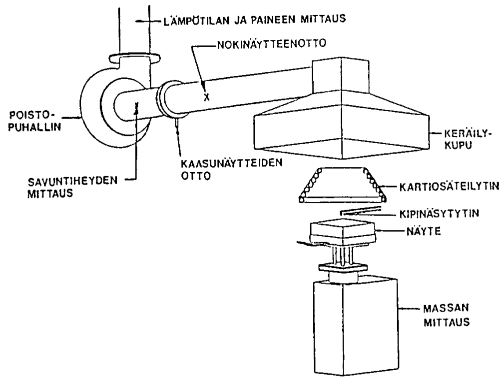
3. Experimental

3.1 Experimental method

The results of the full-scale room corner test performed according to NT FIRE 025/ISO 9705 at different Nordic laboratories in the EUREFIC project have been used in this study. All the tests have been performed with both the walls and the ceiling covered with the product tested. In all cases, the smoke obscuration has been measured in the exhaust duct with white light and a photocell.



a)



b)

Figure 1. a) Room Corner Test (Wickström, Göransson 1992) and Cone Calorimeter Test set-up.

The Room/Corner Test (ISO 9705) is shown Figure 1a. The specimen is mounted on the walls as well as on the ceiling. According to Nordtest Fire 025, the ignition source, a propane gas burner, is operated at two levels. The ignition source output is 100 kW during the first 10 minutes. Thereafter the rate is increased to 300 kW for another 10 minutes. The test is terminated after 20 minutes if flashover has not occurred before. Flashover is defined as a HRR of 1000 kW (from ignition source and product) which coincides with flames emerging from the doorway. HRR is obtained by oxygen consumption calorimetry.

The Cone Calorimeter tests (Fig. 1b) have been performed according to ASTM E 1354 since the equivalent ISO 5660 does not include smoke measurements. All products have been tested by using the recommended stainless steel retainer frame and with low density fibre blanket as backing material according to the standard (Östman & Tsantaridis 1993).

3.2 Smoke parameters

The same smoke parameters and units have been used both for the full-scale and the Cone Calorimeter data. The smoke production rate, SPR, has been calculated according to NT Fire 025 since most of the full-scale data are given in that way in the FDMS-files of the EUREFIC-program;

$$\dot{S} = D\ V_f = 10\ (1/L)\log(I_o/I)\ V_f, \tag{3}$$

where

$\dot{S}$	is smoke production rate SPR on large scale (dB m^2/s)
L	is path length (m)
I	is intensity of transmitted light
I_o	is intensity of incident light
D	is smoke intensity (dB)
V_f	is volume flow rate (m^3/s).

Specific extinction area SEA is defined as;

$$SEA = k\ V_f / \dot{m}, \tag{4}$$

where

$\dot{m}$	is mass loss rate of fuel (MLR) (kg/s)
SEA	(m^2/kg).

In Cone Calorimeter FDMS-files the rate of smoke production was not given. Instead, the specific extinction area SEA and mass loss rate MLR were given as a function of time. The smoke production rate had therefore to be calculated using equation

$$\dot{S}_{cone} = \dot{m}\, SEA\, \frac{D}{k}, \tag{5}$$

where

$\dot{S}_{cone}$ is smoke production rate (dB m^2/s) in the Cone Calorimeter

k is the extinction coefficient *ln(10)/10 D.*

Smoke production rate of the Cone Calorimeter per unit area was finally defined from equation

$$\dot{s}'' = \dot{S}_{cone} / A_{cone}, \tag{6}$$

where

$\dot{s}''$ is smoke production rate (spr) of the Cone Calorimeter per unit area (dB /s)
A_{cone} is area of the cone calorimeter specimen 0,01 m^2.

Also, heat release rate of the Cone Calorimeter has been used as per unit area

$$\dot{q}'' = HRR_{cone} \, / \, A_{cone} \,, \tag{7}$$

where

$\dot{s}''$ is heat release rate (hrr) of the Cone Calorimeter per unit area (kW/m^2).

3.3 The experimental data used

The building products of the EUREFIC-programme were used in this study. The heat release rate of the products in the Cone Calorimeter and Room/Corner Test are presented in Figures 2 - 9. The Cone Calorimeter heat release rate and smoke production rate have been given per unit area ($\dot{s}''$, $\dot{q}''$).

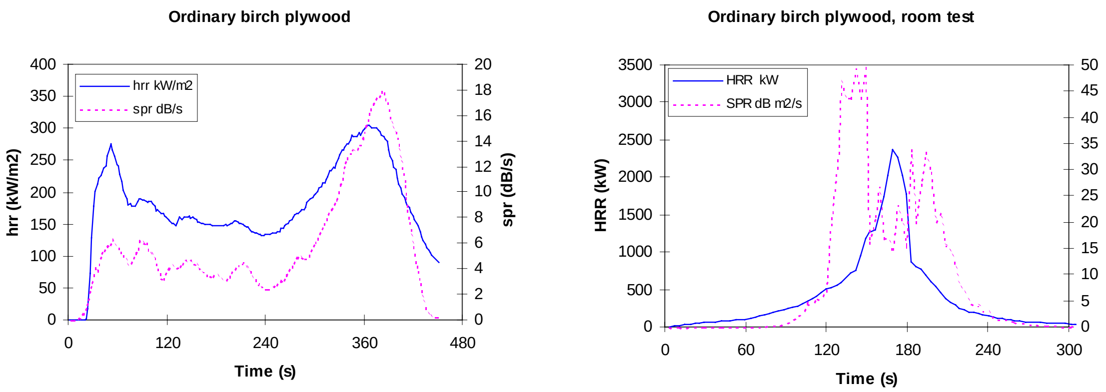


Figure 2. a) The Cone Calorimeter and b) Room/Corner Test results of ordinary birch plywood.

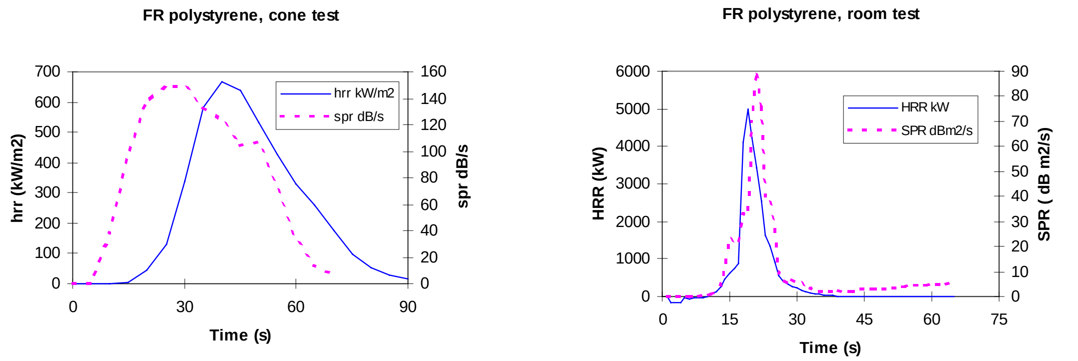


Figure 3. a) The Cone Calorimeter and b) Room/Corner Test results of Styrofoam FR polystyrene.

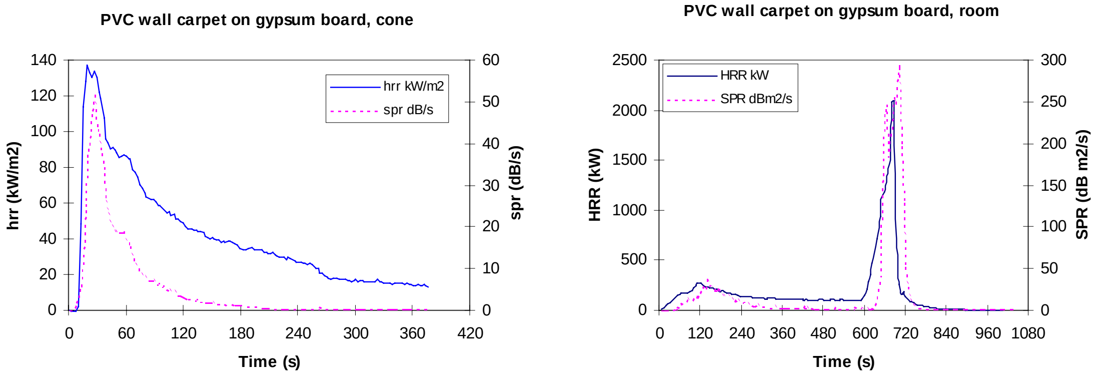


Figure 4. a) The Cone Calorimeter and b) Room/Corner Test results of PVC-wallcarpet on gypsum paper plasterboard.

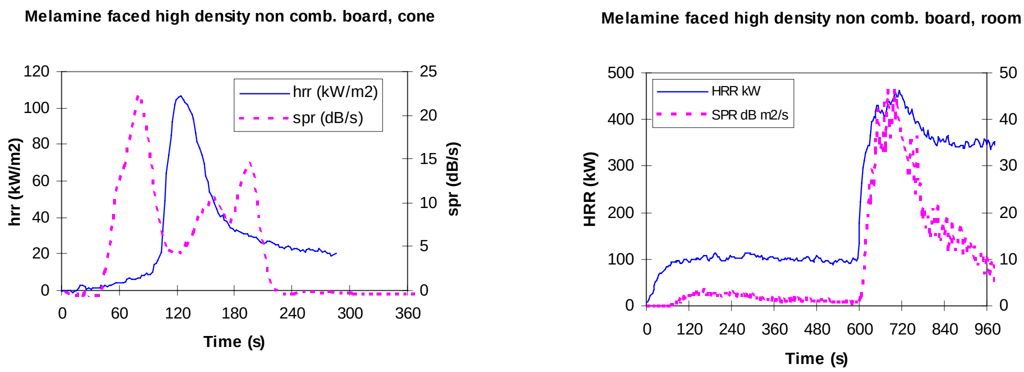


Figure 5. a) The Cone Calorimeter and b) Room/Corner Test results of melamine faced high density non-combustible board.

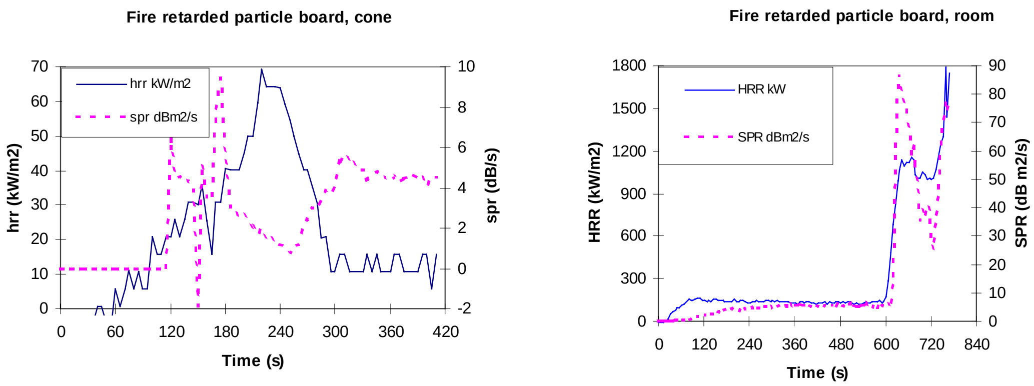


Figure 6.a) The Cone Calorimeter and b) Room/Corner Test results of fire retarded particle board.

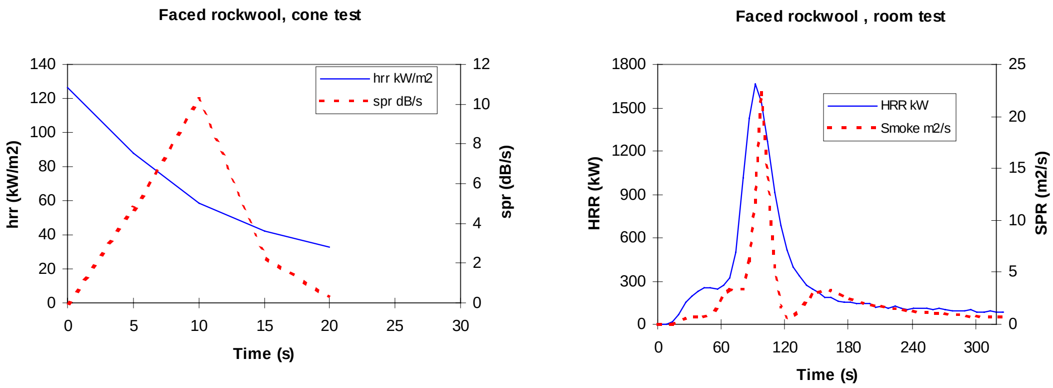


Figure 7. a) The Cone Calorimeter and b) Room/Corner Test results of combustible faced mineral wool.

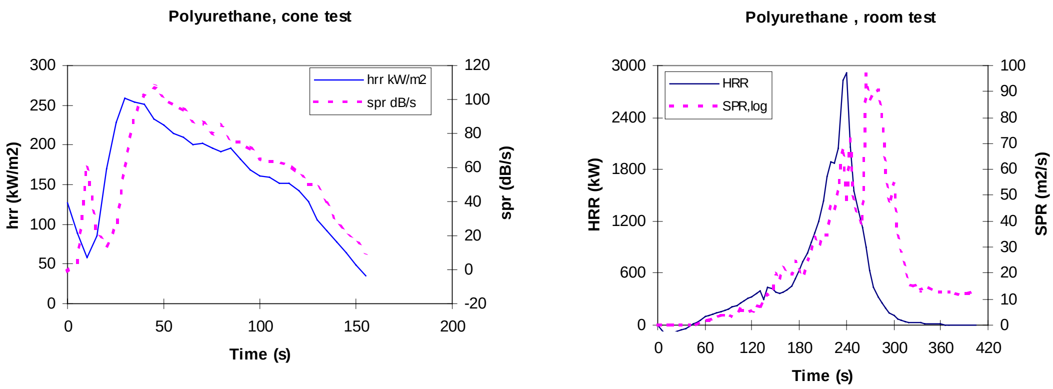


Figure 8. a) The Cone Calorimeter and b) Room/Corner Test results of polyurethane foam.

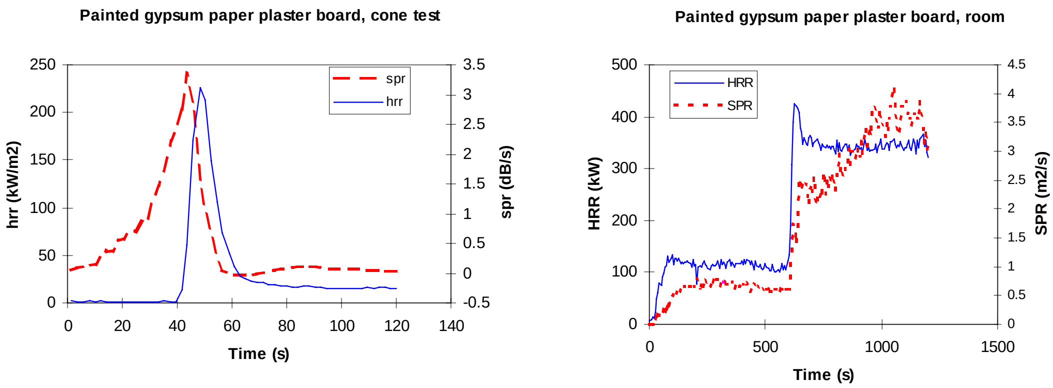


Figure 9. a) The Cone Calorimeter and b) Room/Corner Test results of painted gypsum plaster board.

4. Inverse problem: Solution of the area functions

4.1 Tikhonov regularized Singular Value Decomposition (SVD)

Let the full-scale heat release rate HRR $\dot{Q}(t)$ or the full-scale smoke production rate SPR $\dot{S}(t)$ be described by the convolution product model (1). The continuous unknown time derivatives of burning area $\dot{A}_q(\tau)$ or smoke producing area $\dot{A}_s(\tau)$ are discretized using the piece-wise continuous linear approximations

$$\dot{A}_q(s) = \sum_{r=1}^{2} N_r(s)\ x_r \tag{8a}$$

$$\dot{A}_s(s) = \sum_{r=1}^{2} N_r(s)\ x_r \tag{8b}$$

for $s \in [s_i, s_{i+1}]$. The time interval is discretized into *N-1* sub-intervals [s_i, s_{i+1}] of arbitrary lengths s_{i+1} - s_i , where i = *1, ..., N-1*. The nodal values of the function $x(s_i) = \dot{A}_q(s_i)$ (or $x(s_i) = \dot{A}_s(s_i)$) are gathered into the vector $\mathbf{x}_{Nx1}$ .of unknowns. The basis functions used in eq. (8) are

$$N_1(s) = \frac{s - s_i}{s_{i+1} - s_i} \quad and \quad N_2(s) = \frac{s}{s_{i+1} - s_i}\,. \tag{9}$$

By fulfilling the convolution equation (1) or (2) at *M* collocation points t_j (the measurement sampling times) we obtain an overdetermined discrete system of equations written with the matrix notation as

$$\mathbf{y} = \mathbf{A}\ \mathbf{x} + \eta \tag{10}$$

with $y(t_j) = \dot{Q}\ (t_j)$ or $\dot{S}\ (t_j)$. The discretization nodes s_i do not necessarily coincide with the collocation points t_j. Vector η consists the measurement imperfections.

The unknown is the *(Nx1)* vector $\mathbf{x} = \left(x(s_1) \quad \ldots \quad x(s_i) \quad \ldots \quad x(s_N)\right)^T$. The measured values are the (*Mx1*) vector $\mathbf{y} = \left(y(t_1) \quad \ldots \quad y(t_j) \quad \ldots \quad y(t_M)\right)^T$. We suppose that *M* > *N*, i.e., we have more equations than unknowns.

The elementary contribution from the sub-interval $I^i = [s_i, s_{i+1}]$ to the global matrix **A** in eq. (10) is

$$A_{k,j}^{i} = \int_{s_i}^{s_{i+1}} K(s-t_j)\ N_k(s)\ ds \approx \sum_{r=1}^{Np} w_r K(s_r - t_j) N_k(s_r)\,, \tag{11}$$

where the trapezoidal quadrature is used. When solving problem (1) or (2) the kernel $K(t)$ is either the heat release rate $\dot{q}''$ or smoke production rate $\dot{s}''$ in the Cone Calorimeter. The indices in eq. (11) are *i = 1, ..., N* and *j = 1, ..., M*. The matrix **N** = [$N_1(s)$, $N_2(s)$] is the standard linear basis, q = 1,2 and w_r are the weights of the integration rule.

The ideal data ***y*** inevitably contains measurement errors, which is emphasised by writing it as $\mathbf{y}_\delta$, where $\|\mathbf{y}-\mathbf{y}_\delta\| \le \delta$ is an estimate of the error level in the data. This data error δ may cause instabilities in the estimated parameters $\mathbf{x}^\delta$, due to the nature of the problem, which may be ill-posed, depending on the data and the mathematical model (1). In ill-posed problems the parameters do not unfortunately depend continuously on the data. A cure for this problem is to use the Tikhonov regularization methods (Groetsch 1993, pp. 84 - 90, Tikhonov and Arsenin 1977).

In this method, the unknowns **x** are calculated as the solution of the regularised linear minimisation problem where

$$\min_{\mathbf{x}} \qquad F(\mathbf{x}_\alpha^\delta) = \min_{\mathbf{x}} \left(\left\| \mathbf{A}\mathbf{x}_\alpha^\delta - \mathbf{y}^\delta \right\|^2 + \alpha \left\| \mathbf{x}_\alpha^\delta \right\|^2 \right) \tag{12}$$

with an optimal choice of the regularization parameter α .The first term in the functional (12) enforces the consistency of the solution while the second term enforces its stability. An appropriate balance between the need to describe well the measurement and the need to achieve a stable solution is reached by finding an optimal regularization parameter.

The solution of the minimization of the functional is also known as the minimal norm solution which is best computed using the SVD procedure (Prees et al. 1992) which can be viewed as a generalized Fourier series of the solution. The regularized solution of (12) is written using matrix notation as

$$\mathbf{x}_\alpha = \mathbf{A}_\alpha^{+}\mathbf{y} = \mathbf{V}\mathbf{D}_\alpha^{+}\mathbf{U}^T\mathbf{y}\ , \tag{13}$$

where $\mathbf{A}_\alpha^{+}$ Penrose generalized inverse matrix of **A** and the singular value decomposition of matrix **A** is written as

$$\mathbf{A}_{MxN} = \mathbf{U}_{MxN}\mathbf{D}_{NxN}\mathbf{V}_{NxN}{}^T\,, \tag{14}$$

where **U** and **V** are orthogonal matrices. The positive matrix $\mathbf{D}_{NxN} = diag(d_1 \quad ... \quad d_N)$ where the singular values $d_1 \ge d_2 \ge ... \ge d_{\min(N,M)} \ge 0$, are the eigenvalues of the symmetric matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ of the eq. (10). The generalized inverse matrix is calculated as

$$\mathbf{D}_\alpha^{+} = diag\left(\frac{1}{d_1 + \alpha / d_1}, \quad ..., \quad \frac{1}{d_{\min} + \alpha / d_{\min}} \right). \tag{15}$$

Statistically the regularization parameter can be viewed as the ratio of the variances of the noise to the signal , i.e., $\alpha = \sigma^2_{Noise} / \sigma^2_{Signal}$. Unfortunately this ratio is not always available and therefore one has to estimate the regularization parameters using other methods not relying on this knowledge. The method used in this work is the *L*-curve method. Besides the *L*-curve method, in the absence of *a priori* information about the variances of the noise with the signal, a rough estimate can be used in such a way as to balance consistency and stability in the solution, i.e., $\alpha \left\| \mathbf{x}^{\delta}_{\alpha} \right\|^2 = \left\| \mathbf{A}\mathbf{x}^{\delta}_{\alpha} - \mathbf{y}^{\delta} \right\|^2$. This usually produces a smoother solution than needed. The use of the *L*-curve method is more explicitly explained in Nordtest project 1302-96 report (Baroudi 1996). Since the a posteriori data correction method (Tikhonov regularization of the inverse problem) is dependent on current data, i.e., on the noise to signal ratio, then the choice of the regularization parameter demands some expertise from the user.

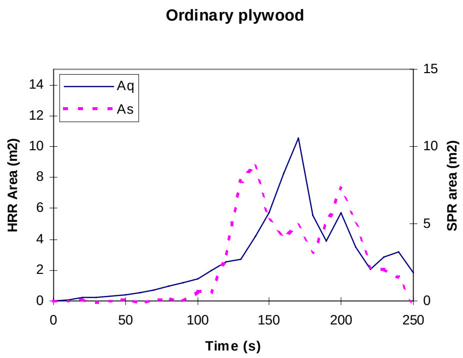
4.2 Calculation of the areas using SVD

Both the time derivative of the burning area $\dot{A}_q(t)$ and smoke producing area $\dot{A}_s(t)$ have been solved by the Tikhonov regularized SVD method described in Chapter 4.1 for the EUREFIC materials (Chapter 3.3) as an inverse solution of equations (1) and (2). The solutions of the areas were obtained by integrating the derivatives

$$A_q(t) = \int_0^t \dot{A}_q(\tau)\ d\tau \qquad (16a)$$

$$A_s(t) = \int_0^t \dot{A}_s(\tau)\ d\tau \qquad (16b)$$

The solved areas for the EUREFIC materials (Chapter 3.3) are given in Figures 10 - 13.



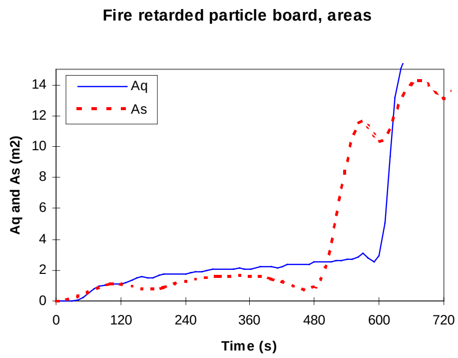


Figure 10. Burning area $A_q(t)$ and smoke producing area $A_s(t)$ of ordinary birch plywood (a) and fire retarded particle board (b).

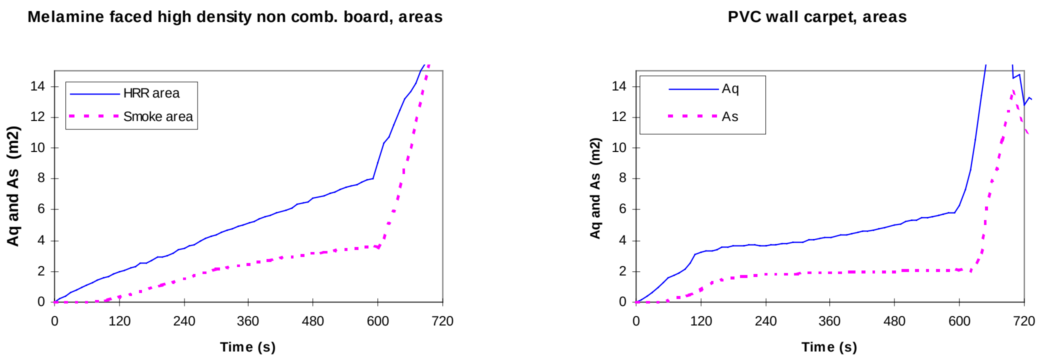


Figure 11. Burning area $A_q(t)$ and smoke producing area $A_s(t)$ of melamine faced high density non-combustible board (a) and PVC wall carpet (b).

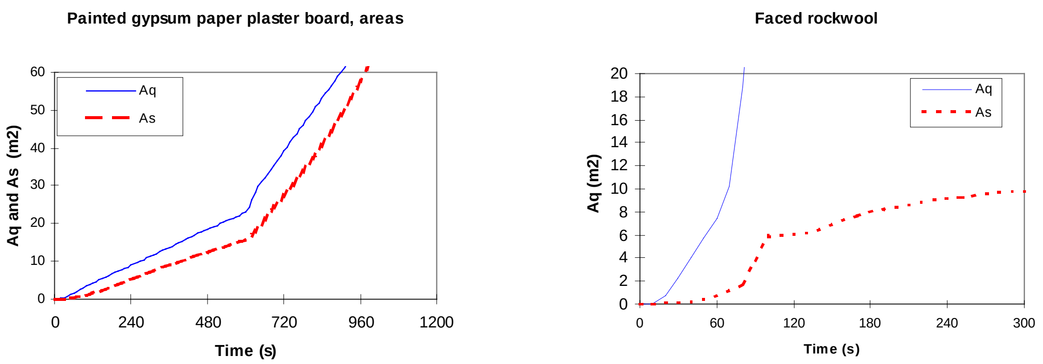


Figure 12. Burning area $A_q(t)$ and smoke producing area $A_s(t)$ of painted gypsum plaster board (a) and faced rockwool (b).

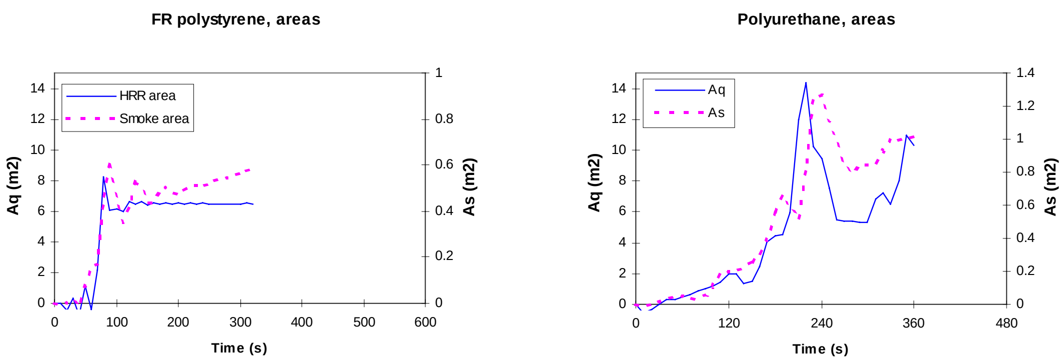


Figure 13. Burning area $A_q(t)$ and smoke producing area $A_s(t)$ of FR polystyrene (a) and polyurethane (b).

4.3 Convolution and deconvolution using Fast Fourier Transforms

A physical problem can be described either in the *time domain,* by the values of some quantity h as a function of time t, e.g. $h(t)$, or else in the *angular frequency domain*, where the process is specified by giving its amplitude $\hat{h}(\omega)$ (generally a complex number indicating phase also) as a function of angular frequency ω, that is $\hat{h}(\omega)$, with $-\infty < \omega < \infty$. One goes back and forth between these two representations by means of the *Fourier transform* equations (Prees et al. 1992);

$$\hat{h}(\omega) = \mathcal{F}(h(t)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(t)\ e^{-i\,\omega\,t} dt \tag{17a}$$

$$h(t) = \mathcal{F}^{-1}(h(\omega)) = \int_{-\infty}^{\infty} \hat{h}(\omega)\ e^{i\,\omega\,t} d\omega \tag{17b}$$

With two functions $h(t)$ and $g(t)$, and their corresponding Fourier transforms $\hat{h}(\omega)$ and $\hat{g}(\omega)$, we can form a combination of special interest. The convolution of two functions, denoted $g*h$, is defined by

$$g*h = \int_{-\infty}^{\infty} g(\tau)\ h(t-\tau)\ d\tau\,. \tag{18}$$

It turns out that the function $g*h$ is one member of a simple transform pair

$$\mathcal{F}(g*h) = \mathcal{F}(\int_{-\infty}^{\infty} g(\tau)\ h(t-\tau)\ d\tau) = 2\pi\ \hat{g}(\omega)\ \hat{h}(\omega)\,. \tag{19}$$

This is the "Convolution Theorem". If we apply this equation to the hereditary integral of equations (1) and (2), we get

$$\hat{\dot{Q}}(\omega) = 2\pi\ \hat{q}''(\omega)\ \ \hat{\dot{A}}_q(\omega) \text{ and} \tag{20a}$$

$$\hat{\dot{S}}(\omega) = 2\pi\ \hat{s}''(\omega)\ \ \hat{\dot{A}}_s(\omega)\,. \tag{20b}$$

These equations give us the solution of the convolution integral in the angular frequency domain. The discrete Fourier transforms and their inverse can be computed with an algorithm called the *fast Fourier transform* (FFT) and inverse (IFFT). This algorithm is adopted in the programming environment MATLAB and in the spreadsheet program EXCEL. The inverse solution of eq. 20 can be obtained easily.

$$\hat{A}_q(\omega) = \frac{\hat{\dot{Q}}(\omega)}{2\pi\ \hat{q}''(\omega)} \Rightarrow \dot{A}_q(t) = \mathcal{F}^{-1}\ (\hat{A}_q(\omega)) \tag{21a}$$

$$\hat{A}_s(\omega) = \frac{\hat{\dot{S}}(\omega)}{2\pi\ \hat{s}''(\omega)} \Rightarrow \dot{A}_s(t) = \mathcal{F}^{-1}\ (\hat{A}_s(\omega)) \tag{21b}$$

With high frequencies, the Fourier transforms $\hat{q}''(\omega)$, $\hat{s}''(\omega)$ of the measured Cone Calorimeter data are very small and small errors in them are so much amplified in the solution that it becomes useless. The high frequencies have to be filtered in some suitable method. The following examples were conducted simply with the spreadsheet program EXCEL applying truncation of the high frequencies (Fig. 14).

$$\hat{A}_s(\omega) = \frac{\hat{\dot{S}}(\omega)}{2\pi\ \hat{s}''(\omega)}\Phi(\omega) \Rightarrow \dot{A}_s(t) = \mathcal{F}^{-1}\ (\hat{A}_s(\omega)), \tag{22}$$

where
$\Phi(\omega) = 1$ when $\omega < \omega_o$
$\Phi(\omega) = 0$ when $\omega > \omega_o$.

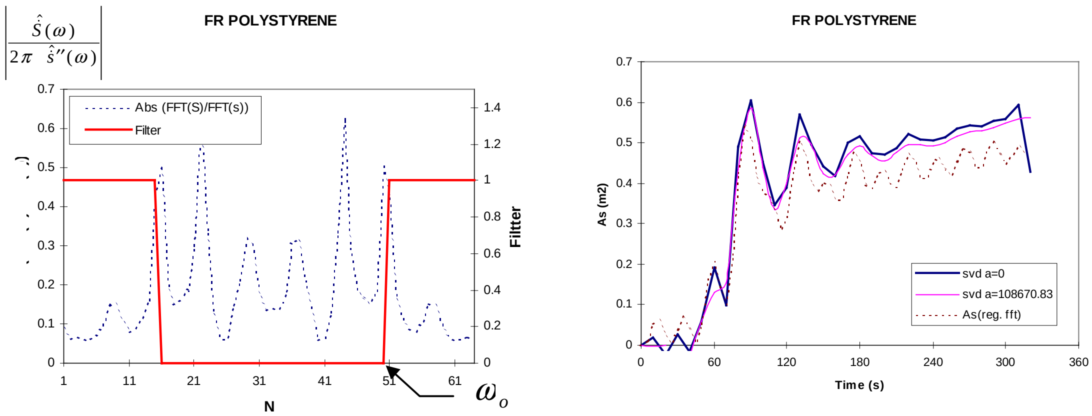


Figure 14. a) The absolute value $\left|\hat{A}_s(\omega)\right| = \left|\frac{\hat{\dot{S}}(\omega)}{2\pi\ \hat{s}''(\omega)}\right|$ *of FR polystyrene and the applied filter as function of number of Fourier coefficients (N), b) Smoke producing area* A_s *calculated by SVD solution with regularization (*α *=108670) and without regularization (*α *=0) compared with the deconvolution with filtered Fourier transformation.*

The results of a convolution calculation (eq.2) with FFT of the EXCEL spreadsheet program are presented in Fig. 15. When the FFT algorithm of EXCEL is used it must be remembered that both the Cone and Room data must have the same time step and same number of measurements. Usually there is not the same amount of data in Cone Calorimeter test results. When FFT is used, the lacking data vector is filled with zeros (Treatment by Zero Padding, see Prees et al. 1992, p. 533).

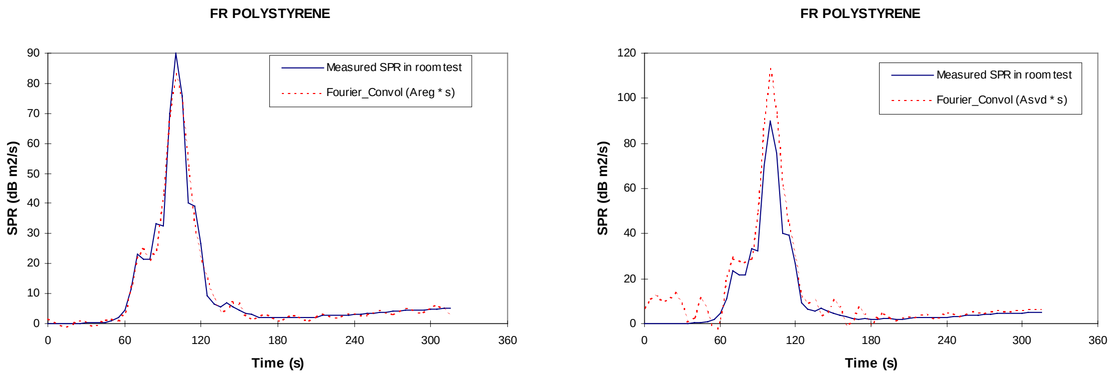


Figure 15. Convolution calculation of SPR Fourier transformation a) A_s calculated with filtered FFT b) A_s calculated with regularized SVD.

4.4 Calculation of the areas using average cone data

We approximate the cone data as a step function with amplitude $\bar{q}''_{t_c}$ of duration t_c (Fig. 16).

$$\dot{q}''_{approx.} = \bar{q}''_{t_c}\,(H(t) - H(t - t_c)), \tag{23}$$

where $H(t - t_c)$ is the Heaviside function.

$$H(t - t_c) = 0 \quad \text{when } 0 \le t \le t_c$$
$$H(t - t_c) = 1 \quad \text{when } t > t_c\,.$$

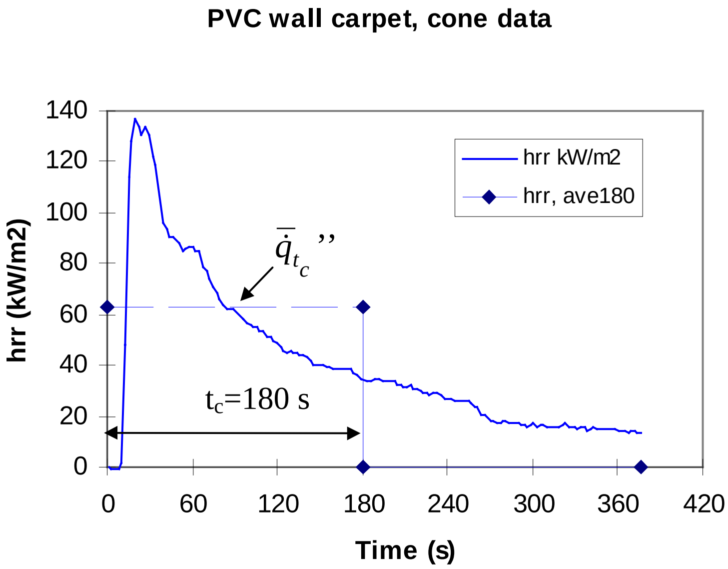


Figure 16. Heat release rate of PVC wall carpet approximated as an average step function (t_c = 180 s).

The amplitude of the step function is calculated as the average of the Cone Calorimeter heat release rate during time $0 \le t \le t_c$

$$\bar{\dot{q}}''_{t_c} = \frac{\int_0^{t_c} \dot{q}''\, dt}{t_c} = \frac{thr_{t_c}}{t_c} \tag{24}$$

Applying eq. 17 to the convolution integral (eq. 1) we get an approximative solution for the heat release rate on large scale

$$\dot{Q}(t) = \bar{\dot{q}}''_{t_c} \int_0^t \dot{A}_q(\tau)d\tau = \bar{\dot{q}}''_{t_c}\ A_q(t) \qquad \text{when } t \le t_c \tag{25a}$$

$$\dot{Q}(t) = \bar{\dot{q}}''_{t_c} A_q(t) - \dot{Q}(t - t_c) \qquad \text{when } t_c \le t \le 2\ t_c \tag{25b}$$

From these equations we can solve the burning area in a simple way

$$A_q(t) = \frac{\dot{Q}(t)}{\bar{\dot{q}}''_{t_c}} \qquad \text{when } t \le t_c \tag{26a}$$

$$A_q(t) = \frac{\dot{Q}(t) + \dot{Q}(t - t_c)}{\bar{\dot{q}}''_{t_c}} \qquad \text{when } t_c \le t \le 2\ t_c \tag{26b}$$

These equations can also be used respectively for the solution of smoke production area.

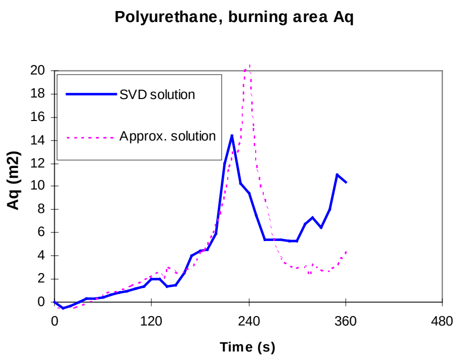
$$\dot{S}(t) = \bar{\dot{s}}''_{t_c} \int_0^t \dot{A}_s(\tau)d\tau = \bar{\dot{s}}''_{t_c}\ A_s(t)\,, \qquad \text{when } t \le t_c \tag{27a}$$

$$\dot{S}(t) = \bar{\dot{s}}''_{t_c}\ A_s(t) - \dot{S}(t - t_c)\,, \qquad \text{when } t_c \le t \le 2\ t_c \tag{27b}$$

$$A_s(t) = \frac{\dot{S}(t)}{\bar{\dot{s}}''_{t_c}}\,, \qquad \text{when } t \le t_c \tag{28a}$$

$$A_s(t) = \frac{\dot{S}(t) + \dot{S}(t - t_c)}{\bar{\dot{s}}''_{t_c}}\,, \qquad \text{when } t_c \le t \le 2\ t_c \tag{28b}$$

This approximation works pretty well (Fig. 17) at the rising part of the Room Corner Test data.



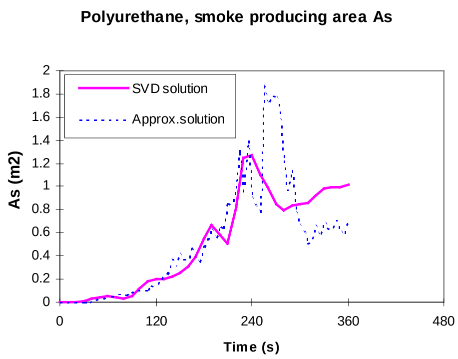


Figure 17. a) Burning area $A_q(t)$ and b) smoke producing area $A_s(t)$ calculated using SVD inverse method (solid line) and approximate average method for polyurethane (dotted line).

4.5 Correlation of burning and smoke producing area

In order to correlate the effect of heat release rate to smoke production rate, the integral of the smoke producing areas has been presented as a function (eq. 29) of the integral of the burning areas for different building materials (Fig. 18).

$$\int_0^t A_s(t)\ dt = f\left(\int_0^t A_q(t)\ dt\right) \tag{29}$$

Let us study what these area integrals mean using the usual parameters of fire engineering. If we calculate the areas using average cone data and integrate the equations (18), we get the following approximative relations:

$$\int_0^t A_q(t)\ \ dt = \frac{\int\ \dot{Q}(t)dt}{\bar{\dot{q}}''_{t_c}} = \frac{THR(t)}{\bar{\dot{q}}''_{t_c}} \qquad \text{when } t \le t_c \tag{30a}$$

$$\int_0^t A_q(t)\ \ dt = \frac{\int_0^t \dot{Q}(t)dt + \int_0^t\ \dot{Q}(t-t_c)dt}{\bar{\dot{q}}''_{t_c}} = \frac{THR(t)+THR(t-t_c)}{\bar{\dot{q}}''_{t_c}} \qquad \text{when } t_c \le t \le 2\ t_c \tag{30b}$$

and respectively for smoke production area

$$\int_0^t A_s(t)\ dt = \frac{\int\ \dot{S}(t)dt}{\bar{\dot{s}}''_{t_c}} = \frac{TSP(t)}{\bar{\dot{s}}''_{t_c}} \qquad \text{when } t \le t_c \tag{31a}$$

$$\int_0^t A_s(t)\ dt = \frac{\int_0^t \dot{S}(t)dt + \int_0^t \dot{S}(t-t_c)dt}{\bar{\dot{s}}''_{t_c}} = \frac{TSP(t)+TSP(t-t_c)}{\bar{\dot{s}}''_{t_c}} \qquad \text{when } t_c \le t \le 2\ t_c . \qquad (31b)$$

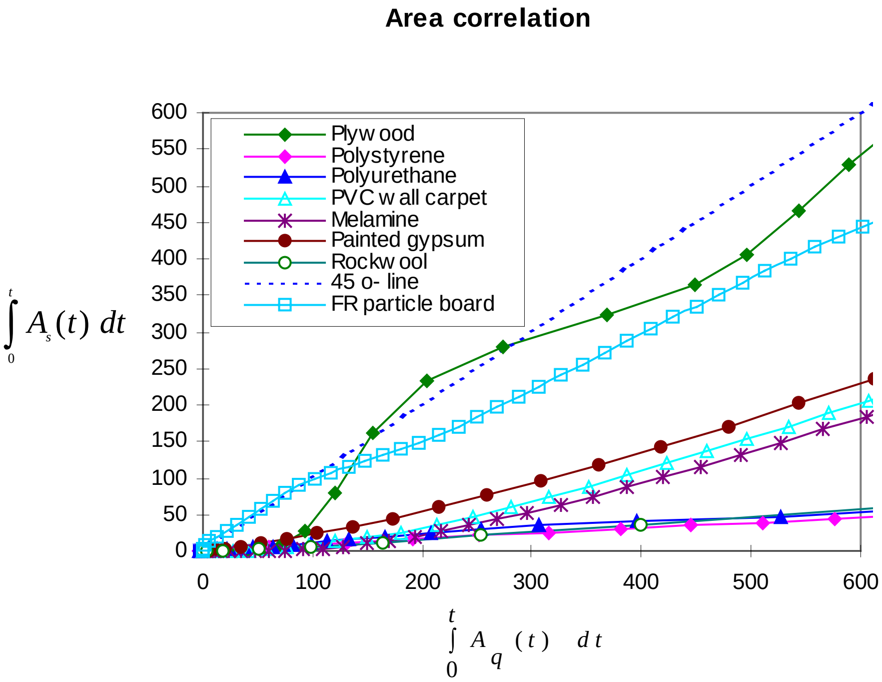


Figure 18. Integral of the smoke producing area as a function of the integral of burning area. Areas are caculated using SVD.

The area correlation curves form a nearly linear functions with three slopes. If we consider these functions as lines with slope k we get following simplification:

$$\int_0^t A_s(t)\ dt = k(t)\ \int_0^t A_q(t)\ dt\ . \qquad (32)$$

Using average approximations (24a) and (25a) we can get the meaning of this correlation

$$\frac{TSP(t)}{\bar{\dot{s}}''_{t_c}} = k(t)\frac{THR(t)}{\bar{\dot{q}}''_{t_c}}. \qquad (33)$$

This can also be presented in a following way

$$\frac{TSP(t)}{THR(t)} = k(t)\frac{\bar{\dot{s}}_{t_c}''}{\bar{\dot{q}}_{t_c}''} = k(t)\frac{tsp(t_c)}{thr(t_c)}, \qquad (34)$$

where

$tsp(t_c)$ is the total smoke production
$thr(t_c)$ is the total heat release rate in the Cone Calorimeter.

From eq. 34 it can be seen that the slope k is actually a scaling factor of total smoke production per total heat release between the bench scale and large scale. From Fig. 19 where the slope k is presented as a function of time, we can observe that the *k(t)* has final values of:

- about 0,8 - 1,0 for wood products (plywood and FR particle board)
- about 0,1 for good insulations (polystyrene, polyurethane and rockwool)
- 0,4 - 0,6 for products with flash over after 10 min (600 s).

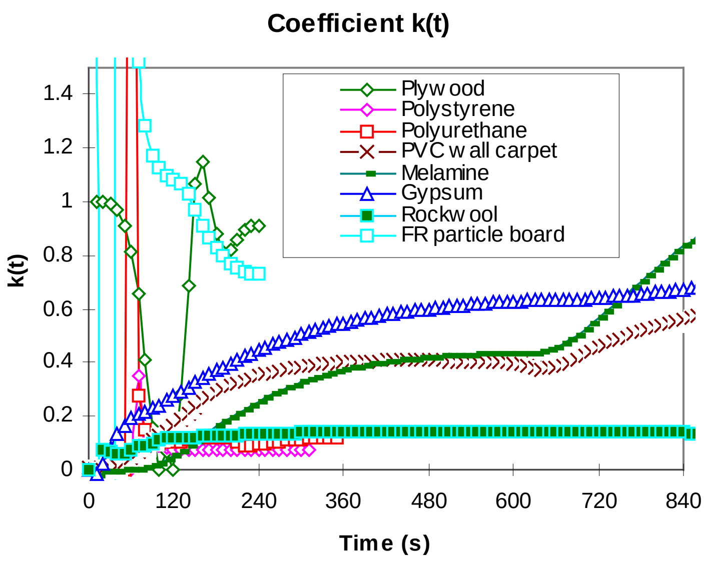


Figure 19. Slope of the $\int_0^t A_s(t)\,dt - \int_0^t A_q(t)\,dt$ *curve for different EUREFIC products.*

This means that the wood products seem to produce nearly the same amount of smoke per heat release on the large scale and bench scale, but the good insulation products produce much less smoke on large scale than in small scale.

5. Discussion and conclusions

To find a suitable correlation and calculation procedure for the smoke production rate on large scale based on the data from small scale seems to be a very difficult task. In order to find better correlation, much larger test material should be analysed. In the light of the modest trial of this study, it seems that one possibility would be the following.

Calculate the HRR on large scale using the Wickström Göransson type of model

- $\dot{Q}(t) = \int_0^t \dot{q}''(t-\tau)\dot{A}_q(\tau)d\tau$.

If the slope k can be assumed a constant, the smoke producing area could be calculated by using equation

- $A_s(t) = k\ A_q(t)$

and the smoke production rate could be calculated from equation

- $\dot{S}(t) = \int_0^t \dot{s}''(t-\tau)k\ \dot{A}_q\ d\tau$.

Another way just using the average Cone data would be the following.

Calculate heat release rate from equation

- $\dot{Q}(t) = \overline{\dot{q}}_{t_c}''A_q\ (t)$

and smoke production rate from equation

- $\dot{S}(t) = \overline{\dot{s}}_{t_c}''k\ A_q(t)$.

Verification of this kind of model would need much deeper and larger analysis than has been possible in this Nordtest programme, but in the light of the correlations found in this study, it might be possible.

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