VTT TIEDOTTEITA

Alpo Ranta-Maunus, Mikael Fonselius,
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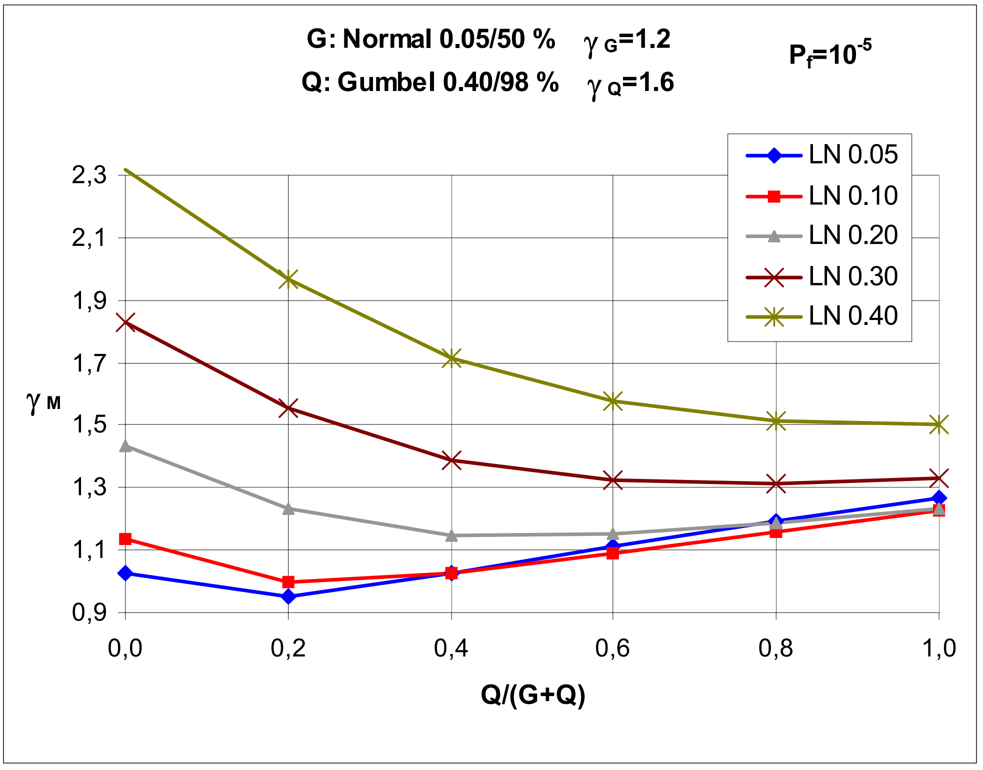
Reliability analysis of timber structures



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Abstract

As part of the European harmonisation of building codes, the determination of design values for loads and materials is important and is the motivation for this research. This report begins with a summary of the probabilistic basis of Eurocodes, analyses the strength distributions of wooden materials, demonstrates the effects of different distribution functions on the calculated safety level and shows some results of the applications of reliability analysis.

When the number of experiments allows, determination of the 5% fractile of strength should be based on the function fitting on the lower tail of the strength values, for instance 10%. All smooth functions fitted to tail data gave good estimates of the 5% fractile. When the 5% fractile was determined from a function fitted to all data, up to 5% error occurred (in one case 9%) when compared to a non-parametric estimate. Three-parameter Weibull distribution gave, in all calculated cases, the 5% fractile within an accuracy of ±3%.

The result of structural reliability analysis depends strongly on the load and strength distribution types used. When fitted functions are used in reliability analysis, it is essential that the fit is good in the lower tail area, the lowest values being most important. When fitted to the same data, a two-parametric Weibull distribution gives the most pessimistic prediction for the tail, with a normal distribution being next, and lognormal and three-parameter Weibull being the most optimistic. In an example, a two-parameter Weibull gave a failure probability 10 times higher than that of a three-parameter Weibull.

The analysis suggests that $\gamma_M = 1.2$ to 1.3 is reasonable for timber structures when $\gamma_G = 1.2$ and $\gamma_Q = 1.5$.

Preface

This publication is the main documentation of VTT's contribution to two projects:

- a Finnish prestudy on the Statistical Determination of the Strength of Wooden Materials
- a Nordic project on Safety in Timber Structures.

The Finnish project was conducted by the authors, and supervised by a management group headed by Jouko Silén (Stora Enso Oyj) as chairman, and having the following members: Antero Järvenpää (Late-Rakenteet Oyj), Tero Nokelainen (Jouni Hakkarainen as deputy, Finnforest Oyj), Jarmo Leskelä (Finnish Forestry Industries Ass.), Pekka Nurro (Wood Focus Finland), Ilmari Absetz (Tekes), Tuija Vihavainen (VTT Building and Transport) and Alpo Ranta-Maunus as secretary (VTT Building and Transport).

The Nordic project is being carried out with collaboration between Lund University, NTI, SBI and VTT. Hans Jorgen Larsen is the project leader. Several organisations are supporting the projects as stipulated in the Acknowledgements chapter.

This project is partly financed by the Nordic Industrial Fund. The Nordic Industrial Fund - Centre for innovation and commercial development is an institution under the Nordic Council of Ministers. The Fund initiates and finances cross-border research and development projects aimed at the Nordic innovation system. Such projects are expected to enhance the competitiveness of Nordic industry and reinforce Nordic business culture while encouraging sustainable development in Nordic society. The Nordic Industrial Fund works closely with the national research financing bodies. Its secretariat is in Oslo.

The roles of the authors were as follows: Mikael Fonselius analysed the material strength data and wrote Chapters 2 and 3. Juha Kurkela made the Excel macros for analysis of fracture probabilities, and performed some of the calculations. Tomi Toratti performed calculations using the Strurel programme and wrote the text concerning system effect. Alpo Ranta-Maunus, who coordinated the work, wrote other parts of the text and performed the other calculations.

Contents

1. Introduction

Discussion on adequate safety levels in the design of buildings has been repeated from time to time. Moreover, the question of the correct ratios of partial safety factors for different building materials has risen again, now that the common European building code is close to completion. Safety factors in building codes are traditionally based on long-term experience. Also, Eurocode states that, as the most common method, numerical values of partial safety factors can be determined on the basis of calibration to the long-term experience of the building industry (EC1, annex C2). As an alternative, the use of statistical evaluation based on probabilistic reliability theory is mentioned. An international model code for probability-based assessment and design of structures is under way by the (IABSE, CIB, fib, ECCS and RILEM) Joint Committee on Structural Safety (Vrouwenvelder 2001) and the existing parts are available from www.jcss.ethz.ch. However, it does not yet include information concerning timber resistance. It is hoped that the assessment of the material strength data in this publication will contribute to the completion of the JCSS code.

This work on the use of probabilistic methods in the development of timber building codes, which is part of the Nordic project, covers a concise literature study as part of the introduction with a special emphasis on Eurocodes, an analysis on material strength data to which VTT has access, and some reliability analyses to demonstrate the effect of selected distribution types and parameters on calculated failure probabilities. Also, a calculation is performed to demonstrate the dependence of safety factors on the coefficient of variation of strength.

1.1 Actions in Eurocode

In Eurocode 1, the characteristic value of a permanent action, G_k, is determined as the mean value, if G does not vary significantly (COV < 0.05). Otherwise, $G_{k,sup}$ is the 95% fractile of the statistical distribution, which can be assumed to be normal.

The annual maximum values are used for variable actions and the characteristic value is, in most cases, based upon a probability of exceedance of 0.02 of its time varying part for a reference period of one year. This is equivalent to a mean return period of 50 years.

The design value of an action is calculated by:

$$F_d = \gamma_f F_{rep} = \gamma_f \psi F_k \qquad (1.1)$$

where F_k is the characteristic value of the action and F_{rep} is the relevant representative value of the action. γ_f is the partial safety factor of the action, which takes account of the possibility of unfavourable deviations of the action values from the representative values and ψ is the combination factor, which is used when effects of simultaneously occurring actions are combined. Each combination of actions should include a leading variable or an accidental action. When a non-leading action is combined with the leading one, the effect of the non-leading action is multiplied by ψ.

Design values of effects of actions are expressed in general terms as:

$$E_d = \gamma_{Sd} E\{\gamma_f \psi F_k, a_{nom}\} \quad (1.2)$$

where a_{nom} is a nominal value of geometric data and γ_{Sd} is the partial safety factor taking account of uncertainties in modelling the actions and in modelling of the effects of the actions. In most cases:

$$E_d = E\{\gamma_F F_{rep}, a_d\} \quad (1.3)$$

with $\gamma_F = \gamma_{Sd} x \gamma_f$ and where a_d is the design value of the geometrical data.

Eurocode recommends the following γ values to be used in the design:

$\gamma_{G,sup}$ =1.35 (Nordic recommendation 1.2)
γ_Q = 1.5 including γ_{Sd} = 1.15

1.2 Resistance in Eurocode

Material strength is represented by a characteristic value defined as the 5% fractile value. The structural stiffness parameters are represented by a mean value, except in the case of instability.

The design value of a material property is expressed as:

$$X_d = \eta \frac{X_k}{\gamma_m} \quad (1.4)$$

where X_k is the characteristic value of the material property
η is the mean value of the conversion factor taking into account the effect of the duration of the load, volume, and scale effects, effects of moisture and temperature and any other relevant parameters;

γ_m is the partial safety factor of the material property which takes account of the possibility of unfavourable deviations of material properties from their characteristic values and the random part of conversion factor η.

The design resistance for material property i is expressed in the following form provided that resistance is a linear function of material strength:

$$R_d = \frac{1}{\gamma_{Rd}} R\{X_{d,i}; a_d\} = R\left\{\eta_i \frac{X_{k,i}}{\gamma_{M,i}}; a_d\right\} \quad (1.5)$$

where $\gamma_{M,i} = \gamma_{Rd} \times \gamma_{m,i,}$ the random part of η_i is included in $\gamma_{m,i}$, and γ_{Rd} is a partial safety factor covering uncertainty in the resistance model, plus geometric deviations if these are not modelled explicitly.

Lognormal or Weibull distributions are usually assumed for material and structural resistance parameters and model uncertainties.

1.3 Reliability formulation

As the resistance (R) and the effect of action (E), against which R provides safety, are random variables, the performance function, g, is also a random variable. A structure is considered safe when

$$g = R - E > 0 \quad (1.6)$$

If the statistical distribution of the performance function is normal, the probability of failure is given by:

$$P_f = P(g \leq 0) = \Phi(-\beta) \quad (1.7)$$

where Φ is the cumulative distribution function of the standardised normal distribution. Numerical values for β giving the probability of failure 10^{-i} are given in Table 1.

Table 1.1. Relation between β and P_f (prEN 1990 Annex C).

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.28	2.32	3.09	3.72	4.27	4.75	5.20

A simple traditional method of calculating β is FORM, the First-Order Reliability Method, as given in many papers (Skov et al. 1976). It is valid for the normally distributed action effect and resistance functions and gives:

$$\beta = \frac{R_{mean} - S_{mean}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{1.8}$$

When the functions are lognormally distributed, Equation (1.8) is used as follows

$$\beta = \frac{\mu_{\ln,R} - \mu_{\ln,S}}{\sqrt{\sigma_{\ln,R}^2 + \sigma_{\ln,S}^2}} \cong \frac{\log R_{mean} - \log S_{mean}}{\sqrt{V_R^2 + V_S^2}} \tag{1.9}$$

When the main uncertainty comes from actions that have statistically independent maxima in each year, the values of β for a different reference period can be calculated using the following expression:

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n \tag{1.10}$$

where β_n is the reliability index for a reference period of n years and β_1 is the reliability index for one year. Please observe that Equation (1.8) gives the survival probability.

According to the Eurocodes, the design values of action effects, E_d, and resistances, R_d, should be defined such that the probability of having a more unfavourable value is as follows:

$$P(E > E_d) = \Phi(+\alpha_E \beta) \tag{1.11}$$

$$P(R \le R_d) = \Phi(-\alpha_R \beta) \tag{1.12}$$

where β is the target reliability index and α_E and α_R are the first-order reliability method weight factors. It is suggested that α_E = -0.7 and α_R = 0.8 for common cases (β = 3.8 on a 50-year basis). Expressions for the calculation of design values of variables with different probability functions are given in Table 1.2.

Table 1.2. Design values for different distribution functions (EC1: C3). μ is the mean value, σ the standard deviation and V the coefficient of variation.

Distribution	Design values
Normal	$\mu-\alpha\beta\sigma$
Lognormal	$\mu \exp(-\alpha\beta V)$ for $V=\sigma/\mu < 0.2$
Gumbel	$u-a^{-1} \ln[-\ln\Phi(-\alpha\beta)]$ where $u = \mu - 0.577/a$; $a = \pi/(\sigma\sqrt{6})$

1.4 Safety levels and studies

Some building codes define the target safety levels as β-index values. It is said that partial factors used in Eurocodes generally lead to a structure with a β-index value greater than 3.8 for a 50-year reference period (4.7 for 1 year). Based on this target reliability, a Nordic study was carried out to compare reliability levels of concrete, steel and timber structures (SAKO). Input data are given in Table 1.3. It was concluded that

- the reliability level varies considerably with the ratio of variable load to total load, having a maximum when the ratio is around 0.2,
- concrete structures have higher reliability levels than steel and glulam structures when the characteristic variable load is less than the permanent load,
- glulam structures have higher reliability levels than steel and concrete structures for load cases with dominating variable action.

Table 1.3. Statistical distributions and coefficients of variation used for the base case in the Nordic study.

Parameter	Coefficient of variation			Distribution type
	Concrete	Steel	Glulam timber	
Actions				
Permanent				
- Self-weight	0.06	0.02	0.06	Normal
- Other	0.10	0.10	0.10	Normal
Variable				
- Environmental	0.40	0.40	0.40	Gumbel
- Imposed	0.20	0.20	0.20	Gumbel
Strength				
Concrete	0.10			Lognormal
Reinforcement	0.04			Lognormal
Structural Steel		0.05		Lognormal
Glulam timber			0.15	Lognormal
Geometry				
Effective depth	0.02			Normal
Beam depth	0.02	0.01	0.01	Normal
Beam width	0.02	0.01	0.01	Normal
Plate thickness		0.04		Normal
Model uncertainties				
R-model	0.05	0.05	0.05	Normal

In connection with a revision of the Danish structural codes, a calibration of safety factors was carried out. The target for the average β was 4.8. The assumptions and results (optimised partial factors) are summarised in Table 1.4 (Sorensen et al. 2001).

Table 1.4. Assumptions and old and optimised partial coefficients.

	Fractile per cent	Coeff. of variation, per cent	Distribution	Partial coefficients Old	Optimised
Self-weight	50		Normal	1.0	1.0 (fixed)
concrete		6			
steel		4			
wood		6			
Other permanent	50	10	Normal	1.0	1.0 (fixed)
Variable action	98		Gumbel		
imposed		20		1.3	1.3
natural		40		1.3	1.5
Concrete	10/5[1)]	15		1.58	1.49
reinforcement	0.1/5[1)]	5	Log-	1.32	1.23
Steel	5	5	normal	1.42	1.29
Wood					
structural	5	20		1.49	1.64
glulam	5	15		1.34	1.51
Model uncertainty	50		Normal		
concrete		5			
steel		3			
wood		5			

[1)] Old codes/new codes.

In the Swedish building code the target level is 4.3 for a one-year reference period or 3.3 for 50 years (safety class 2 structures). A calibration study of partial safety factors was carried out based on the use of normal distributions of all stochastic variables (lognormal for strength). The assumptions used in the analysis are collected in Table 1.5. As a conclusion, the study suggests that γ_M is 1.15 for wooden materials under special quality control, and 1.25 for other timber materials and connections (Thelandersson et al. 1999).

Table 1.5. Input data in Swedish analysis.

Variable	COV	Characteristic value	Partial factor
Permanent load G	0.05	Mean	1.0
Variable load Q	0.40	98% fractile	1.3
Bending strength f	0.20	5% fractile	to be optimised
Geometrical variable a	0.02	Mean	
Model reliability C	0.10	Mean	

Foschi (Foschi et al. 1989) used a level of β = 3 as a target when analysing the reliability of Canadian structures (30-year reference period). The target reliability level in Canada and the USA seems to be lower than that adopted in the Eurocodes and many European countries.

2. Strength distributions of Finnish timber materials

2.1 Statistical distributions

2.1.1 Normal distribution

The probability density function of a normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{2.1}$$

while the cumulative distribution function is given by

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx \tag{2.2}$$

where μ is the mean and σ is the standard deviation.

The normal distribution is valid for all values of x.

The cumulative distribution function, Equation (2.2), cannot be solved in closed form. However, in many commercial computer programmes, $x_{fractile}$ can be solved numerically as a function of different fractiles of $F(x)$, μ and σ. Furthermore, $x_{fractile}$ is given by

$$x_{fractile} = \mu - k\sigma \tag{2.3}$$

where k is tabulated for the most frequently used fractiles, Table 2.1.

Table 2.1. Values of k for the most frequently used fractiles.

Fractile	0.001	0.005	0.010	0.025	0.050	0.100	0.250
Value of k	3.090	2.576	2.326	1.960	1.645	1.282	0.674

From a set of data (x_1, x_2, ... x_n) the estimate of the mean μ_{est} is given by

$$\mu_{est} = \frac{\sum x_i}{n} \tag{2.4}$$

where x_i is the individual value and n is the number of x values. The estimate of the standard deviation σ_{est} is given by

$$\sigma_{est} = \sqrt{\frac{\sum (x_i - \mu_{est})^2}{n-1}} \tag{2.5}$$

If only some of the x values are used, for example 10% of the smallest ones, the estimated mean and the estimated standard deviation is found by iteration.

2.1.2 Lognormal distribution

The probability density function as well as the cumulative distribution function of a lognormal distribution is given by replacing the value of x by the value of ln$x$ in Equations (2.1) and (2.2).

The lognormal distribution is valid for $x > 0$.

For a lognormal distribution, Equation (2.3) is changed to

$$x_{fractile} = e^{\mu_{\ln x} - k\sigma_{\ln x}} \tag{2.6}$$

The estimate mean μ_{est} and the estimate standard deviation σ_{est} shall be calculated for ln$x$ instead of for x in Equations (2.4) and (2.5).

2.1.3 Two-parameter Weibull distribution

The probability density function of a two-parameter Weibull distribution is given by

$$f(x) = \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \tag{2.7}$$

while the cumulative distribution function is given by

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \tag{2.8}$$

where α is the shape parameter and β is the scale parameter.

The two-parameter Weibull distribution is valid for $x \geq 0$. For $x = 0$, the density function $f(0) = 0$ and the cumulative distribution function $F(0) = 0$.

By rearranging Equation (2.8) to the form

$$\ln x = \ln \beta + \frac{1}{\alpha} \ln(-\ln(1 - F(x))) \tag{2.9}$$

the logarithm of x can be solved using the linear regression equation

$$\ln x = A + B \ln(-\ln(1 - F(x))) \tag{2.10}$$

where the constant A gives an estimate for $\ln\beta$ and the slope B gives an estimate for $1/\alpha$. The value of $F(x)$ is found by ranking the x values in ascending order and dividing the rank number n_{rank} by the number of x values n_{tot}. Finally, the inverse of two times the number of x values is subtracted from the value obtained. Hence, $F(x)$ is given by

$$F(x) = \frac{n_{rank}}{n_{tot}} - \frac{1}{2n_{tot}} \tag{2.11}$$

If only some of the x values are used, for example 10% of the smallest ones, the estimated shape and scale parameters are found using the same regression analysis method.

2.1.4 Three-parameter Weibull distribution

The probability density function of a three-parameter Weibull distribution is given by

$$f(x) = \frac{\alpha}{(\beta - \varepsilon)^{\alpha}} (x - \varepsilon)^{\alpha - 1} e^{-\left(\frac{x-\varepsilon}{\beta-\varepsilon}\right)^{\alpha}} \tag{2.12}$$

while the cumulative distribution function is given by

$$F(x) = 1 - e^{-\left(\frac{x-\varepsilon}{\beta-\varepsilon}\right)^{\alpha}} \quad (2.13)$$

where α is the shape parameter, β is the scale parameter and ε is the location parameter.

The three-parameter Weibull distribution is valid for all $x \geq \varepsilon$. For $x = \varepsilon$, the density function $f(\varepsilon) = 0$ and the cumulative distribution function $F(\varepsilon) = 0$.

From a set of data as well as from a part of those data, the estimated shape, scale and location parameters are found by iteration.

2.1.5 Non-parametric distribution

The cumulative distribution function of a non-parametric distribution is found by ranking the x values in ascending order and dividing the rank number n_{rank} by the number of x values n_{tot}. The inverse of two times the number of x values is subtracted from the value obtained. Hence, $F(x)$ is given by Equation (2.11).

2.2 Sawn timber

2.2.1 Data

The data used represents sawn spruce (*Picea abies*) and pine (*Pinus silvestris*). Spruce was sampled from six different locations in Finland as well as from one location in Sweden. Pine was sampled from one location in Finland as well as from one location in Sweden. Both spruce and pine were randomly sampled from ungraded lots of sawn timber.

Prior to testing, all sawn timber was machine-graded using a Raute Timgrader machine. Raute Timgrader is a traditional bending machine. The load to bend a timber member to a pre-set deflection is measured along the member length at intervals of about 100 mm. The smallest load value describes the weakest cross-section of the member and gives the basis for grading.

Table 2.1. Sawn timber used in the analysis. The width, b, depth, h, moisture content, ω, density, ρ, bending strength, f, and modulus of elasticity, E, are the mean values of each series.

Species	Series	Number	b mm	h mm	ω %	ρ kg/m^3	f N/mm^2	E N/mm^2	Data source
Spruce	S-1	589	42	146	14.7	448	45.2	13000	VTT 1995
Spruce	S-2	150	72	221	14.6	415	38.1	10800	VTT 1996
Spruce	S-3	149	35	97	14.6	457	46.9	12800	VTT 1996
Spruce	S-4	172	45	172	12.2	435	42.8	11900	VTT 1997
Spruce	S-5	167	35	120	11.4	456	44.3	12200	VTT 1997
Spruce	S-97	122	45	95	11.5	497	39.6	-	TRÄTEK 1990
Spruce	S-98	80	45	145	11.0	479	42.0	-	TRÄTEK 1990
Spruce	S-99	79	45	190	10.3	462	34.0	-	TRÄTEK 1990
Pine	P-1	188	42	146	13.6	508	48.5	12800	VTT 1995
Pine	P-97	100	45	95	15.0	471	37.9	10400	TRÄTEK 1986
Pine	P-98	99	45	145	15.3	475	38.1	10100	TRÄTEK 1986
Pine	P-99	100	45	190	15.1	477	38.8	10000	TRÄTEK 1986

Additionally, all sawn timber sampled from Finland was visually graded according to the Nordic grading rules given in INSTA 142. This grading was carried out by a person responsible for the practical training of graders. Furthermore, the grading was carried out in laboratory facilities with no time limit.

It should be noted that the visual grading was not carried out as a part of the production process, where the time for grading is greatly limited, but in laboratory facilities. Secondly, the grader had a higher skill level than most graders. Hence, the visually graded timber is representative for applications of the grading rules but certainly not representative for applications of practical grading.

After grading, all the timber specimens were loaded in edgewise bending to failure according to the test method given in EN 408. In addition to bending strength and modulus of elasticity (true), density and moisture content were determined.

Before analysis, all individual bending strength values were adjusted to a reference depth of 150 mm according to EN 384. Furthermore, all individual modulus of elasticity and density values were adjusted to a moisture content of 12% according to EN 384.

A summary of the data used in the analysis is given in Table 2.1.

2.2.2 Analysis of spruce with a depth of 150 mm

The reference depth given in EN 384 and Eurocode 5 is 150 mm for sawn timber. Hence, the reference spruce material is represented by series S-1 and S-98 given in Table 2.1. Since detailed background information on series S-98 is not available, this series was, for now, excluded from the analysis.

In series S-1, the depth of all the 589 specimens was about 146 mm which is close to the reference depth. Hence, the effect of size on bending strength did not affect the results. The mean moisture content of the specimens was 14.7% while the standard deviation was 1.5%. This resulted in somewhat lower bending strength values than for specimens of 12% moisture content.

The bending strength, f, is plotted against the modulus of elasticity, E, in Figure 2.1. Using linear regression analysis the relationship is given by

$$f = -2.66 + 0.00392E \tag{2.14}$$

The coefficient of correlation (r) is 0.804. In Figure 2.1 and Equation (2.14) the bending strength as well as the modulus of elasticity given from the tests were used without any depth or moisture content adjustments.

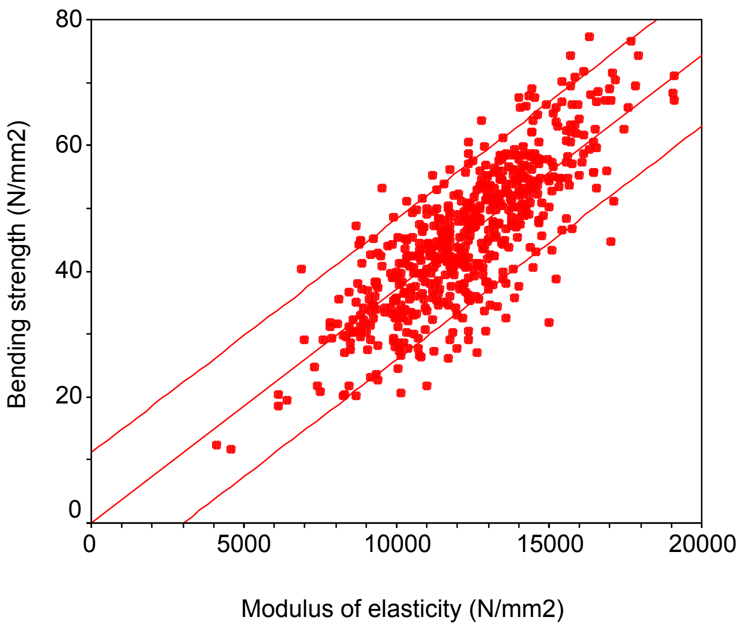


Figure 2.1. The relationship between bending strength and modulus of elasticity for spruce with a depth of 150 mm, series S-1. The linear regression line and the 90% confidence interval are included.

Table 2.2. Density, ρ, bending strength, f, and modulus of elasticity, E, for spruce with a depth of 150 mm, series S-1.

Graded	Number	ρ		f		E	
		Mean	COV	Mean	COV	Mean	COV
		kg/m^3	%	N/mm^2	%	N/mm^2	%
Ungraded	589	448	9.0	45.2	25.2	13000	18.8
Visually	367	455	8.2	49.4	19.8	13700	15.7
Machine	496	456	7.6	47.5	21.6	13600	15.4

All the specimens were analysed as ungraded, visually graded and machine-graded. The density, bending strength and modulus of elasticity of these data sets are summarised in Table 2.2. In addition, the tail data represented by 10% of the weakest specimens were analysed.

The bending strength was modelled using normal, lognormal and two- as well as three-parameter Weibull distribution functions. The estimated parameters of these functions are given in Table 2.3. Furthermore, the model and the data are plotted in Figures 2.2–2.4. The estimated bending strengths at different fractiles are given in Table 2.4.

Since the number of specimens is high enough, 367 for visually and 496 for machine-graded specimens, it is reasonable to use the 5% fractile given by the non-parametric distribution as the reference value. The estimated values divided by the reference value are given in Table 2.5. Based on these ratios the following conclusions can be drawn:

- The lognormal distribution, modelled from all the data, overestimates the 5% fractile of bending strength by about 5%.
- The normal and two- as well as three-parameter Weibull distributions, modelled from all the data, result in good estimates for the 5% fractile of bending strength.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of bending strength.

To compare the tails of the used models to the non-parametric distribution, the estimated fractile values are divided by the corresponding non-parametric values, Table 2.5. The confidence of this analysis may be criticised since only two values are below the 0.5% fractile. However, the following conclusions may be considered:

- The lognormal distribution, modelled from all the data, overestimates the bending strength values for the tail fractiles below 5%.

- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the bending strength values for the tail fractiles below 5%.

- The three-parameter Weibull distribution, modelled from all the data, results in good estimates for the bending strength values for the tail fractiles below 5%.

- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%.

Table 2.3. The modelled distribution functions for spruce with a depth of 150 mm, series S-1.

Distribution function (no. of cases)	Data	Parameters in the distribution function				
		μ	σ	α	β	ε
Ungraded (589)						
Normal	All	45.25	11.40	-	-	-
Normal	Tail	47.69	12.62	-	-	-
Lognormal	All	3.775	0.292	-	-	-
Lognormal	Tail	4.130	0.517	-	-	-
Two-parameter Weibull	All	-	-	4.464	49.60	-
Two-parameter Weibull	Tail	-	-	3.205	68.24	-
Three-parameter Weibull	All	-	-	3.748	49.35	7.02
Three-parameter Weibull	Tail	-	-	-	-	-
Visually graded to C24 (367)						
Normal	All	49.41	9.78	-	-	-
Normal	Tail	46.09	7.60	-	-	-
Lognormal	All	3.880	0.205	-	-	-
Lognormal	Tail	3.909	0.242	-	-	-
Two-parameter Weibull	All	-	-	6.329	53.04	-
Two-parameter Weibull	Tail	-	-	9.524	45.97	-
Three-parameter Weibull	All	-	-	3.137	52.70	21.48
Three-parameter Weibull	Tail	-	-	-	-	-
Machine graded to C30 (496)						
Normal	All	47.48	10.27	-	-	-
Normal	Tail	44.07	7.98	-	-	-
Lognormal	All	3.836	0.226	-	-	-
Lognormal	Tail	3.895	0.284	-	-	-
Two-parameter Weibull	All	-	-	5.714	51.26	-
Two-parameter Weibull	Tail	-	-	8.475	43.90	-
Three-parameter Weibull	All	-	-	3.155	50.95	17.95
Three-parameter Weibull	Tail	-	-	4.466	47.93	12.82

Table 2.4. The modelled bending strengths for spruce with a depth of 150 mm, series S-1.

Distribution function (no. of cases)	Data	Bending strength for different fractiles				
		0.001	0.005	0.010	0.050	0.100
Ungraded (589)						
Normal	All	10.0	15.9	18.7	26.5	30.6
Normal	Tail	8.7	15.2	18.3	26.9	31.5
Lognormal	All	17.7	20.5	22.1	27.0	30.0
Lognormal	Tail	12.6	16.4	18.7	26.6	32.1
Two-parameter Weibull	All	10.6	15.1	17.7	25.5	30.0
Two-parameter Weibull	Tail	7.9	13.1	16.2	27.0	33.8
Three-parameter Weibull	All	13.7	17.3	19.4	26.2	30.2
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	3.9	15.3	20.0	27.5	30.4
Visually graded to C24 (367)						
Normal	All	19.2	24.2	26.7	33.3	36.9
Normal	Tail	22.6	26.5	28.4	33.6	36.4
Lognormal	All	25.7	28.6	30.1	34.6	37.2
Lognormal	Tail	23.6	26.7	28.4	33.5	36.6
Two-parameter Weibull	All	17.8	23.0	25.6	33.2	37.2
Two-parameter Weibull	Tail	22.3	26.4	28.4	33.7	36.3
Three-parameter Weibull	All	24.9	27.3	28.7	33.6	36.7
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	20.9	27.0	29.1	33.1	36.3
Machine graded to C30 (496)						
Normal	All	15.7	21.0	23.6	30.6	34.3
Normal	Tail	19.4	23.5	25.5	30.9	33.8
Lognormal	All	23.0	25.9	27.4	32.0	34.7
Lognormal	Tail	20.4	23.7	25.4	30.8	34.2
Two-parameter Weibull	All	15.3	20.3	22.9	30.5	34.6
Two-parameter Weibull	Tail	19.4	23.5	25.5	30.9	33.7
Three-parameter Weibull	All	21.6	24.1	25.6	30.8	34.1
Three-parameter Weibull	Tail	20.3	23.5	25.4	30.9	34.0
Non-parametric	All	20.6	22.9	26.0	30.5	33.9

Table 2.5. The ratio of modelled to non-parametric bending strength values for spruce with a depth of 150 mm, series S-1.

Distribution function (no. of cases)	Data	Model per non-parametric				
		0.001	0.005	0.010	0.050	0.100
Visually graded to C24 (367)						
Normal	All	-	0.90	0.92	1.01	1.02
Normal	Tail	-	0.98	0.98	1.02	1.00
Lognormal	All	-	1.06	1.03	1.05	1.02
Lognormal	Tail	-	0.99	0.98	1.01	1.01
Two-parameter Weibull	All	-	0.85	0.88	1.00	1.02
Two-parameter Weibull	Tail	-	0.98	0.98	1.02	1.00
Three-parameter Weibull	All	-	1.01	0.99	1.02	1.01
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	-	1	1	1	1
Machine graded to C30 (496)						
Normal	All	-	0.92	0.91	1.00	1.01
Normal	Tail	-	1.03	0.98	1.01	1.00
Lognormal	All	-	1.13	1.05	1.05	1.02
Lognormal	Tail	-	1.03	0.98	1.01	1.01
Two-parameter Weibull	All	-	0.89	0.88	1.00	1.02
Two-parameter Weibull	Tail	-	1.03	0.98	1.01	0.99
Three-parameter Weibull	All	-	1.05	0.98	1.01	1.01
Three-parameter Weibull	Tail	-	1.03	0.98	1.01	1.00
Non-parametric	All	-	1	1	1	1

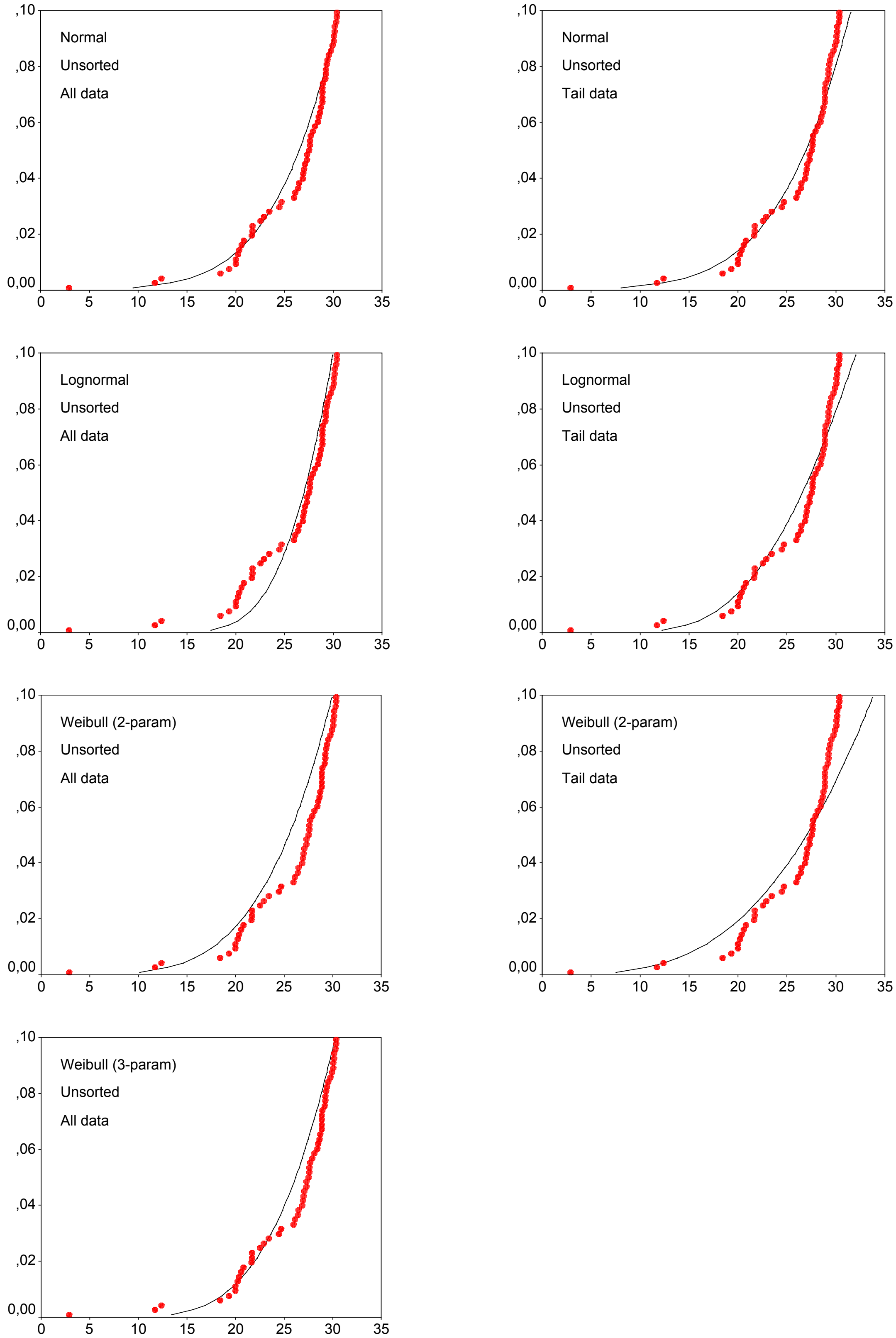


Figure 2.2. Cumulative distributions of bending strength for ungraded spruce with a depth of 150 mm, series S-1.

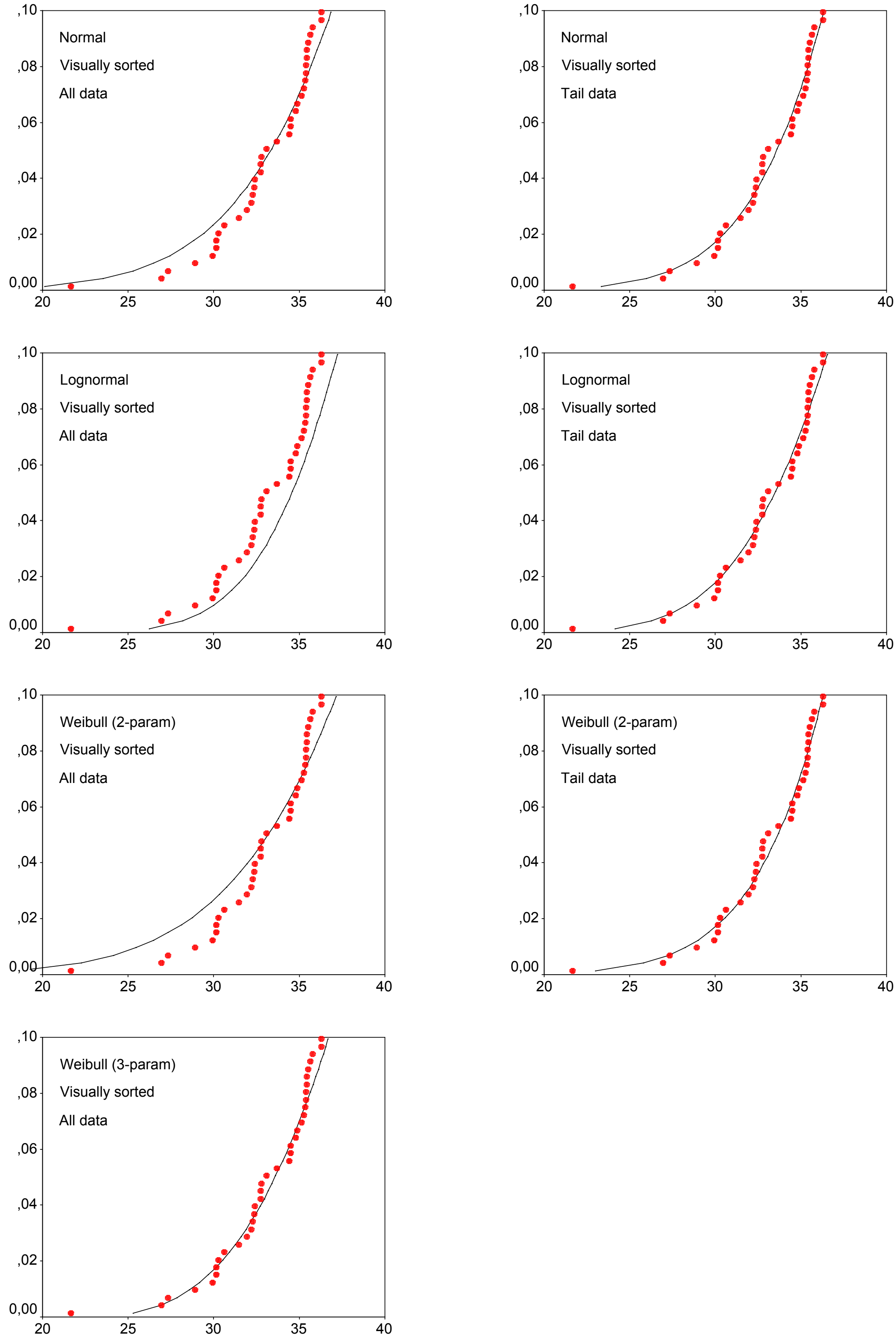


Figure 2.3. Cumulative distributions of bending strength for visually graded spruce with a depth of 150 mm, series S-1. The spruce was graded to class T2 and better given in INSTA 142. T2 corresponds to the European strength class C24.

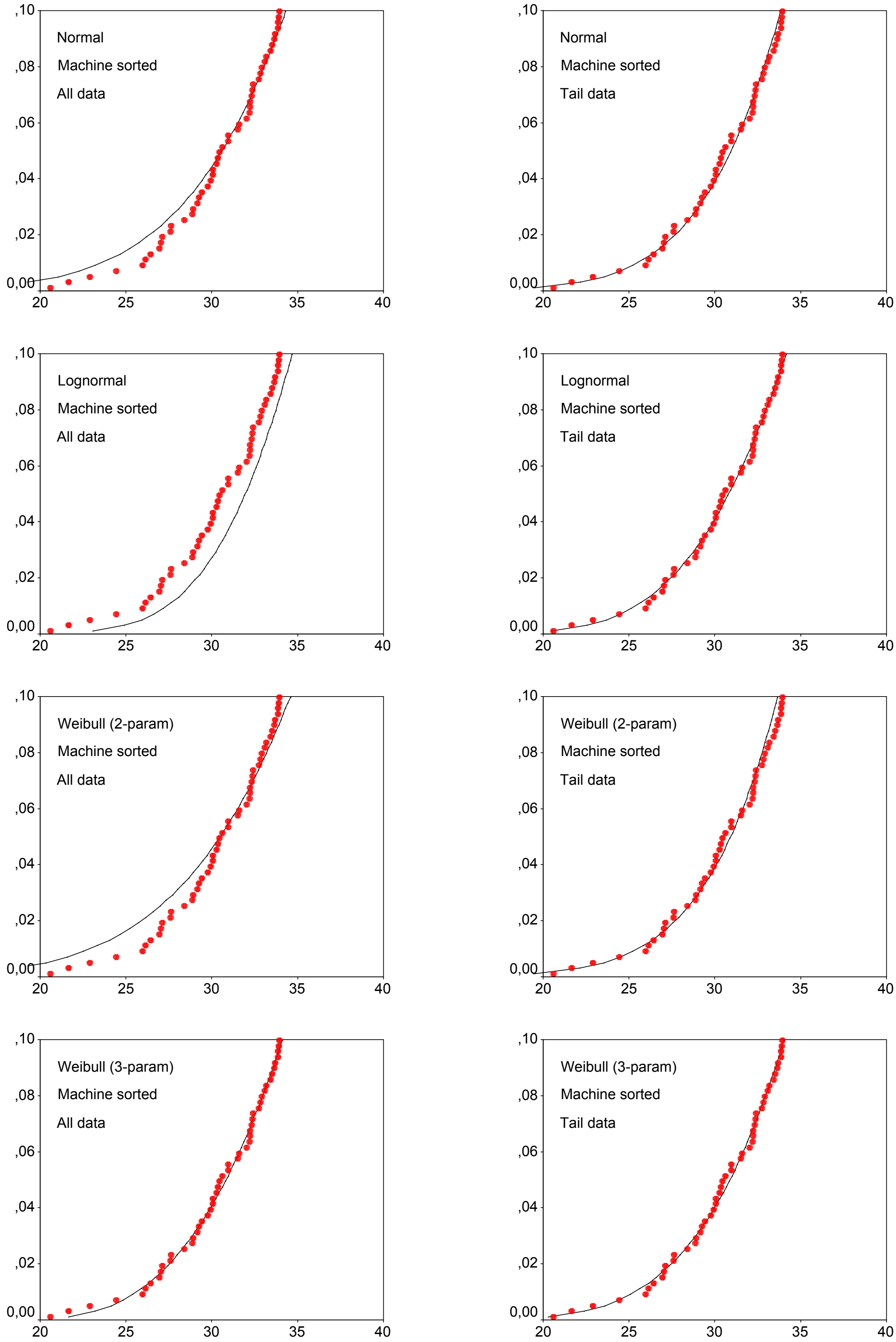


Figure 2.4. Cumulative distributions of bending strength for machine-graded spruce with a depth of 150 mm, series S-1. The spruce was graded by Raute Timgrader to the European strength class C30.

2.2.3 Analysis of spruce

In series S-1 to S-99, the depths of the 1508 specimens were between 95 and 221 mm. The tested bending strength values were adjusted to a reference depth of 150 mm according to EN 384. Since this adjustment is not complete the effect of size on bending strength will affect the results for both ungraded and visually graded spruce. The effect of size on bending strength will not affect the results for machine-graded spruce since the complete effect of it was already considered when the machine was approved. The mean moisture content of the specimens was 13.3% while the standard deviation was 2.0%. This results in somewhat lower bending strength values than for specimens of 12% moisture content.

All the specimens were analysed as ungraded, visually graded and machine-graded. The density, bending strength and modulus of elasticity of these data sets are summarised in Table 2.6. In addition, the tail data represented by 10% of the weakest specimens were analysed.

The bending strength was modelled using normal, lognormal and two- as well as three-parameter Weibull distribution functions. The estimated parameters of these functions are given in Table 2.7. Furthermore, the model and the data are plotted in Figures 2.5–2.7. The estimated bending strengths at different fractiles are given in Table 2.8.

In estimating the 5% fractile of the 986 machine-graded specimens, Figure 2.7, the same conclusions as those drawn for spruce with a depth of 150 mm can also be drawn. The conclusions were:

- The lognormal distribution, modelled from all the data, overestimate the 5% fractile of bending strength.
- The normal and two- as well as three-parameter Weibull distributions, modelled from all the data, result in good estimates for the 5% fractile of bending strength.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of bending strength.

Table 2.6. Density, ρ, bending strength, f, and modulus of elasticity, E, for spruce, series S-1 to S-99.

Graded	Number	ρ		f		E	
		Mean	COV	Mean	COV	Mean	COV
		kg/m^3	%	N/mm^2	%	N/mm^2	%
Ungraded	1508	451	9.7	43.1	27.1	12400	19.8
Visually	781	447	8.8	47.3	21.2	13000	18.1
Machine	986	465	8.4	47.8	21.0	13400	15.3

The following conclusions can be drawn for estimation of the tails of the models used, Figure 2.7:

- The lognormal distribution, modelled from all the data, overestimates the bending strength values for the tail fractiles below 5%. This is the same conclusion as that drawn for spruce with a depth of 150 mm.
- The two-parameter Weibull distribution, modelled from all the data, underestimates the bending strength values for the tail fractiles below 5%. This is the same conclusion as that drawn for spruce with a depth of 150 mm.
- The normal and three-parameter Weibull distributions, modelled from all the data, result in good estimates for the bending strength values for the tail fractiles below 5%.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%. This is the same conclusion as that drawn for spruce with a depth of 150 mm.

Table 2.7. The modelled distribution functions for spruce, series S-1 to S-99.

Distribution function (no. of cases)	Data	Parameters in the distribution function				
		μ	σ	α	β	ε
Ungraded (1508)						
Normal	All	43.10	11.68	-	-	-
Normal	Tail	42.26	11.21	-	-	-
Lognormal	All	3.721	0.309	-	-	-
Lognormal	Tail	4.049	0.544	-	-	-
Two-parameter Weibull	All	-	-	4.167	47.47	-
Two-parameter Weibull	Tail	-	-	3.610	54.22	-
Three-parameter Weibull	All	-	-	3.705	47.30	4.27
Three-parameter Weibull	Tail	-	-	-	-	-
Visually graded to C24 (781)						
Normal	All	47.30	10.03	-	-	-
Normal	Tail	45.22	8.30	-	-	-
Lognormal	All	3.833	0.220	-	-	-
Lognormal	Tail	3.921	0.288	-	-	-
Two-parameter Weibull	All	-	-	5.882	50.96	-
Two-parameter Weibull	Tail	-	-	8.197	45.42	-
Three-parameter Weibull	All	-	-	2.994	50.60	19.81
Three-parameter Weibull	Tail	-	-	7.536	45.81	2.18
Machine graded to C30 (986)						
Normal	All	47.79	10.01	-	-	-
Normal	Tail	45.44	8.53	-	-	-
Lognormal	All	3.844	0.220	-	-	-
Lognormal	Tail	3.944	0.305	-	-	-
Two-parameter Weibull	All	-	-	5.882	51.52	-
Two-parameter Weibull	Tail	-	-	8.197	45.11	-
Three-parameter Weibull	All	-	-	3.338	51.25	17.50
Three-parameter Weibull	Tail	-	-	3.165	54.19	16.51

Table 2.8. The modelled bending strengths for spruce, series S-1 to S-99.

Distribution function (no. of cases)	Data	Bending strength for different fractiles				
		0.001	0.005	0.010	0.050	0.100
Ungraded (1508)						
Normal	All	7.0	13.0	15.9	23.9	28.1
Normal	Tail	7.6	13.4	16.2	23.8	27.9
Lognormal	All	15.9	18.6	20.1	24.8	27.8
Lognormal	Tail	10.7	14.1	16.2	23.4	28.6
Two-parameter Weibull	All	9.0	13.3	15.7	23.3	27.7
Two-parameter Weibull	Tail	8.0	12.5	15.2	23.8	29.1
Three-parameter Weibull	All	10.9	14.6	16.7	23.6	27.7
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	6.9	12.8	16.6	23.9	28.0
Visually graded to C24 (781)						
Normal	All	16.3	21.5	24.0	30.8	34.4
Normal	Tail	19.6	23.8	25.9	31.6	34.6
Lognormal	All	23.4	26.2	27.7	32.2	34.9
Lognormal	Tail	20.7	24.0	25.8	31.4	34.9
Two-parameter Weibull	All	15.7	20.7	23.3	30.8	34.8
Two-parameter Weibull	Tail	19.6	23.8	25.9	31.6	34.5
Three-parameter Weibull	All	22.9	25.1	26.4	31.2	34.3
Three-parameter Weibull	Tail	19.6	23.8	25.9	31.6	34.5
Non-parametric	All	20.1	23.5	25.3	31.6	34.5
Machine graded to C30 (986)						
Normal	All	16.9	22.0	24.5	31.3	35.0
Normal	Tail	19.1	23.5	25.6	31.4	34.5
Lognormal	All	23.7	26.5	28.0	32.5	35.2
Lognormal	Tail	20.1	23.5	25.4	31.3	34.9
Two-parameter Weibull	All	15.9	20.9	23.6	31.1	35.1
Two-parameter Weibull	Tail	19.4	23.6	25.7	31.4	34.3
Three-parameter Weibull	All	21.8	24.4	26.0	31.4	34.7
Three-parameter Weibull	Tail	20.8	23.6	25.3	31.3	35.0
Non-parametric	All	21.2	22.8	25.6	31.3	34.9

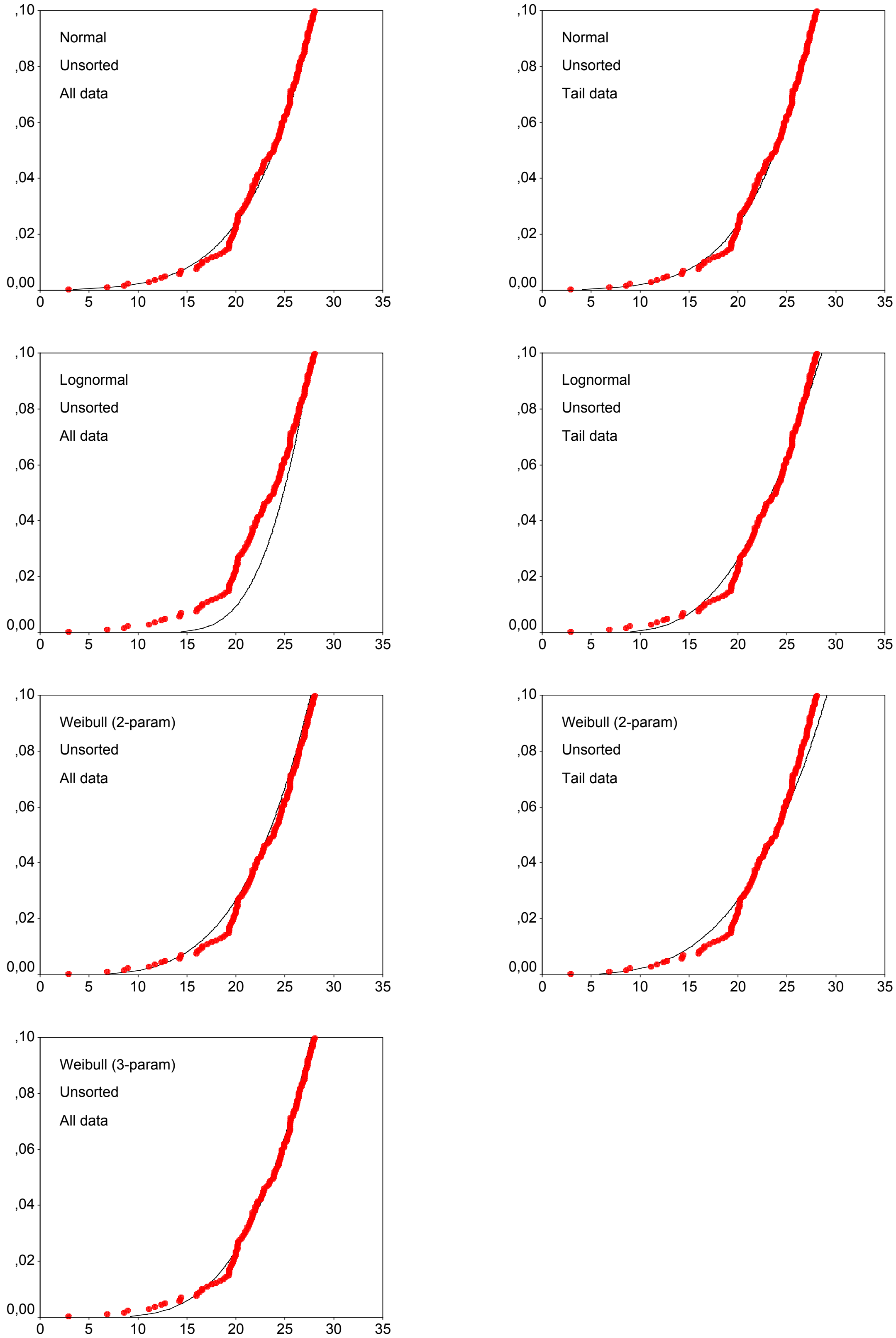


Figure 2.5. Cumulative distributions of bending strength for ungraded spruce, series S-1 to S-99.

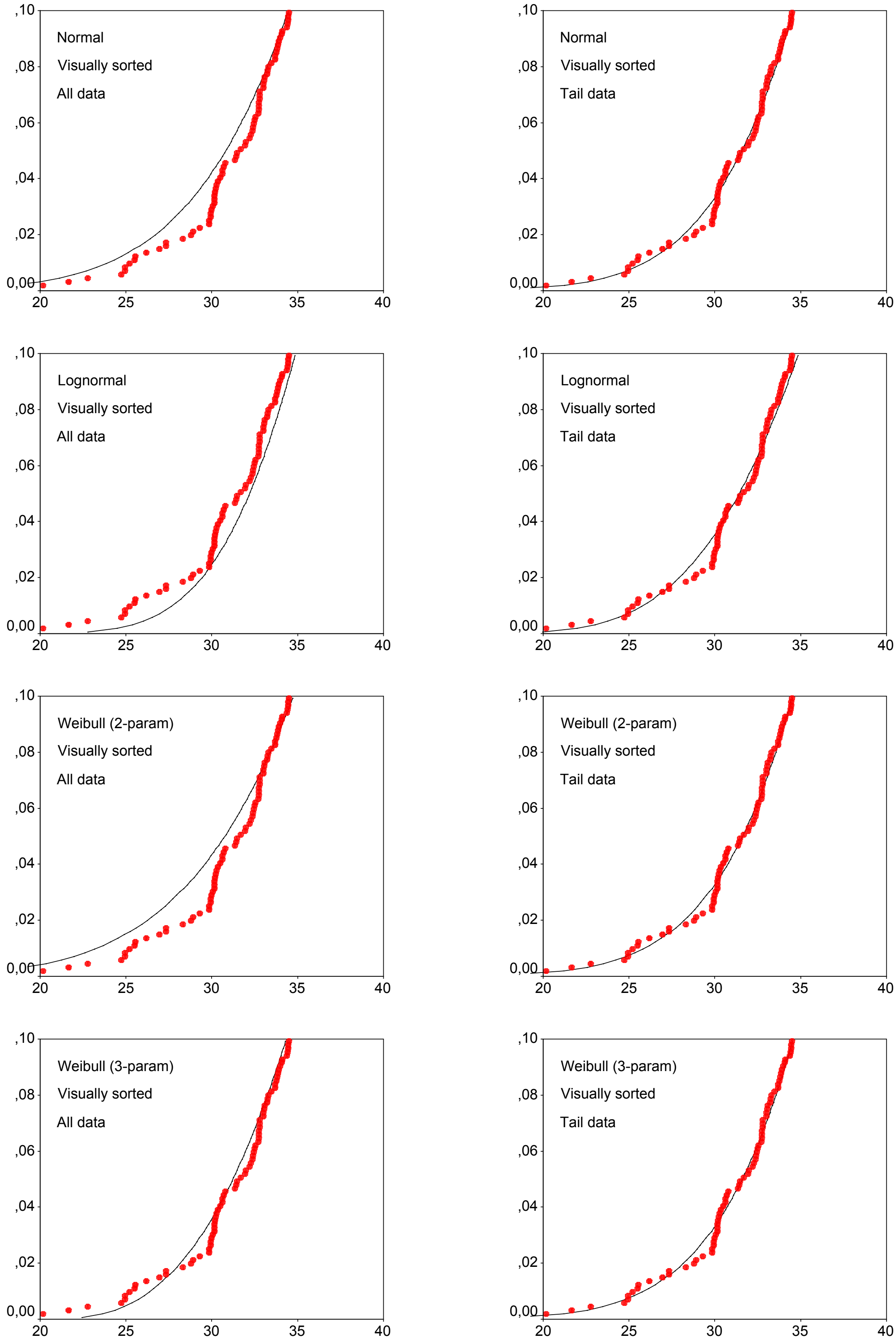


Figure 2.6. Cumulative distributions of bending strength for visually graded spruce, series S-1 to S-99. The spruce was graded to class T2 and better given in INSTA 142. T2 corresponds to the European strength class C24.

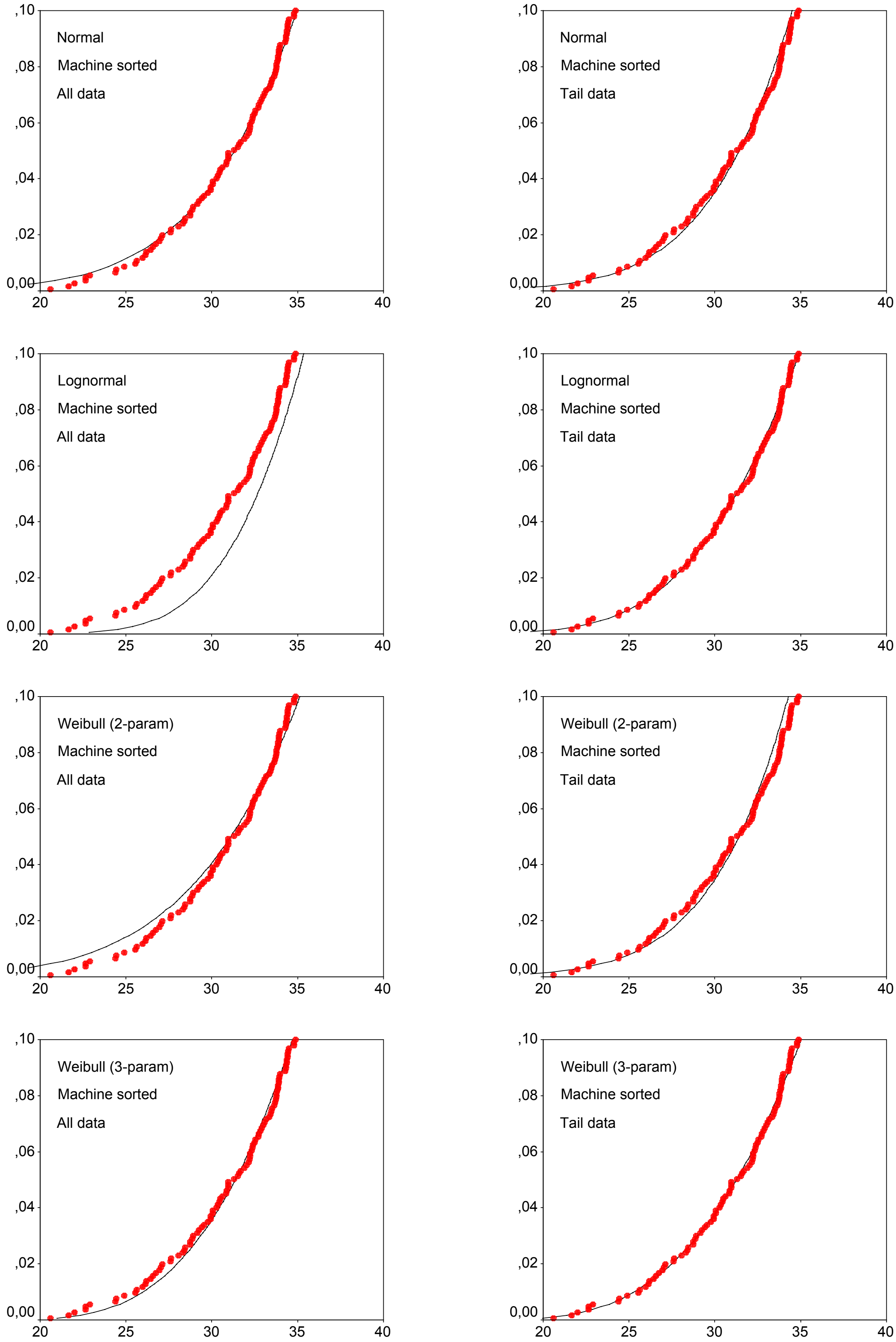


Figure 2.7. Cumulative distributions of bending strength for machine-graded spruce, series S-1 to S-99. The spruce was graded by Raute Timgrader to the European strength class C30.

2.2.4 Analysis of spruce and pine

In series S-1 to S-99 and P-1 to P-99, the depths of the 1995 specimens were between 95 and 221 mm. The tested bending strength values were adjusted to a reference depth of 150 mm according to EN 384. Since this adjustment is not complete the effect of size on bending strength will affect the results for both ungraded and visually graded spruce. The effect of size on bending strength will not affect the results for machine-graded spruce since the complete effect of it was already considered when the machine was approved. The mean moisture content of the specimens was 13.6% while the standard deviation was 1.9%. This results in somewhat lower bending strength values than for specimens of 12% moisture content.

All the specimens were analysed as ungraded, visually graded and machine-graded. The density, bending strength and modulus of elasticity of these data sets are summarised in Table 2.9. In addition, the tail data represented by 10% of the weakest specimens were analysed.

The bending strength was modelled using normal, lognormal and two- as well as three-parameter Weibull distribution functions. The estimated parameters of these functions are given in Table 2.10. Furthermore, the model and the data are plotted in Figures 2.8–2.10. The estimated bending strengths at different fractiles are given in Table 2.11.

In estimating the 5% fractile of the 1327 machine-graded specimens, Figure 2.10, the same conclusions as those drawn for spruce with a depth of 150 mm can also be drawn. The conclusions were:

- The lognormal distribution, modelled from all the data, overestimates the 5% fractile of bending strength.
- The normal and two- as well as three-parameter Weibull distributions, modelled from all the data, result in good estimates for the 5% fractile of bending strength.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of bending strength.

Table 2.9. Density, ρ, bending strength, f, and modulus of elasticity, E, for spruce and pine, series S-1 to S-99 and P-1 to P-99.

Graded	Number	ρ		f		E	
		Mean	COV	Mean	COV	Mean	COV
		kg/m^3	%	N/mm^2	%	N/mm^2	%
Ungraded	1995	460	10.5	42.9	28.6	12100	22.0
Visually	902	458	10.9	48.4	21.9	13200	18.0
Machine	1327	475	9.5	47.8	22.3	13100	16.9

The following conclusions can be drawn for estimation of the tails of the models used, Figure 2.10:

- The lognormal distribution, modelled from all the data, overestimates the bending strength values for the tail fractiles below 5%. This is the same conclusion as that drawn for spruce with a depth of 150 mm.
- The two-parameter Weibull distribution, modelled from all the data, underestimates the bending strength values for the tail fractiles below 5%. This is the same conclusion as that drawn for spruce with a depth of 150 mm.
- The normal and three-parameter Weibull distributions, modelled from all the data, result in reasonable estimates for the bending strength values for the tail fractiles below 5%.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%. This is the same conclusion as that drawn for spruce with a depth of 150 mm.

Table 2.10. The modelled distribution functions for spruce and pine, series S-1 to S-99 and P-1 to P-99.

Distribution function (no. of cases)	Data	Parameters in the distribution function				
		μ	σ	α	β	ε
Ungraded (1995)						
Normal	All	42.89	12.28	-	-	-
Normal	Tail	39.23	9.88	-	-	-
Lognormal	All	3.712	0.320	-	-	-
Lognormal	Tail	3.938	0.496	-	-	-
Two-parameter Weibull	All	-	-	4.016	47.28	-
Two-parameter Weibull	Tail	-	-	4.132	47.23	-
Three-parameter Weibull	All	-	-	3.297	47.11	6.13
Three-parameter Weibull	Tail	-	-	-	-	-
Visually graded to C24 (902)						
Normal	All	48.38	10.61	-	-	-
Normal	Tail	45.30	8.18	-	-	-
Lognormal	All	3.854	0.227	-	-	-
Lognormal	Tail	3.918	0.280	-	-	-
Two-parameter Weibull	All	-	-	5.714	52.20	-
Two-parameter Weibull	Tail	-	-	8.403	45.42	-
Three-parameter Weibull	All	-	-	2.925	51.84	19.89
Three-parameter Weibull	Tail	-	-	8.131	45.55	0.86
Machine graded to C30 (1327)						
Normal	All	47.80	10.64	-	-	-
Normal	Tail	45.98	9.30	-	-	-
Lognormal	All	3.841	0.235	-	-	-
Lognormal	Tail	3.979	0.341	-	-	-
Two-parameter Weibull	All	-	-	5.495	51.73	-
Two-parameter Weibull	Tail	-	-	7.092	46.67	-
Three-parameter Weibull	All	-	-	3.280	51.45	16.10
Three-parameter Weibull	Tail	-	-	4.399	50.77	9.70

Table 2.11. The modelled bending strengths for spruce and pine, series S-1 to S-99 and P-1 to P-99.

Distribution function (no. of cases)	Data	Bending strength for different fractiles				
		0.001	0.005	0.010	0.050	0.100
Ungraded (1995)						
Normal	All	4.9	11.3	14.3	22.7	27.2
Normal	Tail	8.7	13.8	16.2	23.0	26.6
Lognormal	All	15.2	18.0	19.4	24.2	27.2
Lognormal	Tail	11.1	14.3	16.2	22.7	27.2
Two-parameter Weibull	All	8.5	12.6	15.0	22.6	27.0
Two-parameter Weibull	Tail	8.9	13.1	15.5	23.0	27.4
Three-parameter Weibull	All	11.2	14.4	16.3	22.8	26.8
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	7.6	14.4	16.9	22.6	26.9
Visually graded to C24 (902)						
Normal	All	15.6	21.1	23.7	30.9	34.8
Normal	Tail	20.0	24.2	26.3	31.8	34.8
Lognormal	All	23.4	26.3	27.8	32.5	35.3
Lognormal	Tail	21.2	24.5	26.2	31.7	35.1
Two-parameter Weibull	All	15.6	20.7	23.3	31.0	35.2
Two-parameter Weibull	Tail	20.0	24.2	26.3	31.9	34.7
Three-parameter Weibull	All	22.9	25.1	26.5	31.5	34.7
Three-parameter Weibull	Tail	20.0	24.2	26.2	31.9	34.7
Non-parametric	All	20.1	24.7	25.5	32.1	34.9
Machine graded to C30 (1327)						
Normal	All	14.9	20.4	23.0	30.3	34.2
Normal	Tail	17.2	22.0	24.3	30.7	34.1
Lognormal	All	22.5	25.4	27.0	31.6	34.5
Lognormal	Tail	18.6	22.2	24.2	30.5	34.5
Two-parameter Weibull	All	14.7	19.7	22.4	30.1	34.3
Two-parameter Weibull	Tail	17.6	22.1	24.4	30.7	34.0
Three-parameter Weibull	All	20.4	23.1	24.8	30.4	33.9
Three-parameter Weibull	Tail	18.2	22.0	24.1	30.6	34.3
Non-parametric	All	18.5	22.0	23.6	30.6	34.3

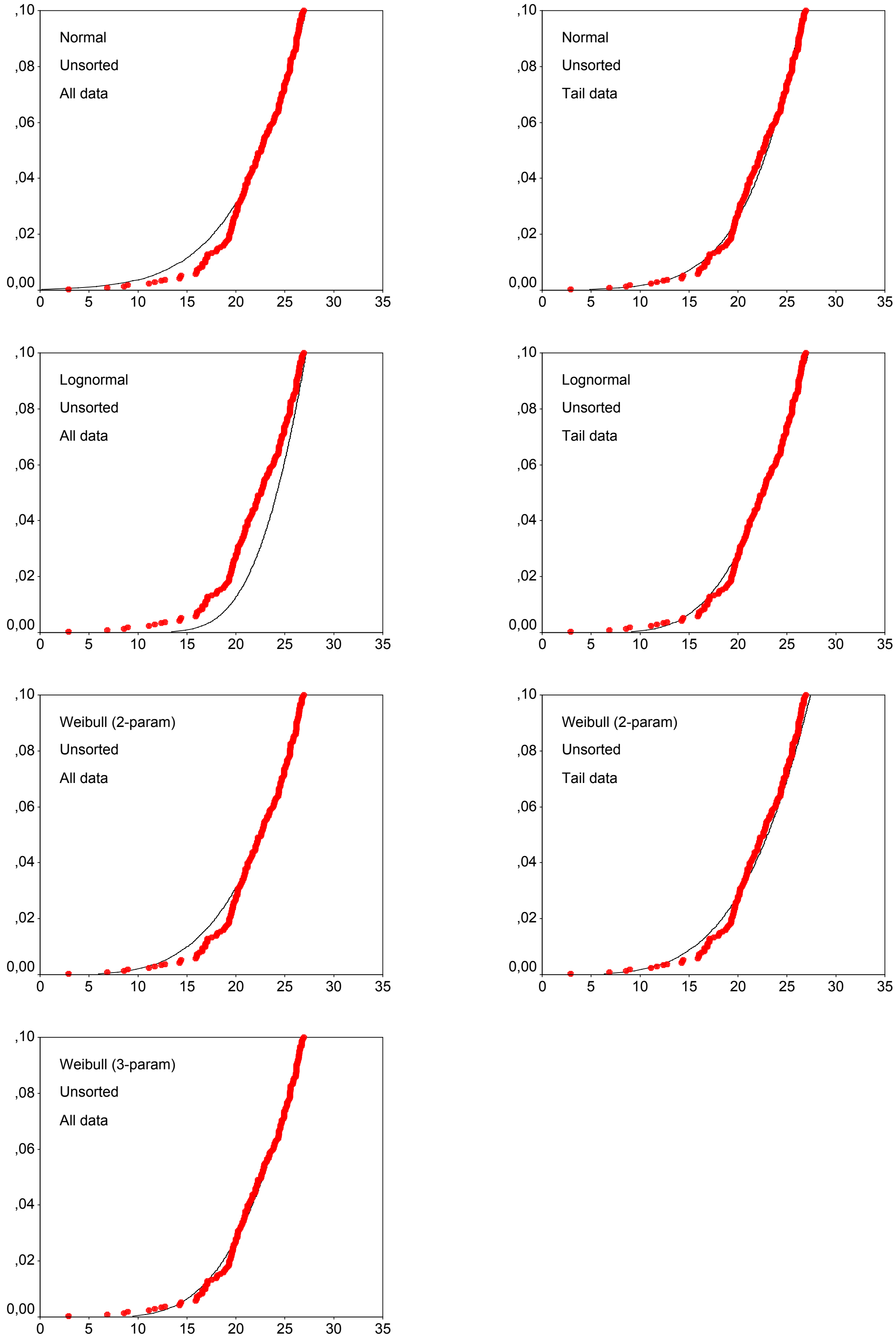


Figure 2.8. Cumulative distributions of bending strength for ungraded spruce and pine series S-1 to S-99 and P-1 to P-99.

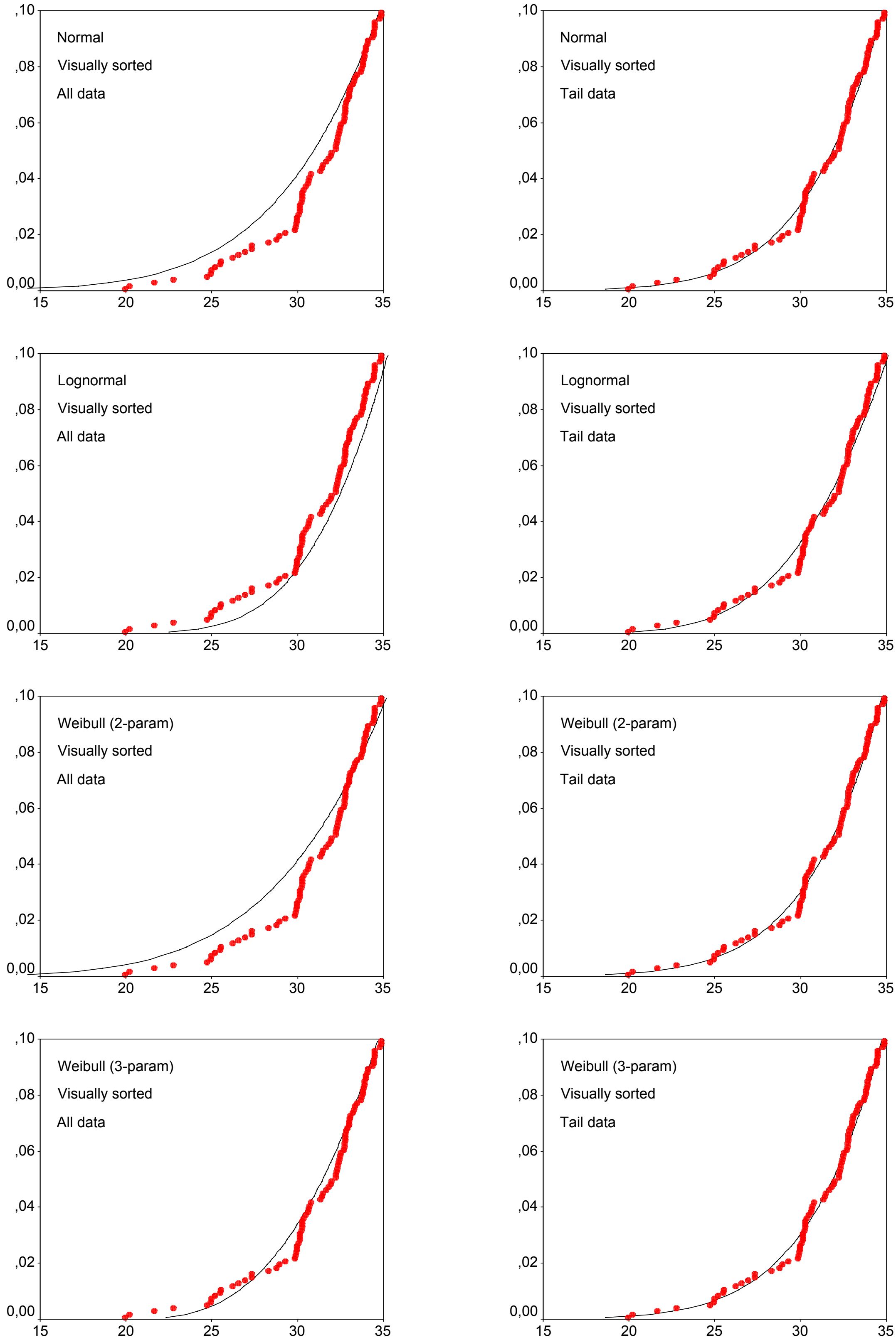


Figure 2.9. Cumulative distributions of bending strength for visually graded spruce and pine, series S-1 to S-99 and P-1 to P-99. The spruce was graded to class T2 and better given in INSTA 142. T2 corresponds to the European strength class C24.

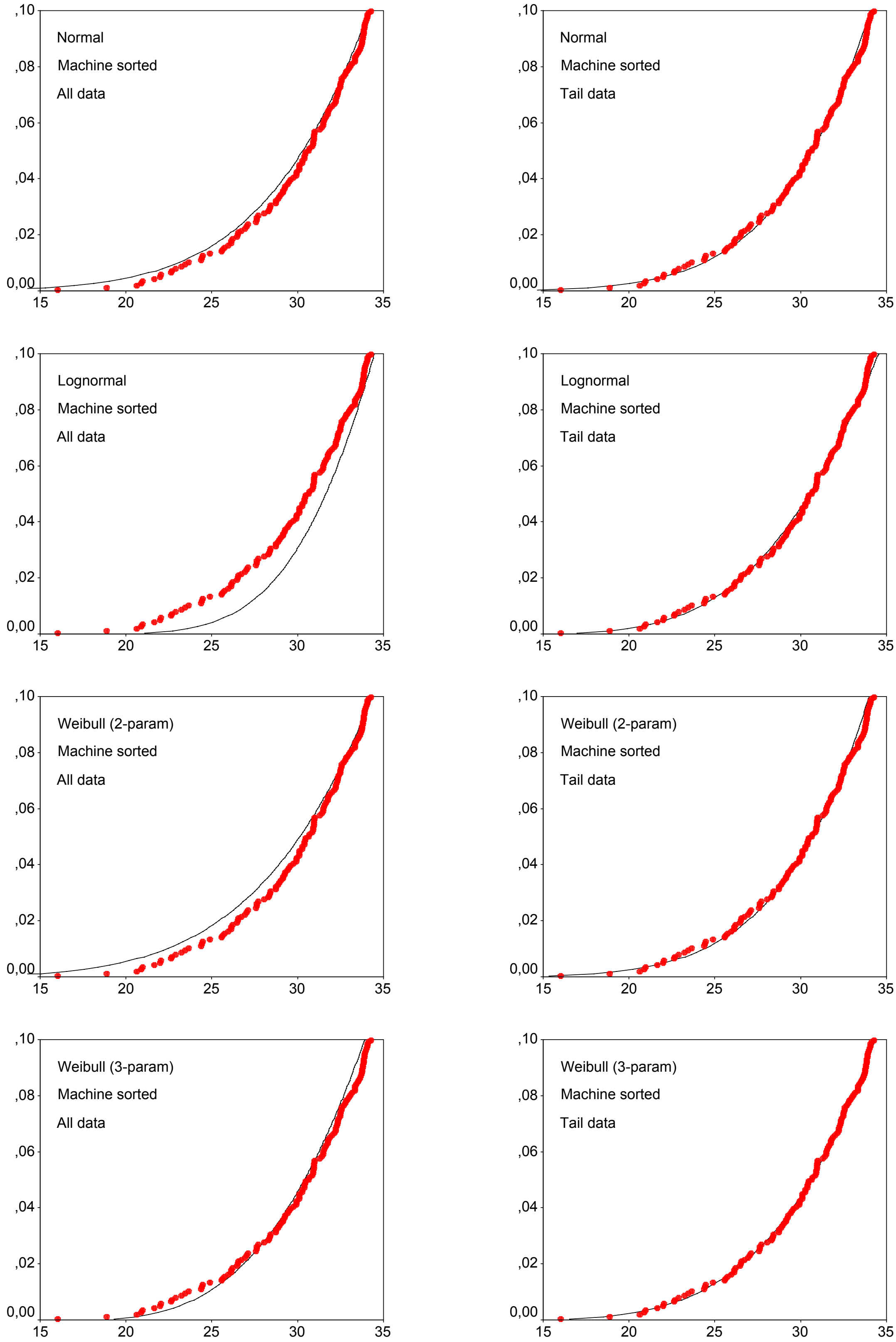


Figure 2.10. Cumulative distributions of bending strength for machine-graded spruce and pine, series S-1 to S-99 and P-1 to P-99. The spruce was graded by Raute Timgrader to the European strength class C30.

2.3 Kerto-laminated veneer lumber

2.3.1 Data

The data used represents Kerto-laminated veneer lumber manufactured by Finnforest at the mill located in Lohja, Finland. In addition to data obtained from internal quality control between 1995 and 1998, the data obtained from external quality control between 1993 and 1999 were used.

The 3.0 mm thick veneers used in the production process of Kerto-laminated veneer lumber are rotary peeled from spruce (*Picea abies*). In the automatic sorting process all veneers with a density below a pre-set limit value are rejected. In order to obtain a more uniform quality of the characteristics of the product, the veneers are in turn placed in different stacks. The size and location of knots and other discontinuities are then spread.

Both edgewise and flatwise bending tests were carried out according to the test method given in EN 408. In edgewise bending, the depths of the specimens were 100 mm and the widths (thickness of the panel) were between 27 and 75 mm. In flatwise bending, the depths of the specimens were between 27 and 75 mm and the widths were 100 mm.

A summary of the data used in the analysis is given in Table 2.12. For the external quality control tests, the specimens are pre-conditioned in humidity rooms where the relative humidity is 65% and the temperature is 20 °C. No conditioning is carried out and the moisture content is about 8% at testing for internal quality control. A lower moisture content results in higher bending strength values. On the other hand, testing of fresh specimens may also effect bending strength values.

Table 2.12. Moisture content, ω, density, ρ, bending strength, f, and modulus of elasticity, E, of Kerto-LVL used in the analysis.

LVL	Series		ω %	ρ kg/m^3	f_{edge} N/mm^2	E_{edge} N/mm^2	f_{flat} N/mm^2	E_{flat} N/mm^2	Data source
Kerto	K-EQC	Mean	9.7	508	58.6	13500	60.7	13800	VTT-QC
		COV	8.4	4.0	10.2	8.4	12.7	10.4	
		Number	372	372	372	372	372	372	
Kerto	K-IQC	Mean	-	-	60.1	-	64.3	-	FF-QC
		COV	-	-	9.6	-	14.4	-	
		Number	-	-	1968	-	1963	-	

2.3.2 Analysis of external quality control results

The bending strength, f_{edge}, is plotted against the modulus of elasticity, E_{edge}, in Figure 2.11. Using linear regression analysis the relationship is given by

$$f_{edge} = 7.84 + 0.00376 E_{edge} \tag{2.15}$$

The coefficient of correlation (r) is 0.717. This relationship is based on the 372 specimens tested in the external quality control process.

Fonselius (1997) has previously reported the relationship for Kerto-laminated veneer lumber

$$f_{edge} = 2.46 + 0.00410 E_{edge} \tag{2.16}$$

of which the coefficient of correlation is 0.823. This relationship is based on 150 specimens of which 15 represent Kerto-laminated veneer lumber of low density, 430 kg/m^3.

Equations (2.15) and (2.16) result in practically the same values for bending strength for modulus of elasticity values between 10000 and 18000 N/mm^2.

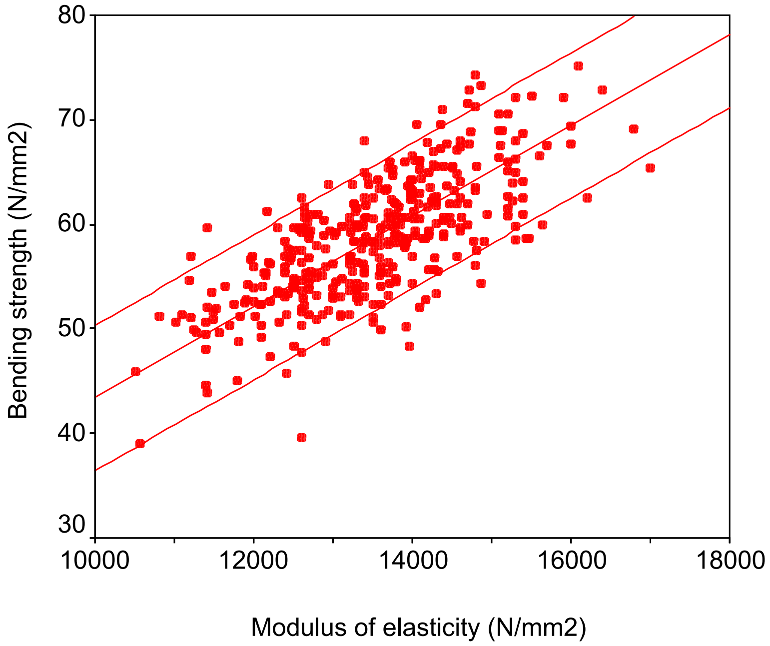


Figure 2.11. Dependence between bending strength and modulus of elasticity for Kerto-LVL, series K-EQC. The linear regression line and the 90% confidence interval are included.

2.3.3 Analysis of internal quality control results

Both edgewise and flatwise bending strength was modelled using normal, lognormal and two- as well as three-parameter Weibull distribution functions. In addition to all of the data, the tail data, represented by 10% of the weakest specimens, were also analysed. The estimated parameters of these functions are given in Table 2.13. Furthermore, the model and the data are plotted in Figures 2.12–2.13. The estimated bending strengths at different fractiles are given in Table 2.14.

Since the number of specimens is adequate, 1968 for edgewise bent and 1963 for flatwise bent specimens, it is reasonable to use the 5% fractile given by the non-parametric distribution as the reference value. The estimated values divided by the reference value are given in Table 2.15. Based on these ratios the following conclusions can be drawn:

- The lognormal and three-parameter Weibull distributions, modelled from all the data, result in good estimates for the 5% fractile of bending strength.
- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the 5% fractile of bending strength by about 2%.
- The normal, lognormal and two-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of bending strength.

To compare the tails of the models used to the non-parametric distribution, the estimated fractile values are divided by the corresponding non-parametric values, Table 2.15. The following conclusions can be drawn:

- The lognormal distribution, modelled from all the data, result in good estimates for the bending strength values for the tail fractiles below 5%.
- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the bending strength values for the tail fractiles below 5%.
- The three-parameter Weibull distribution, modelled from all the data, overestimates the bending strength values for the tail fractiles below 5%.
- The normal, lognormal and two-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%.

Table 2.13. The modelled distribution functions for Kerto-LVL, series K-IQC.

Distribution function (no. of cases)	Data	Parameters in the distribution function				
		μ	σ	α	β	ε
Bending strength, edgewise (1968)						
Normal	All	60.11	5.77	-	-	-
Normal	Tail	58.84	4.59	-	-	-
Lognormal	All	4.092	0.095	-	-	-
Lognormal	Tail	4.089	0.092	-	-	-
Two-parameter Weibull	All	-	-	13.945	62.36	-
Two-parameter Weibull	Tail	-	-	26.288	57.45	-
Three-parameter Weibull	All	-	-	2.530	61.83	46.53
Three-parameter Weibull	Tail	-	-	-	-	-
Bending strength, flatwise (1963)						
Normal	All	64.29	9.24	-	-	-
Normal	Tail	60.52	5.98	-	-	-
Lognormal	All	4.153	0.142	-	-	-
Lognormal	Tail	4.128	0.123	-	-	-
Two-parameter Weibull	All	-	-	9.346	67.69	-
Two-parameter Weibull	Tail	-	-	19.608	58.97	-
Three-parameter Weibull	All	-	-	2.317	66.88	44.12
Three-parameter Weibull	Tail	-	-	-	-	-

Table 2.14. The modelled bending strengths for Kerto-LVL, series K-IQC.

Distribution function (no. of cases)	Data	Bending strength for different fractiles				
		0.001	0.005	0.010	0.050	0.100
Bending strength, edgewise (1968)						
Normal	All	42.3	45.2	46.7	50.6	52.7
Normal	Tail	44.7	47.0	48.2	51.3	53.0
Lognormal	All	44.6	46.9	48.0	51.2	53.0
Lognormal	Tail	44.9	47.1	48.2	51.3	53.0
Two-parameter Weibull	All	38.0	42.7	44.8	50.4	53.1
Two-parameter Weibull	Tail	44.2	47.0	48.2	51.3	52.7
Three-parameter Weibull	All	47.5	48.4	49.0	51.3	52.8
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	44.7	47.0	48.1	51.3	53.0
Bending strength, flatwise (1963)						
Normal	All	35.7	40.5	42.8	49.1	52.4
Normal	Tail	42.0	45.1	46.6	50.7	52.9
Lognormal	All	41.0	44.1	45.7	50.4	53.0
Lognormal	Tail	42.4	45.2	46.6	50.7	53.0
Two-parameter Weibull	All	32.3	38.4	41.4	49.3	53.2
Two-parameter Weibull	Tail	41.5	45.0	46.6	50.7	52.6
Three-parameter Weibull	All	45.3	46.4	47.2	50.4	52.7
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	41.2	44.6	46.6	50.3	53.0

Table 2.15. The ratio of modelled to non-parametric bending strength values for Kerto-LVL, series K-IQC.

Distribution function (no. of cases)	Data	Model per non-parametric				
		0.001	0.005	0.010	0.050	0.100
Bending strength, edgewise (1968)						
Normal	All	0.95	0.96	0.97	0.99	0.99
Normal	Tail	1.00	1.00	1.00	1.00	1.00
Lognormal	All	1.00	1.00	1.00	1.00	1.00
Lognormal	Tail	1.00	1.00	1.00	1.00	1.00
Two-parameter Weibull	All	0.85	0.91	0.93	0.98	1.00
Two-parameter Weibull	Tail	0.99	1.00	1.00	1.00	0.99
Three-parameter Weibull	All	1.06	1.03	1.02	1.00	1.00
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	1	1	1	1	1
Bending strength, flatwise (1963)						
Normal	All	0.87	0.91	0.92	0.98	0.99
Normal	Tail	1.02	1.01	1.00	1.01	1.00
Lognormal	All	1.00	0.99	0.98	1.00	1.00
Lognormal	Tail	1.03	1.01	1.00	1.01	1.00
Two-parameter Weibull	All	0.78	0.86	0.89	0.98	1.00
Two-parameter Weibull	Tail	1.01	1.01	1.00	1.01	0.99
Three-parameter Weibull	All	1.10	1.04	1.01	1.00	0.99
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	1	1	1	1	1

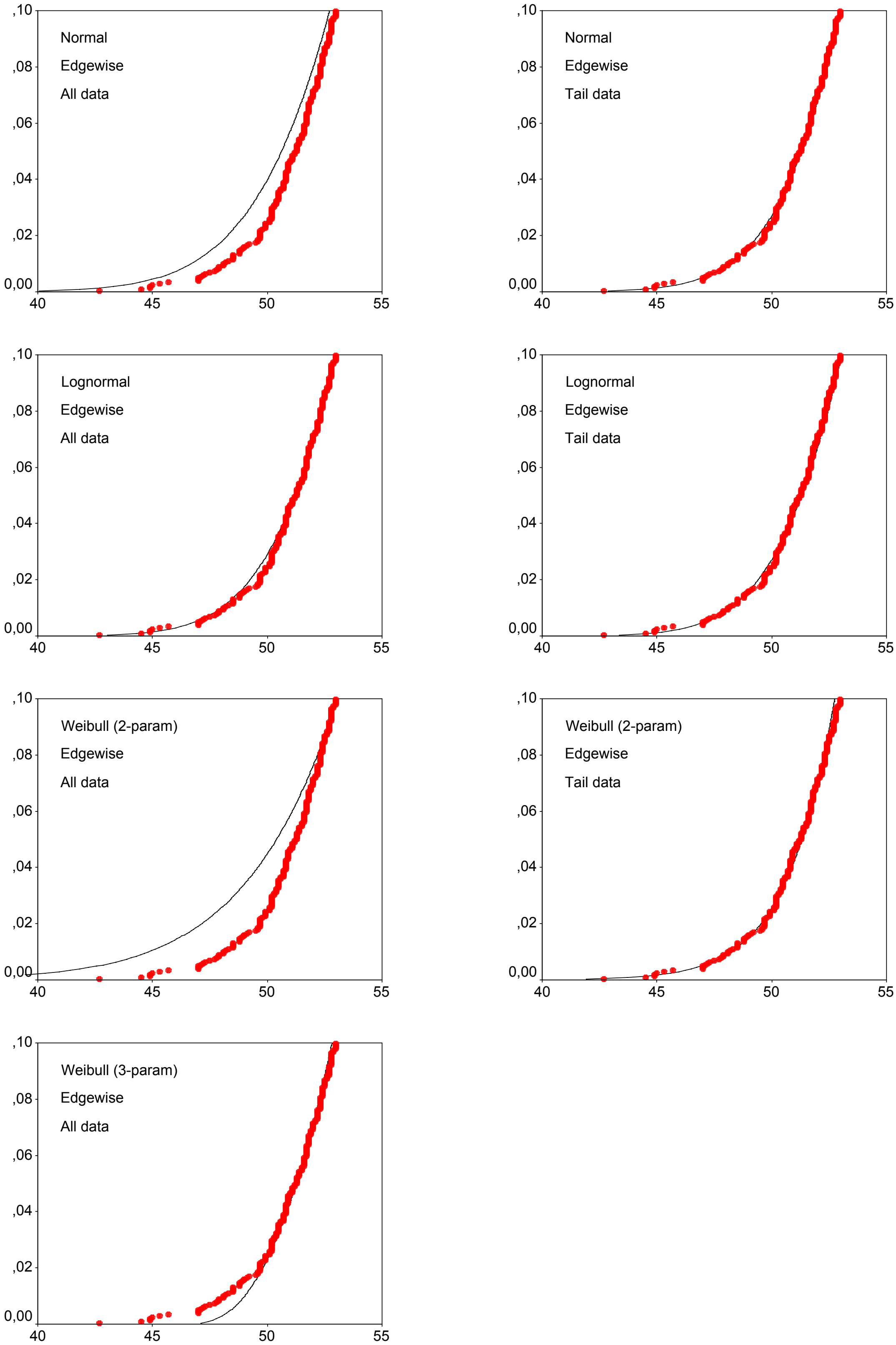


Figure 2.12. Cumulative distributions of edgewise bending strength for Kerto-LVL, series K-IQC.

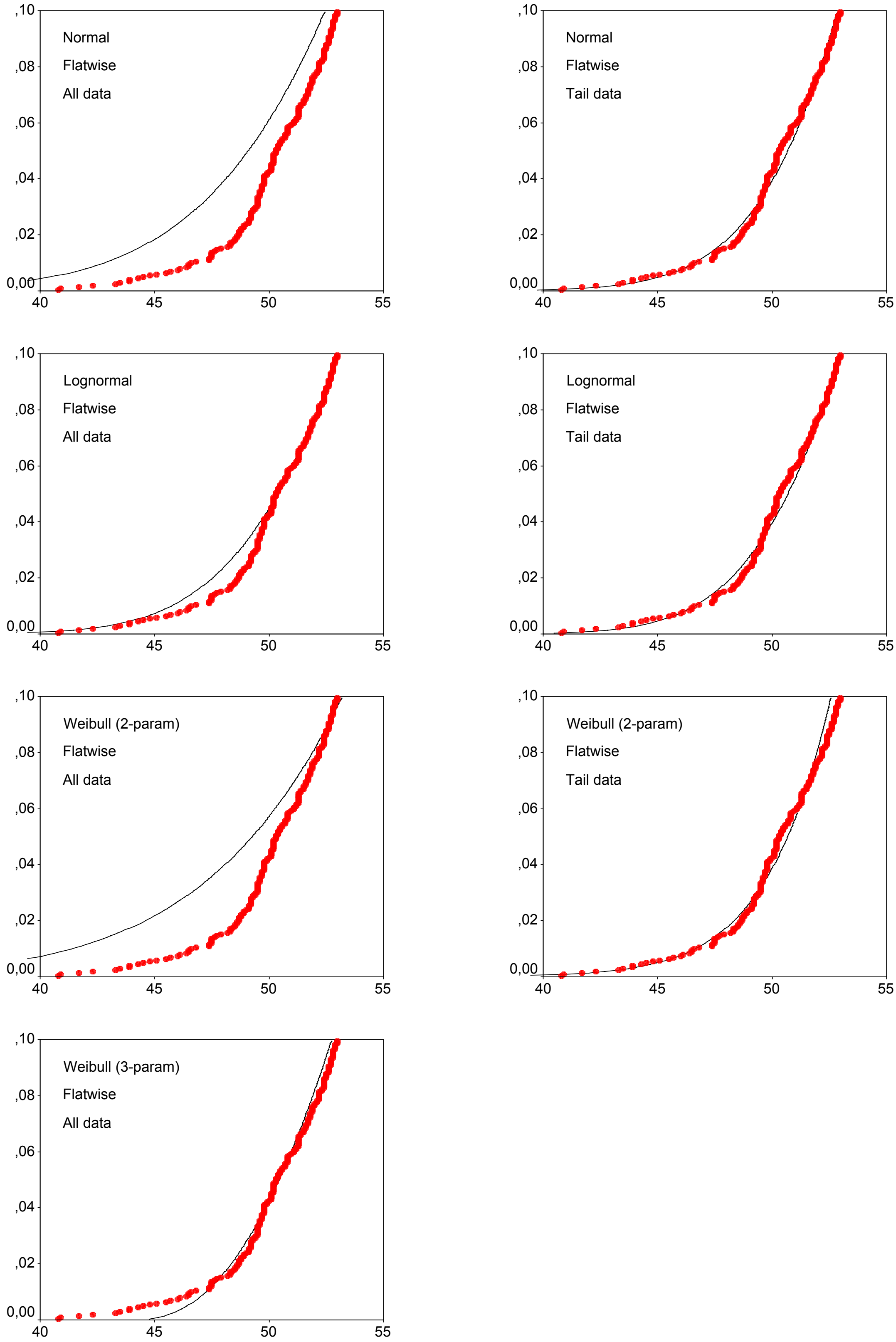


Figure 2.13. Cumulative distributions of flatwise bending strength for Kerto-LVL, series K-IQC.

2.4 Plywood

2.4.1 Data

The data used represent birch and conifer plywood. Birch plywood was produced from 1.4 mm thick birch (*Betula pendula*) veneers. Conifer plywood was produced from 1.4 as well as from 3.0 mm thick spruce (*Picea abies*) veneers. The birch plywood was sampled from twelve different mills in Finland while the conifer plywood was sampled from eight different mills in Finland. The thick plywood was sampled between 1995 and 1999 from one mill in Finland. The thickness of the plywood was between 9 and 30 mm.

Prior to testing, all plywood was conditioned in humidity rooms where the relative humidity was 65% and the temperature was 20 °C.

Flatwise bending tests were carried out according to the test method given in EN 789. In addition to bending strength and modulus of elasticity (true), density and moisture content were determined.

Before analysis, the bending strength, f_{pli}, of the load-bearing veneers was calculated from

$$f_{pli} = \frac{W}{W_{eff}} f \tag{2.17}$$

where W is the section modulus of the full cross-section, W_{eff} is the section modulus of the load-bearing veneers of the cross-section and f is the bending strength determined for the full cross-section.

Additionally, the modulus of elasticity, E_{pli}, of the load-bearing veneers was calculated from

$$E_{pli} = \frac{I}{I_{eff}} E \tag{2.18}$$

where I is the second moment of area of the full cross-section, I_{eff} is the second moment of area of the load-bearing veneers of the cross-section and E is the modulus of elasticity determined for the full cross-section.

Table 2.16. Moisture content, ω, density, ρ, bending strength, f, and modulus of elasticity, E, of plywood -veneers used in the analysis.

Species	Veneer thickness mm	Series		ω %	ρ kg/m^3	f_{pli} N/mm^2	E_{pli} N/mm^2	Data source
Birch	1.4	B-14	Mean	10.5	679	90.9	17900	VTT 2000
			COV	4.8	3.4	13.2	10.6	
			Number	108	108	224	224	
Spruce	1.4	S-14	Mean	11.8	523	56.5	13700	VTT 2000
			COV	4.2	4.4	19.8	16.9	
			Number	72	72	173	173	
Spruce	3.0	S-30	Mean	10.2	463	49.2	12500	VTT 1995
			COV	3.9	4.7	20.7	18.2	
			Number	180	180	281	281	

In Equations (2.17) and (2.18) the cross-veneers are assumed to be non-load-bearing veneers. This calculation method excludes the effect of different veneer lay-ups of plywood on the bending properties. It is to be noted that the veneer properties given are valid only for veneers as a part of plywood products. The coefficient of variation is considerably larger for single veneers.

A summary of the data used in the analysis is given in Table 2.16.

2.4.2 Analysis

The bending strengths for 1.4 mm thick birch and spruce as well as for 3.0 mm thick spruce plywood veneers were modelled using normal, lognormal and two- as well as three-parameter Weibull distributions. In addition to all of the data, the tail data, represented by 10% of the weakest specimens, were also analysed. The estimated parameters of these functions are given in Table 2.17. Furthermore, the model and data are plotted in Figures 2.14–2.16. The estimated bending strengths at different fractiles are given in Table 2.18.

The amount of data from each location and thickness is too small to give a representative sample for 1.4 mm thick birch and spruce plywood veneers. Hence, no conclusions should be drawn on the distribution functions.

The following conclusions can be drawn for estimation of the 5% fractile of the 281 spruce plywood veneers of 3.0 mm thickness, Figure 2.16:

- The lognormal and three-parameter Weibull distributions, modelled from all the data, result in good estimates for the 5% fractile of bending strength.

- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the 5% fractile of bending strength.

- The normal, lognormal and two-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of bending strength.

The following conclusions can be drawn for estimation of the tails of the models used, Figure 2.16:

- The lognormal distribution, modelled from all the data, results in good estimates for the bending strength values for the tail fractiles below 5%.

- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the bending strength values for the tail fractiles below 5%.

- The three-parameter Weibull distribution, modelled from all the data, overestimates the bending strength values for the tail fractiles below 5%.

- The normal, lognormal and two-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%.

Table 2.17. The modelled distribution functions for 1.4 mm thick birch and spruce as well as 3.0 mm thick spruce plywood veneers, series B-14, S-14 and S-30.

Distribution function (no. of cases)	Data	Parameters in the distribution function				
		μ	σ	α	β	ε
Birch plywood (224)						
Normal	All	90.92	12.02	-	-	-
Normal	Tail	91.70	11.66	-	-	-
Lognormal	All	4.501	0.133	-	-	-
Lognormal	Tail	4.560	0.169	-	-	-
Two-parameter Weibull	All	-	-	9.804	95.58	-
Two-parameter Weibull	Tail	-	-	13.961	89.84	-
Three-parameter Weibull	All	-	-	2.843	94.77	59.49
Three-parameter Weibull	Tail	-	-	-	-	-
Spruce plywood (173)						
Normal	All	56.48	10.71	-	-	-
Normal	Tail	52.29	7.07	-	-	-
Lognormal	All	4.016	0.192	-	-	-
Lognormal	Tail	4.015	0.189	-	-	-
Two-parameter Weibull	All	-	-	6.849	60.34	-
Two-parameter Weibull	Tail	-	-	13.661	50.55	-
Three-parameter Weibull	All	-	-	2.616	59.75	30.50
Three-parameter Weibull	Tail	-	-	-	-	-
Spruce plywood of thick veneers (281)						
Normal	All	49.24	10.19	-	-	-
Normal	Tail	46.06	7.28	-	-	-
Lognormal	All	3.875	0.207	-	-	-
Lognormal	Tail	3.908	0.232	-	-	-
Two-parameter Weibull	All	-	-	6.329	52.83	-
Two-parameter Weibull	Tail	-	-	10.611	45.11	-
Three-parameter Weibull	All	-	-	2.287	52.07	27.30
Three-parameter Weibull	Tail	-	-	-	-	-

Table 2.18. The modelled bending strengths for 1.4 mm thick birch and spruce as well as 3.0 mm thick spruce plywood veneers, series B-14, S-14 and S-30.

Distribution function (no. of cases)	Data	Bending strength for different fractiles				
		0.001	0.005	0.010	0.050	0.100
Birch plywood (224)						
Normal	All	53.8	60.0	63.0	71.1	75.5
Normal	Tail	55.7	61.7	64.6	72.5	76.8
Lognormal	All	59.7	64.0	66.1	72.4	76.0
Lognormal	Tail	56.7	61.8	64.5	72.4	77.0
Two-parameter Weibull	All	47.2	55.7	59.8	70.6	76.0
Two-parameter Weibull	Tail	54.8	61.5	64.7	72.7	76.6
Three-parameter Weibull	All	62.6	65.0	66.5	71.9	75.5
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	-	60.3	61.4	73.1	75.8
Spruce plywood (173)						
Normal	All	23.4	28.9	31.6	38.9	42.8
Normal	Tail	30.4	34.1	35.8	40.7	43.2
Lognormal	All	30.7	33.8	35.5	40.5	43.4
Lognormal	Tail	30.9	34.1	35.7	40.6	43.5
Two-parameter Weibull	All	22.0	27.8	30.8	39.1	43.4
Two-parameter Weibull	Tail	30.5	34.3	36.1	40.7	42.9
Three-parameter Weibull	All	32.6	34.4	35.5	39.9	42.9
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	-	35.2	35.8	38.9	43.9
Spruce plywood of thick veneers (281)						
Normal	All	17.8	23.0	25.5	32.5	36.2
Normal	Tail	23.6	27.3	29.1	34.1	36.7
Lognormal	All	25.4	28.3	29.8	34.3	37.0
Lognormal	Tail	24.3	27.4	29.0	34.0	37.0
Two-parameter Weibull	All	17.7	22.9	25.5	33.0	37.0
Two-parameter Weibull	Tail	23.5	27.4	29.2	34.1	36.5
Three-parameter Weibull	All	28.5	29.7	30.6	34.1	36.6
Three-parameter Weibull	Tail	-	-	-	-	-
Non-parametric	All	-	27.4	29.1	33.6	37.3

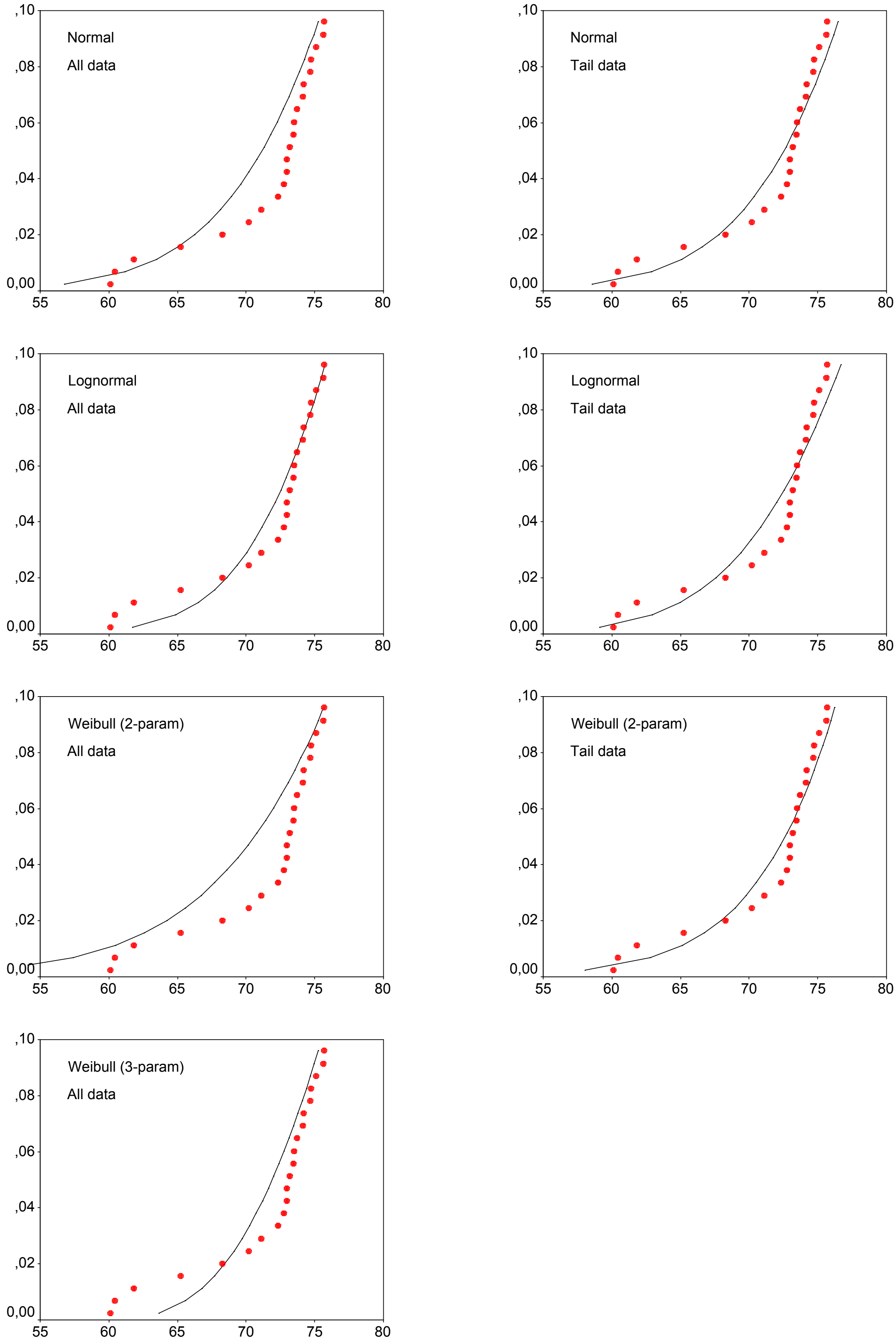


Figure 2.14. Cumulative distributions of bending strength for 1.4 mm thick birch plywood veneers, series B-14.

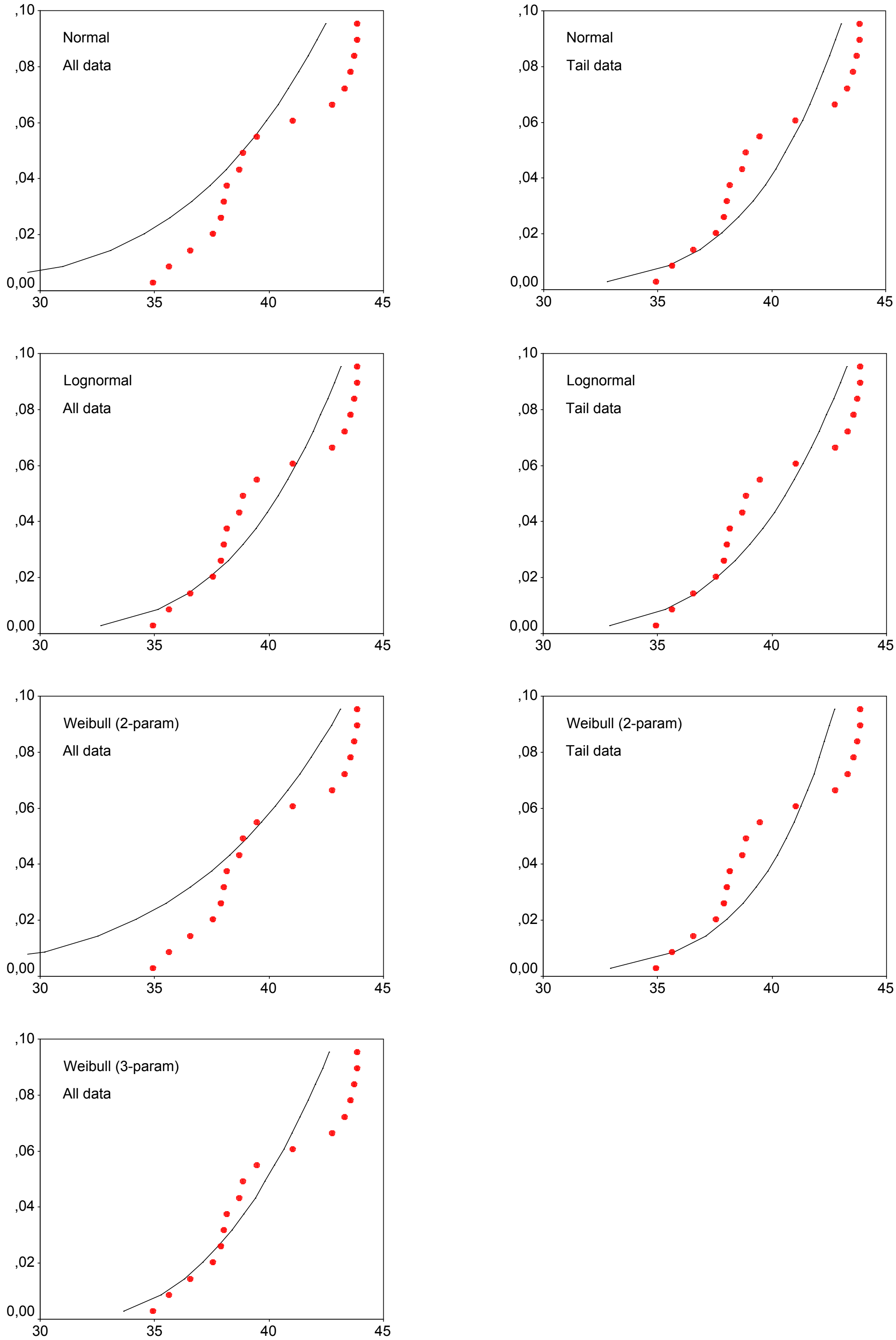


Figure 2.15. Cumulative distributions of bending strength for 1.4 mm thick spruce plywood veneers, series S-14.

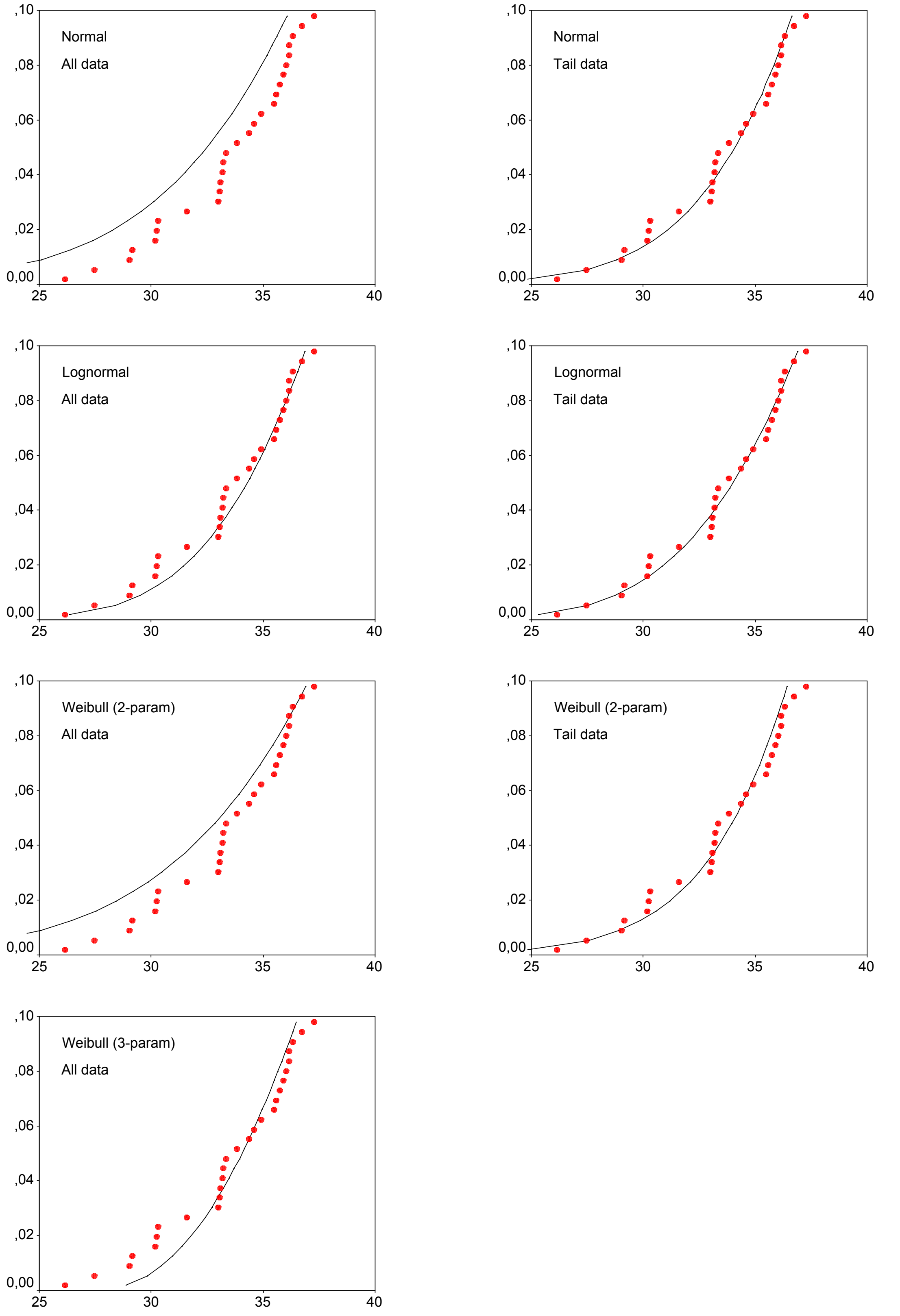


Figure 2.16. Cumulative distributions of bending strength for 3.0 mm thick spruce plywood veneers, series S-30.

2.5 Small-diameter round timber

2.5.1 Data

The data used represents small-diameter round timber of spruce (*Picea abies* and *Picea sitchensis*) and pine (*Pinus silvestris*). Spruce was sampled from two locations in Finland, two locations in Austria and one location in the United Kingdom. Pine was sampled from four locations in Finland and one location in the United Kingdom.

Both bending and compression parallel to the grain tests were carried out following, as closely as possible, the test method given in EN 408. In addition to bending strength and modulus of elasticity (true), density and moisture content were determined. Further details on the data, test method and results are given by Ranta-Maunus (1999).

Before analysis, all individual density values were adjusted to a moisture content of 12% according to EN 384. No other adjustments were carried out.

A summary of the data used in the analysis is given in Table 2.19. The mean diameter of the specimens was 123 mm. The mean moisture content of the specimens was 16.1%. This results in lower strength and modulus of elasticity values than those of specimens of 12% moisture content.

Table 2.19. Density, ρ, bending strength, f, and modulus of elasticity, E, of small-diameter round timber used in the analysis.

Property	Number	ρ		f		E	
		Mean	COV	Mean	COV	Mean	COV
		kg/m^3	%	N/mm^2	%	N/mm^2	%
Bending	660	467	12.7	56.2	21.3	12300	26.4
Compression	575	469	13.2	26.9	23.3	10700	28.3

2.5.2 Analysis

Both bending and compression strength was modelled using normal, lognormal and two- as well as three-parameter Weibull distribution functions. In addition to all of the data, the tail data, represented by 10% of the weakest specimens, were also analysed. The estimated parameters of these functions are given in Table 2.20. Furthermore, the model and the data are plotted in Figures 2.17–2.18. The estimated strengths at different fractiles are given in Table 2.21.

In estimating the 5% fractile of the 660 bending specimens, Figure 2.17, the same conclusions as those drawn for spruce with a depth of 150 mm can be drawn. The conclusions were:

- The lognormal distribution, modelled from all the data, overestimates the 5% fractile of bending strength.
- The normal and two- as well as three-parameter Weibull distributions, modelled from all the data, result in good estimates for the 5% fractile of bending strength.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of bending strength.

The following conclusions can be drawn for estimation of the tails of the models used, Figure 2.17:

- The lognormal distribution, modelled from all the data, overestimates the bending strength values for the tail fractiles below 5%.
- The normal and two- as well as three-parameter Weibull distributions, modelled from all the data, result in reasonable estimates for the bending strength values for the tail fractiles below 5%.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%.

The following conclusions can be drawn for estimation of the 5% fractile of the 575 compression specimens, Figure 2.18:

- The lognormal and three-parameter Weibull distributions, modelled from all the data result in good estimates for the 5% fractile of compression strength.
- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the 5% fractile of compression strength.
- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest specimens, result in good estimates for the 5% fractile of compression strength.

The following conclusions can be drawn for estimation of the tails of the models used, Figure 2.18:

- The lognormal and three-parameter Weibull distributions, modelled from all the data, result in reasonable estimates for the compression strength values for the tail fractiles below 5%.

- The normal and two-parameter Weibull distributions, modelled from all the data, underestimate the compression strength values for the tail fractiles below 5%.

- The normal, lognormal and two- as well as three-parameter Weibull distributions, modelled from 10% of the weakest data, result in good estimates for the bending strength values for the tail fractiles below 5%.

Table 2.20. The modelled distribution functions for small-diameter round timber.

Distribution function (no. of cases)	Data	Parameters in the distribution function				
		μ	σ	α	β	ε
Bending strength (660)						
Normal	All	56.25	12.00	-	-	-
Normal	Tail	54.89	10.92	-	-	-
Lognormal	All	4.006	0.223	-	-	-
Lognormal	Tail	4.144	0.328	-	-	-
Two-parameter Weibull	All	-	-	5.780	60.70	-
Two-parameter Weibull	Tail	-	-	7.194	55.76	-
Three-parameter Weibull	All	-	-	3.077	60.24	22.57
Three-parameter Weibull	Tail	-	-	5.557	57.91	7.17
Compression strength (575)						
Normal	All	26.88	6.26	-	-	-
Normal	Tail	22.64	2.95	-	-	-
Lognormal	All	3.265	0.232	-	-	-
Lognormal	Tail	3.170	0.178	-	-	-
Two-parameter Weibull	All	-	-	5.682	28.96	-
Two-parameter Weibull	Tail	-	-	14.085	21.96	-
Three-parameter Weibull	All	-	-	2.136	28.52	14.20
Three-parameter Weibull	Tail	-	-	2.808	26.39	13.12

Table 2.21. The modelled bending strengths for small-diameter round timber.

Distribution function (no. of cases)	Data	Bending strength for different fractiles				
		0.001	0.005	0.010	0.050	0.100
Bending strength (660)						
Normal	All	19.2	25.3	28.3	36.5	40.9
Normal	Tail	21.1	26.8	29.5	36.9	40.9
Lognormal	All	27.6	30.9	32.7	38.1	41.3
Lognormal	Tail	22.9	27.1	29.4	36.8	41.4
Two-parameter Weibull	All	18.4	24.3	27.4	36.3	41.1
Two-parameter Weibull	Tail	21.3	26.7	29.4	36.9	40.8
Three-parameter Weibull	All	26.6	29.3	31.0	36.9	40.7
Three-parameter Weibull	Tail	21.8	26.7	29.3	36.9	41.0
Non-parametric	All	23.0	24.3	29.9	36.6	41.2
Compression strength (575)						
Normal	All	7.5	10.8	12.3	16.6	18.9
Normal	Tail	13.5	15.0	15.8	17.8	18.9
Lognormal	All	12.8	14.4	15.3	17.9	19.4
Lognormal	Tail	13.7	15.1	15.7	17.8	19.0
Two-parameter Weibull	All	8.6	11.4	12.9	17.2	19.5
Two-parameter Weibull	Tail	13.4	15.1	15.8	17.8	18.7
Three-parameter Weibull	All	14.8	15.4	15.9	17.8	19.2
Three-parameter Weibull	Tail	14.3	15.1	15.7	17.7	19.1
Non-parametric	All	14.3	14.9	15.8	17.8	19.2

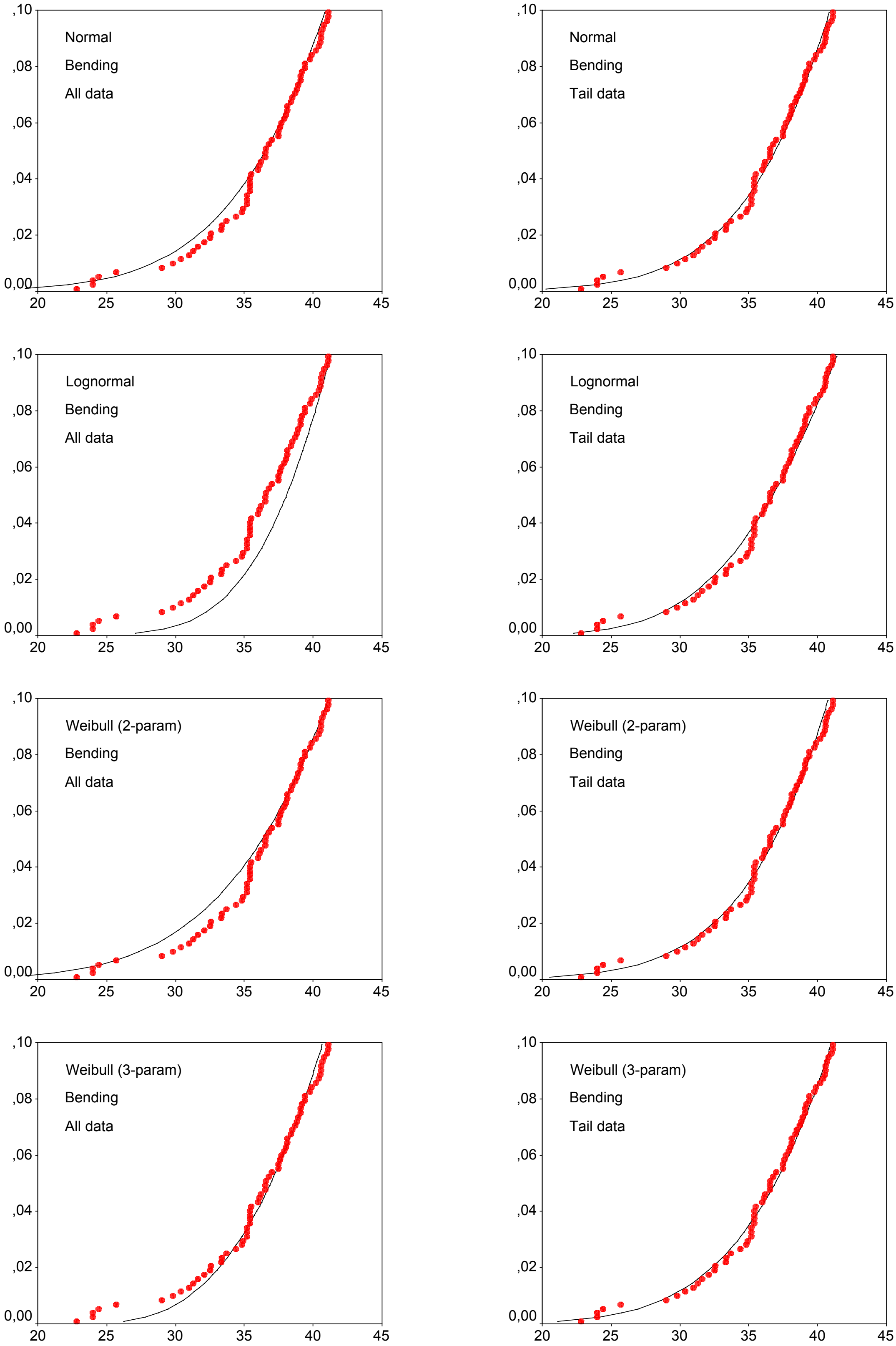


Figure 2.17. Cumulative distributions of bending strength for small-diameter round timber.

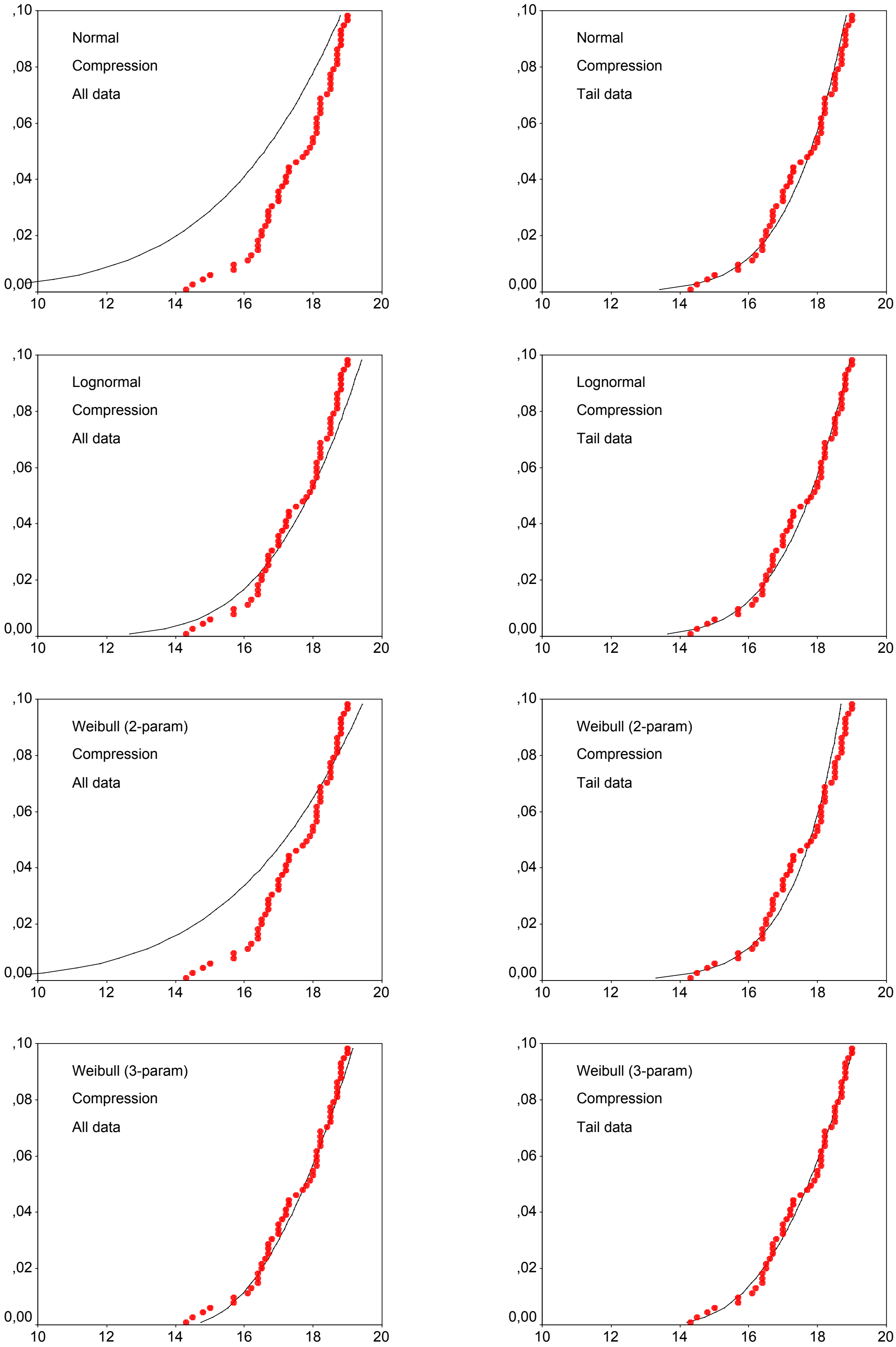


Figure 2.18. Cumulative distributions of compression strength for small-diameter round timber.

2.6 Summary

Strength data on sawn timber, Kerto-laminated veneer lumber, plywood and small-diameter round timber were analysed. A summary of the strength results is given in Table 2.22.

Strength was modelled using normal, lognormal and two- as well as three-parameter Weibull distribution functions. In addition to all of the data, the tail data, represented by 10% of the weakest specimens, were also analysed. The model was judged using two criteria:

- How well the 5% fractile is estimated.
- How well the values below the 5% fractile are estimated.

Any of the models based on the tail data gave good estimates for both the 5% fractile and the values below that. A summary of the evaluation for the models based on all the data is given in Tables 2.23–2.24.

Fitted distribution functions can be characterized by giving mean value and COV instead of the usual parameters. Appendix A gives the equations needed to calculate mean value (m) and standard deviation (σ) from the distribution parameters, and

$$COV = \frac{\sigma}{m}$$

Table 2.25 summarizes COV values calculated from the distributions fitted to all data and to the tail. Quality of the fitting of those functions fitted to all data in tail area is characterized by a few words and COV´s of those functions are close to the COV obtained from the test. The tail-fitted functions give a different COV, when the all data-curve does not fit well to the tail data.

Table 2.22. Summary of strength results.

Material	Remarks	Mean N/mm^2	COV %	5%-frac N/mm^2	Number -
Sawn timber					
Spruce, edgewise,	Ungraded	45.2	25	27.5	589
depth = 150 mm	Visually graded, C24	49.4	20	33.1	367
	Machine graded, C30	47.5	22	30.5	496
Sawn timber	Ungraded	42.9	29	22.6	1995
Spruce and pine,	Visually graded, C24	48.4	22	32.1	902
edgewise	Machine graded, C30	47.8	22	30.6	1327
Laminated veneer lumber					
Kerto, edgewise	External quality control	58.6	10	49.9	372
	Internal quality control	60.1	10	51.3	1968
Kerto, flatwise	External quality control	60.7	13	48.1	372
	Internal quality control	64.3	14	50.3	1963
Plywood					
1.4 mm thick birch plies, flatwise		90.9	13	73.1	224
1.4 mm thick spruce plies, flatwise		56.5	20	38.9	173
3.0 mm thick spruce plies, flatwise		49.2	21	33.6	281
Small-diameter round timber					
Spruce and pine	Bending	56.2	21	36.6	660
Spruce and pine	Compression	26.9	23	17.8	575

Table 2.23. Evaluation of model used for estimation of 5% fractile. Optim means optimistic (model overestimates strength) while Pess means pessimistic(model underestimates strength).

Material	Remarks	Normal %	Lognorm. %	We-2 %	We-3 %
Sawn timber					
	Ungraded	-	-	-	-
	Visually graded, C24	Pess/OK	Optim	Pess/OK	OK
	Machine graded, C30	OK	Optim	OK	OK
Laminated veneer lumber					
Kerto, edgewise	Internal quality control	Pess	OK	Pess	OK
Kerto, flatwise	Internal quality control	Pess	OK	Pess	OK
Plywood					
1.4 mm thick birch plies, flatwise		-	-	-	-
1.4 mm thick spruce plies, flatwise		-	-	-	-
3.0 mm thick spruce plies, flatwise		Pess	OK	Pess	OK
Small-diameter round timber					
Spruce and pine	Bending	OK	Optim	OK	OK
Spruce and pine	Compression	Pess	OK	Pess	OK

Table 2.24. Evaluation of model used for estimation of tail fractiles below 5%. Optim means optimistic (model overestimates strength) while Pess means pessimistic (model underestimates strength).

Material	Remarks	Normal %	Lognorm. %	We-2 %	We-3 %
Sawn timber					
	Ungraded	-	-	-	-
	Visually graded, C24	Pess/OK	Optim	Pess	OK
	Machine graded, C30	Pess/OK	Optim	Pess	OK
Laminated veneer lumber					
Kerto, edgewise	Internal quality control	Pess	OK	Pess	Optim
Kerto, flatwise	Internal quality control	Pess	OK	Pess	Optim
Plywood					
1.4 mm thick birch plies, flatwise		-	-	-	-
1.4 mm thick spruce plies, flatwise		-	-	-	-
3.0 mm thick spruce plies, flatwise		Pess	OK	Pess	Optim
Small-diameter round timber					
Spruce and pine	Bending	OK	Optim	OK	OK
Spruce and pine	Compression	Pess	OK	Pess	OK

Table 2.25. Summary of COV values calculated from fitted distribution functions. Sawn timber is machine-graded.

Material	COV in test	COV of models fitted to all	Fitting quality in tail area	COV of models fitted to tail	See
Spruce 150 mm	21.6 %	N: 22 LN: 23 W2: 20 W3: 22	pessimistic optimistic pessimistic a bit optimistic	N: 18 LN: 29 W2 14 W3: 18	Table 2.3
Spruce all	21.0	N: 21 LN: 22 W2: 20 W3: 21	a bit pessimistic very optimistic pessimistic a bit optimistic	N: 19 LN: 31 W2: 15 W3: 23	Table 2.7
Spruce & pine	22.3	N: 22 LN: 24 W2: 21 W3: 22	a bit pessimistic very optimistic pessimistic a bit optimistic	N: 20 LN: 35 W2: 17 W3: 20	Table 2.10
LVL edgewise	9.6	N: 10 LN: 10 W2: 9 W3: 10	a bit pessimistic good very pessimistic very optimistic	N: 8 LN: 9 W2: 5 W3: -	Table 2.13
LVL flatwise	14.4	N: 14 LN: 14 W2: 13 W3: 14	very pessimistic a bit pessimistic very pessimistic very optimistic	N: 10 LN: 12 W2: 6 W3: -	Table 2.13
Plywood birch	13.2	N: 13 LN: 13 W2: 12 W3: 13		N: 13 LN: 17 W2: 9 W3: -	Table 2.17
Plywood spruce 1.4 mm	19.8	N: 19 LN: 19 W2: 17 W3: 19		N: 14 LN: 19 W2: 9 W3: -	Table 2.17
Plywood 3 mm	20.7	N: 21 LN: 21 W2: 18 W3: 21		N: 16 LN: 23 W2: 11 W3:	Table 2.17
Small-diameter round timber, bending	21.3	N: 21 LN: 23 W2: 20 W3: 21	a bit pessimistic very optimistic a bit pessimistic very optimistic	N: 20 LN: 34 W2: 16 W3: 18	Table 2.20
Small-diameter round timber, compression	23.3	N: 23 LN: 24 W2: 20 W3: 23	very pessimistic a bit pessimistic very pessimistic a bit optimistic	N: 13 LN: 18 W2: 9 W3: 18	Table 2.20

3. Determination of characteristic 5% fractile values

3.1 Standardised methods

3.1.1 ISO 12491

ISO 12491 is a material-independent standard in which general principles for the application of statistical methods to be used in quality control are given. Whether or not the estimation of fractiles is a part of quality control is a matter of contention. However, the classical approach to the estimation of fractiles is described.

Guidance for determination of different fractiles using different confidence levels for a normal distribution function is given in Section 6 of ISO 12491. In addition to a recommended simple technique, a technique based on the Bayesian approach is given.

Using both the simple technique and the technique based on the Bayesian approach the characteristic strength value, f_k, defined as the 5% fractile value of the strength property, is given by

$$f_k = f_{mean} - k_n f_{stdev} \tag{3.1}$$

where f_{mean} is the mean value and f_{stdev} is the standard deviation. k_n depends on the number of tests, the confidence level used and whether the coefficient of variation is known or unknown before testing. Values for k_n using a confidence level of 75% are given in Table 3.1.

Table 3.1. Values for k_n to be used in Equation (3.1), ISO 12491.

Number of tests	3	4	6	8	10	20	30	50	100	∞
f_{stdev} is unknown										
Simple technique	3.15	2.68	2.34	2.19	2.10	1.93	1.87	1.81	1.76	1.64
Bayesian approach	-	2.63	2.18	2.00	1.92	1.77	1.73	-	-	1.64

No guidance is given for a lognormal distribution function. However, the guidance given for a normal distribution function can be used by replacing the variable f_i by $\ln f_i$. Hence, the mean value, f_{mean}, and standard deviation, f_{stdev}, in Equation (3.1) shall be replaced by $(\ln f)_{mean}$ and $(\ln f)_{stdev}$. The characteristic strength value is then given by

$$f_k = e^{(\ln f)_{mean} - k_n (\ln f)_{stdev}} \tag{3.2}$$

3.1.2 Eurocode 1

According to Section 5 of Part 1 of Eurocode 1, the characteristic strength value is defined as the 5% fractile value. However, the confidence level is not given. Guidance for determination of characteristic values for a normal distribution function is given in Section 3.2 of Appendix D of Part 1 of Eurocode 1. It is stated that this guidance leads to almost the same result as classical statistics with confidence levels equal to 0.75.

The characteristic strength value, f_k, defined as the 5% fractile value of the strength property, is given by Equation (3.1). The values for k_n to be used are given as the Bayesian values in Table 3.1.

3.1.3 Eurocode 5

As in Eurocode 1, the characteristic strength value is defined as the 5% fractile value in Section 3.1 of Part 1-1 of Eurocode 5. Guidance for determination of characteristic values using a confidence level of 84.1% is given in Appendix A of Part 1-1 of Eurocode 5 (1993). This method should not be used in cases covered by other European standards or in cases where the assumption of a lognormal distribution function is not appropriate. Furthermore, this method will probably be replaced by the method given in EN TC 124.bbb.

The characteristic strength value, f_k, defined as the 5% fractile value of the strength property, is given by

$$f_k = k_n f_{mean} \tag{3.3}$$

where f_{mean} is the mean value and k_n is given by

$$k_n = e^{\left(0.15 - \left(2.645 + \frac{1}{\sqrt{n}}\right) f_{\text{cov}}\right)} \tag{3.4}$$

where f_{cov} is the coefficient of variation and n is the sample size. The value of f_{cov} shall not be taken as less than 0.10. The sample size shall not be less than 30.

3.1.4 EN 1058

Guidance for determination of characteristic values for wood-based panel products for structural purposes is given in EN 1058. However, this method will probably be replaced by the method given in EN TC 124.bbb.

The characteristic strength value, f_k, defined as the 5% fractile value of the strength property, is given by Equation (3.3). The value for k_n to be used is given by

$$k_n = e^{0.15 - k f_{\mathrm{cov}}} \tag{3.5}$$

where f_{cov} is the coefficient of variation. The value of f_{cov} shall not be taken as less than 0.10. k depends on the number of tests. Values for k are given in Table 3.2. The sample size shall not be less than 32.

Table 3.2. Values for k to be used in Equation (3.5), EN 1058.

Number of tests	32	36	40	60	80	100	∞
k	3.10	3.07	3.04	2.95	2.91	2.88	2.65

3.1.5 EN TC 124.bbb

The forthcoming standard EN TC 124.bbb is based on Annex A of Eurocode 5 and EN 1058. A more general method is given and some errors are corrected. The characteristic strength value is defined as the 5% fractile value using a confidence level of 84.1% and a lognormal distribution function.

The characteristic strength value, f_k, defined as the 5% fractile value of the strength property, is given by Equation (3.2). The values for k_n to be used are given in Table 3.3.

Table 3.3. Values for k_n to be used in Equation (3.2), EN TC 124.bbb.

Number of tests	3	5	10	15	20	30	50	100	∞
k_n	4.11	2.91	2.34	2.16	2.07	1.98	1.89	1.81	1.65

3.1.6 EN 384

Guidance for determination of characteristic values for visually and machine strength graded timber is given in EN 384.

One to five samples shall be selected from the timber population. The sample size shall not be less than 40. From each sample, the 5% fractile, f_{05}, shall be determined by ranking all the test values in ascending order. Since the 5% fractile is defined as the test value for which 5% of the values are lower, the confidence level used is 50%.

The characteristic strength value, f_k, defined as the 5% fractile value of the strength property, is given by

$$f_k = \min \begin{cases} (f_{05})_{mean} k_s k_v \\ 1.2 (f_{05})_{\min} k_s k_v \end{cases} \tag{3.6}$$

where $(f_{05})_{mean}$ is the mean value of the 5% fractile values for each sample, weighted according to the number of specimens in each sample. $(f_{05})_{min}$ is the minimum value of the 5% fractile values of the samples. k_s is a factor to adjust for the number and size of the samples. k_s is equal to 1.0 for five samples. k_v is a factor to be taken as 1.00 for visually graded timber and 1.12 for machine-graded timber.

3.1.7 ASTM D 2915

Guidance for determination of different fractiles using different confidence levels for a normal and lognormal distribution function as well as for a non-parametric distribution is given in Section 4 and Appendix X5 of ASTM D 2915.

The characteristic strength value f_k, defined as the 5% fractile value of the strength property, is given by Equations (3.1) and (3.2) for a normal and a lognormal distribution function, respectively. The values for k_n resulting in a 75% confidence level are given as the simple technique values in Table 3.1.

Order statistics can be used for a non-parametric distribution. The strength values shall be ranked in ascending order and the characteristic value, defined as the 5% fractile value of the strength property, is directly given by the *m*:th strength value. *m* depends on the number of tests and the confidence level used. The values for *m* resulting in a 75% confidence level are given in Table 3.4.

Table 3.4. Values for m to be used in order statistics, ASTM D 2915.

Number of tests	28	53	78	102	125	148	193	237	668	1089
m	1	2	3	4	5	6	8	10	30	50

3.2 Case studies

3.2.1 Sawn timber

As a first case study, the characteristic bending strength, defined as the 5% fractile, for sawn timber is analysed. Sawn timber is represented by the 986 spruce specimens machine-graded to the European strength class C30. This population is presented in detail in Section 2.2.3. The mean value is 47.8 N/mm^2 while the coefficient of variation is 21.0%. Furthermore, the 5% fractile value based on a non-parametric analysis is 31.3 N/mm^2. The 5% fractile is well estimated using a normal distribution function for all the data but overestimated by 4% using a lognormal distribution function for the same data.

The characteristic value determined in accordance with EN 384 is given in Table 3.5. For analysis the population was split into five samples according to the depth of the specimens.

A summary of the characteristic values determined in accordance with some of the standardised methods is given in Table 3.6.

As expected the ISO 12491 method based on a normal distribution function, as well as the ASTM D 2915 method based on order statistics, result in characteristic values equal to the value given in Section 2. Due to the large amount of specimens the effect of different confidence levels can be neglected.

Furthermore, the method to be used in Europe, namely EN 384, overestimates the characteristic value by 11%. However, this is no surprise since the k_v factor which is 1.12 for machine-graded timber is not based on statistical facts. It is incorrect to multiply an estimate of the 5% fractile value by 1.12 and then state that the result is the characteristic value defined as the 5% fractile value.

Table 3.5. Determination of characteristic bending strength of 986 specimens of sawn timber in accordance with EN 384.

Sample	Depth h mm	f_{05} N/mm^2	Number -
Sample 1	h < 110	31.6	166
Sample 2	110 ≤ h < 130	33.8	96
Sample 3	130 ≤ h < 170	30.9	537
Sample 4	170 ≤ h < 190	29.2	130
Sample 5	190 ≤ h	28.1	57
$(f_{05})_{mean}$	all	30.9	986
$1.2 \cdot (f_{05})_{min}$	all	33.7	986
f_k	all	34.6	986

Table 3.6. Characteristic bending strength of 986 specimens of sawn timber in accordance with some standardised methods.

Method	f_k N/mm^2	f_k / 31.3 %
ISO 12491 - Normal - Simple*	30.9	99
EN 384	34.6	111
ASTM D 2915 - Non-parametric	30.8	98
Non-parametric - Section 2	31.3	100

* This method is equal to ASTM D 2915 - Normal

3.2.2 Kerto-laminated veneer lumber

As a second case study the characteristic bending strength, defined as the 5% fractile, for Kerto-laminated veneer lumber is analysed. Kerto-laminated veneer lumber is represented by the 1968 edgewise bent specimens. This population is presented in detail in Section 2.3. The mean value is 60.1 N/mm^2 while the coefficient of variation is 9.6%. Furthermore, the 5% fractile value based on a non-parametric analysis is 51.3 N/mm^2. The 5% fractile is well estimated using a lognormal distribution function for all the data but underestimated by 1% using a normal distribution function for the same data.

A summary of the characteristic values determined in accordance with some of the standardised methods is given in Table 3.7.

As expected, the ISO 12491 method based on a lognormal distribution function results in a characteristic value equal to the value given in Section 2. Furthermore, the method based on Eurocode 5 overestimates the characteristic value by 4%. Due to the large amount of specimens the effect of different confidence levels can be neglected.

In real life, it is unrealistic to assess a characteristic value from a sample size of many hundreds of tested specimens. In Europe, a more realistic sample size is between 30 and 60. To analyse this situation, 30 specimens were randomly selected from all the 1968 specimens. The mean value, f_{mean}, for these 30 specimens was 60.3 N/mm^2 while the coefficient of variation, f_{cov}, was 11.7%. Furthermore, the mean value, $\ln(f)_{mean}$, was 4.09 and the standard deviation, $\ln(f)_{stdev}$, was 0.113. A summary of the characteristic values is given in Table 3.8.

According to Section 2.3, the strength of laminated veneer lumber is lognormally distributed. Hence, it is reasonable to use the strength 48.4 N/mm^2 given by the ISO 12491 method for a lognormal distribution function as a reference value. Compared to this method the ISO 12491 method for a normal distribution function underestimates the characteristic value by 3%. Furthermore, the method given in Eurocode 5 overestimates the value by 4%. Usually a non-parametric method results in conservative values but in this case the value given by ASTM D 2915 overestimates the characteristic strength by 7%. This overestimation may not be representative of the method.

A comparison to the strength 47.1 N/mm^2 given by the ISO 12491 method for a normal distribution function is also included in Table 3.8.

Table 3.7. Characteristic bending strength of 1968 specimens of Kerto-laminated veneer lumber in accordance with some standardised methods.

Method	f_k N/mm^2	f_k / 51.3 %
ISO 12491 - Normal - Simple*	50.5	98
ISO 12491 - Lognormal - Simple**	51.0	99
Eurocode 5	53.5	104
Non-parametric - Section 2	51.3	100

* This method is equal to ASTM D 2915 - Normal

** This method is equal to ASTM D 2915 - Lognormal

Table 3.8. Characteristic bending strength of 30 specimens of Kerto-laminated veneer lumber in accordance with some standardised methods.

Method	f_k N/mm^2	f_k / 48.4 %	f_k / 47.1 %
ISO 12491 - Normal - Simple*	47.1	97	100
ISO 12491 - Normal - Bayesian**	48.1	99	102
ISO 12491 - Lognormal - Simple***	48.4	100	103
ISO 12491 - Lognormal - Bayesian	49.1	101	104
Eurocode 5	50.3	104	107
EN 1058	48.6	100	103
EN TC 124.bbb	47.8	99	101
ASTM D 2915 - Non-parametric	51.6	107	110

* This method is equal to ASTM D 2915 - Normal
** This method is equal to Eurocode 1
*** This method is equal to ASTM D 2915 - Lognormal

3.2.3 Plywood

As a third case study, the characteristic bending strength, defined as the 5% fractile, for plywood produced from 3.0 mm thick spruce veneers is analysed. This plywood is represented by the veneers of the 281 flatwise bent specimens. This population is presented in detail in Section 2.4. The mean value of the veneers is 49.2 N/mm^2 while the coefficient of variation is 20.7%. Furthermore, the 5% fractile value based on a non-parametric analysis is 33.6 N/mm^2. The 5% fractile is overestimated by 2% using a lognormal distribution function for all the data but underestimated by 3% using a normal distribution function for the same data.

A summary of the characteristic values determined in accordance with some of the standardised methods is given in Table 3.9.

The ISO 12491 method based on a lognormal distribution function and the Eurocode 5 method result in characteristic values close to the value given in Section 2. However the ISO 12491 method based on a normal distribution function underestimates the characteristic value by 4%.

As a special case in this third case study, 100 specimens were randomly selected from all the 281 specimens. The mean value, f_{mean}, for these 100 specimens was 49.2 N/mm^2 while the coefficient of variation, f_{cov}, was 20.7%. Furthermore, the mean value, $\ln(f)_{mean}$, was 3.85 and the standard deviation, $\ln(f)_{stdev}$, was 0.211. A summary of the characteristic values is given in Table 3.10.

According to Section 2.4 the strength of plywood is lognormally distributed. Hence, it is reasonable to use the strength 32.4 N/mm^2 given by the ISO 12491 method for a lognormal distribution function as a reference value. Compared to this method all other methods except the EN TC 124.bbb method underestimate the characteristic value by 5 to 8%.

A comparison with the strength 29.7 N/mm^2 given by the ISO 12491 method for a normal distribution function is also included in Table 3.10.

Table 3.9. Characteristic bending strength of 281 specimens of spruce plywood in accordance with some standardised methods.

Method	f_k N/mm^2	f_k / 33.2 %
ISO 12491 - Normal - Simple*	31.8	96
ISO 12491 - Lognormal - Simple**	34.0	102
Eurocode 5	32.7	98
Non-parametric - Section 2	33.2	100

* This method is equal to ASTM D 2915 - Normal
** This method is equal to ASTM D 2915 - Lognormal

Table 3.10. Characteristic bending strength of 100 specimens of spruce plywood in accordance with some standardised methods.

Method	f_k N/mm^2	f_k / 32.4 %	f_k / 29.7 %
ISO 12491 - Normal - Simple*	29.7	92	100
ISO 12491 - Lognormal - Simple**	32.4	100	109
Eurocode 5	30.7	95	103
EN 1058	29.9	92	101
EN TC 124.bbb	32.1	99	108
ASTM D 2915 - Non-parametric	30.3	94	102

* This method is equal to ASTM D 2915 - Normal
** This method is equal to ASTM D 2915 - Lognormal

3.3 Summary

The methods for determination of characteristic strength value, defined as the 5% fractile value, given in ISO 12491, Eurocode 1, Eurocode 5, EN 1058, EN 384 and ASTM D 2915 are summarised. The Eurocode 5 and EN 1058 methods will be replaced by the method given in EN TC 124.bbb. This method which is based on a lognormal distribution function is also included.

Three case studies were carried out and the results of the relevant methods for determination of characteristic values are compared to each other.

The method to be used for sawn timber is based on a non-parametric distribution function. A method based on a normal distribution function is sufficient at least for the analysed case. The k_v factor which is 1.12 for machine-graded timber cannot be defended statistically.

The forthcoming European method given in EN TC 124.bbb results in reasonable 5% fractile values for Kerto-laminated veneer lumber. Whether the method should be based on an 84.1% or a 75% confidence level needs to be discussed. The method given in Eurocode 5 overestimates the 5% fractile value for the coefficient of variation values by just over 10%.

The forthcoming European method given in EN TC 124.bbb results in reasonable 5% fractile values for plywood. The method given in EN 1058 underestimates the 5% fractile value for coefficient of variation values by about 20%.

4. Calculation of the failure probability for different load-material combinations

4.1 Calculation method based on discrete probability distributions

Probability of failure can be calculated by

$$P_f = \int_0^\infty f_S(x) F_R(x) dx \tag{4.1}$$

where f_S is the probability density function of load effect, and F_R is the cumulative probability function of resistance. Here the numerical integration of discrete probability distributions is used and is written as

$$P_f = \sum_{i=1}^{n} F_{R,i}\, f_{S,i}\, \Delta x \tag{4.2}$$

The practical calculation was made by programming an Excel macro, in which f_S and F_R are given numerically in columns. The load distribution was adjusted by a factor to adjust the ratio of resistance and action effect so as to reach a decided safety factor or probability of failure. The output of the calculation includes characteristic values of strength and stress, the total safety factor and P_f. The total safety factor is calculated in the case of a single load as

$$\gamma = \frac{f_{0.05}}{\sigma_k} \tag{4.3}$$

where $f_{0.05}$ is the 5% fractile of the given strength distribution and σ_k is the stress caused by the characteristic load, the fractile of the characteristic load being different for dead and variable loads. Here, strength distribution is thought to be given corresponding to the load duration in question. Tested strength values should be multiplied by a factor, k_{mod}, according to the load duration and service conditions. This would, however, not change the results of the parametric studies to be made. Failure probabilities can be transferred to safety indices β (see Equation (1.7), and Table 1.1).

Combinations of two loads are also analysed. For simplicity, the parameters of the two load distributions are selected so that the stress caused by the characteristic load, σ_k, is the same for both loads. The loads are then combined using different values of α, the ratio of variable load to total load:

$$\alpha = \sigma_{Qk} / (\sigma_{Gk} + \sigma_{Qk}) \tag{4.4}$$

The failure probability is calculated using combined distribution for stress. The strength is then adjusted by a factor, m, such that the target value of P_f is achieved. Finally, the material safety factor γ_M is solved from the design equation:

$$(1-\alpha)\gamma_G \sigma_k + \alpha \gamma_Q \sigma_k = \frac{m f_{0.05}}{\gamma_M} \tag{4.5}$$

where γ_G and γ_Q are the given safety factors for the loads.

The basics of adding and multiplying statistical distributions are given in Appendix A.

The equations of the probability distribution functions commonly used for material strength and load effects are given in Appendix A. The cumulative strength distribution equations are illustrated in Figure 4.1, where they have the same 5 percentile value. Similarly, load density functions of normal and Gumbel distributions are compared in Figure 4.2. The tails of distributions are most interesting from the reliability analysis point of view: a lognormal distribution of strength means that there are less cases with extremely weak strength values than when material obeys a normal or two-parameter Weibull distribution, when the COV calculated from the distribution parameters is the same. Also the upper tails of load distributions are different: the probability of having a load exceeding the 98th percentile by 50% is much higher in the case of a Gumbel distribution than in the case of a normal distribution.

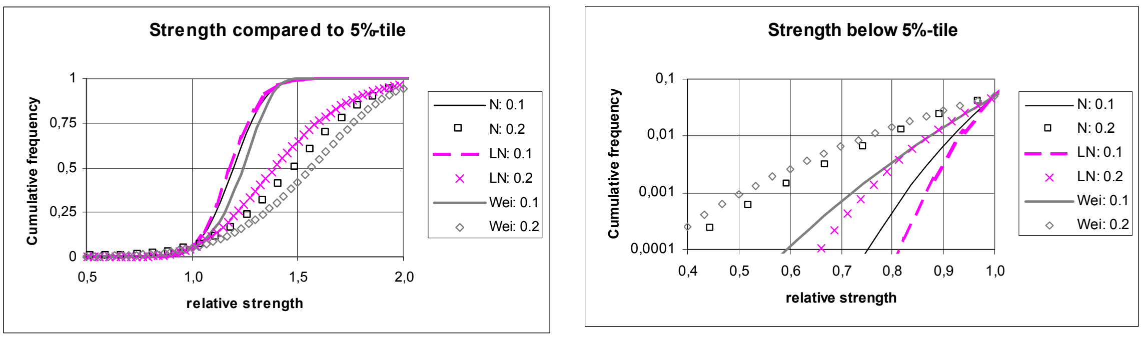


Figure 4.1. Comparison of normal, lognormal and two-parameter Weibull distributions when coefficient of variation is 10 or 20%.

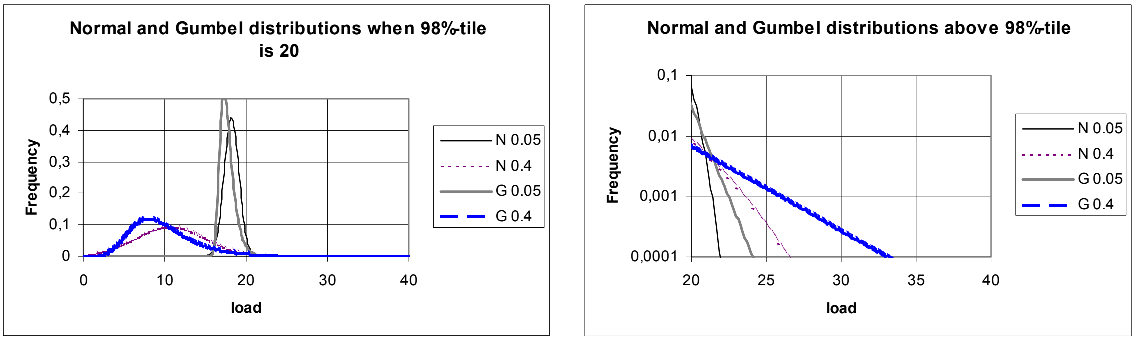


Figure 4.2. Comparison of loads following normal and Gumbel distributions when the 98 percentile value is 20. N means normal and G Gumbel distribution, the number following N or G is the coefficient of variation.

4.2 Accuracy of the calculation

The accuracy of numerical integration was analysed using two different step-lengths of discrete stress and strength distributions. Results are shown in Table 4.1 where the Monte Carlo simulation results of the same problem are also shown (Svensson 2000). The relative difference between the methods increases when the safety level increases. Accuracy with $\Delta\sigma = 0.1$ MPa seems adequate for practical purposes, and this step length has been used in the calculations.

Table 4.1. Comparison of the accuracy of the numerical integration. Failure probabilities at three intended β-index levels. Number of simulations is 10^7 for $\beta = 3$, 10^8 for $\beta = 4$, and 10^9 for $\beta = 5$.

Method	Load		Strength		Probability of failure		
	Distribution	COV	Distribution	COV	$\beta = 3$	$\beta = 4$	$\beta = 5$
$\Delta\sigma = 1$ MPa	normal	0.4	lognormal	0.2	0.001310	0.0000320	0.00000030
$\Delta\sigma = 0.1$ MPa					0.001275	0.0000306	0.00000029
Monte Carlo					0.001210	0.0000277	0.00000025
$\Delta\sigma = 1$ MPa	Gumbel	0.4	lognormal	0.2	0.001377	0.0000320	0.00000029
$\Delta\sigma = 0.1$ MPa					0.001336	0.0000311	0.00000028
Monte Carlo					0.001320	0.0000308	0.00000024

For verification purposes the results of the numerical integration were also compared to results computed by other software, namely the Strurel package using the Comrel program (SORM method). In the numerical integration method, the step in the integration was $\Delta\sigma$ = 0.1 MPa. Two cases were studied as shown in Table 4.2. The results demonstrate that these two methods give the same results within reasonable accuracy and thus verify the result of the numerical integration.

Table 4.2. Comparison of computed probability of failure values computed by the numerical integration and Comrel software.

Input values	Numerical integration	Comrel
Case 1 strength lognormal (mean: 56.5, COV: 26%) load normal (mean: 10.0, COV: 40%) k_{mod} =1.0	$1.9058 \cdot 10^{-6}$	$1.9139 \cdot 10^{-6}$
Case 2 strength ~ lognormal (mean: 56.5 COV: 26%) load ~ Gumbel (u: 4.4, α: 0.6 COV: 40%) k_{mod} =0.8	$0.7190 \cdot 10^{-6}$	$0.7086 \cdot 10^{-6}$

4.3 Sensitivity studies

4.3.1 Effect of load distribution function: one variable load

Two distribution types, normal and Gumbel for actions, and normal and lognormal for resistance were used in the calculation of the total safety factor needed for a certain probability of failure in the case of a single action. In this calculation, characteristic values of resistance were 5 percentiles and those of actions were 98 percentiles, and the total safety factor is the ratio of these two. The values of safety factors needed for a probability of failure of 10^{-6}, 10^{-5} and 10^{-4} are given in Tables 4.3, 4.4 and 4.5, respectively. Results are illustrated in Figure 4.3. It can be concluded that by having the same characteristic values and same coefficient of variation but different shapes of distribution functions very different results can be obtained. The combination of a normal load and a lognormal strength distribution gives a favourable result: γ = 1.76 results in $P_f = 10^{-6}$ when the COV of the load is 40% and the COV of strength is 20%. With the same COV values, a combination of a Gumbel load and a normal strength distribution ends up with the result: γ= 1.87 which results in $P_f = 10^{-4}$.

If the target failure probability level for a year is 10^{-6} and a lognormal distribution is used for strength and a Gumbel distribution with a COV = 40% for variable actions, we end up with γ = 2.31 for a COV = 10% and γ = 2.39 for a COV = 20%. This combination of target safety level and selection of load distribution type leads to a considerably higher safety factor than the present values in most standards (Eurocode 5: $\gamma_m \gamma_Q$ =1.3 × 1.5 = 1.95).

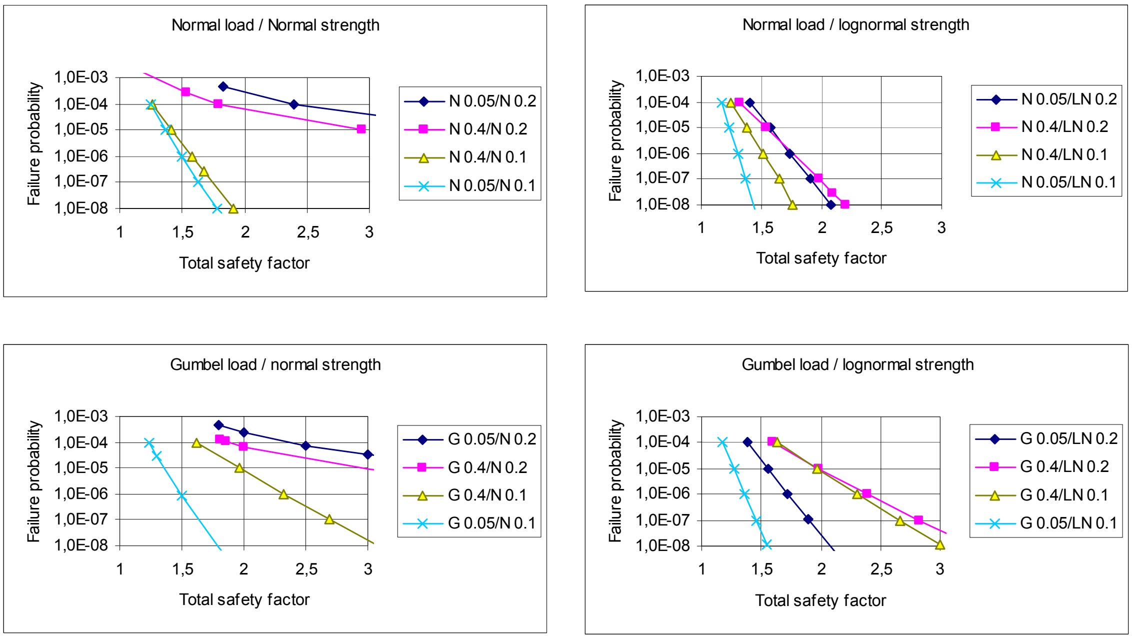


Figure 4.3. Calculated correspondence of P_f and γ when a load is normally or Gumbel distributed and the strength is normally or lognormally distributed. COV is given in the legend after the symbol of distribution N, LN or G. The characteristic load is the 98% fractile.

Table 4.3. Total safety factors corresponding to probability of failure level 10^{-6} (β=4.75) for different combinations of load and strength distribution. NA (not applicable) means that the target probability can-not be achieved using a reasonable safety factor.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05	1.49	NA	1.31	1.74
	0.4	1.58	NA	1.52	1.76
Gumbel	0.05	1.49	NA	1.36	1.73
	0.4	2.32	NA	2.31	2.39

Table 4.4. Total safety factors corresponding to a probability of failure level of 10^{-5} (β=4.27) for different combinations of load and strength distribution.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05	1.37	4.17	1.24	1.58
	0.4	1.42	2.94	1.38	1.54
Gumbel	0.05	1.36	4.07	1.27	1.55
	0.4	1.97	2.92	1.96	1.98

Table 4.5. Total safety factors corresponding to a probability of failure level of 10^{-4} (β=3,72) for different combinations of load and strength distribution.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05	1.25	2.40	1.17	1.41
	0.4	1.26	1.80	1.24	1.32
Gumbel	0.05	1.24	2.34	1.18	1.38
	0.4	1.62	1.87	1.63	1.60

4.3.2 Effect of self-weight definition

The Eurocode gives two possibilities for the determination of the characteristic value of self-weight: mean value if the COV< 0.05 or the 95 percentile, if the coefficient of variation is larger. A normal distribution can be used for a dead load. In the following, the effect on the safety factor is compared when the characteristic value of a dead load is a mean or the 95 percentile. Values for the 98th percentile are also given in Tables 4.6 and 4.7 for comparison purposes. The results show that in all calculated cases the use of a mean value as the characteristic dead load requires a safety factor 8 to 9% higher than that required with the use of the 95th percentile. The difference between the 95th and 98th percentile is equivalent to a 2% difference in safety factor. However, when strength follows a normal distribution and the COV ≥ 0.2 we end up, with unreasonably high safety coefficients. Boxed values of safety factors in Eurocode give $\gamma_m\gamma_G$ =1.3 × 1.35 = 1.76 which is close to the value obtained for the combination of P_f = 10^{-6}, and a lognormal strength distribution with a COV = 0.2.

Table 4.6. Total safety factors corresponding to a probability of failure level of 10^{-6} (β=4.75) for a dead load when the characteristic value is a mean or the 95% or 98% fractile and the COV = 5%.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	Mean	1.65	13.7	1.44	1.92
	95%-tile	1.52	12.6	1.33	1.78
	98%-tile	1.49	NA	1.31	1.74

Table 4.7. Total safety factors corresponding to a probability of failure level of 10^{-5} (β=4.27 for a dead load when the characteristic value is a mean or the 95% or 98% fractile and the COV = 5%.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	Mean	1.51	4.60	1.37	1.74
	95%-tile	1.39	4.25	1.26	1.61
	98%-tile	1.37	4.17	1.24	1.58

4.3.3 Tail effects

The effect of the lower tail of strength distribution is analysed using different truncated distributions. Basically the same load distributions are used as in Paragraph 4.3.1. Strength distributions are truncated in such a way that all values below the truncation point (1% or 5% fractile) are transferred to have the strength of the truncation point. Thus the cumulative distribution above truncation is unchanged. Tables 4.8 and 4.9 give the factor indicating the change of failure probabilities due to the truncation of the tail. The conclusion is that the effect of truncation of the tail depends strongly on the type of distribution and on the COV. This can be explained by Figure 4.4 where the tails of the distributions are shown in an example where $f_{0.05}$ is 20 and σ_k is 40 MPa. Obviously, the truncation is most effective when the tails cross at high levels of P_f.

When COV of strength distribution is large (>20%) we can conclude that strength values below 5% fractile contribute the major part of failure probability, in case of normally distributed load more than 99% of it. When COV of strength is 10% and load distribution type is Gumbel, also values above 5% fractile have practical effect on P_f.

Table 4.8. Probability of failure ×10^{-6} for a strength distribution truncated at the 5% (1%) fractile when the original strength distribution gives $P_f = 10^{-6}$. Other parameters as in Table 4.3.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05			$10^{-13}(10^{-7})$	(10^{-36})
	0.4			0.19 (0.51)	0.00034 (0.02)
Gumbel					
	0.4	0.63 (0.83)	NA	0.76 (0.92)	0.24 (0.55)

Table 4.9. Probability of failure ×10^{-5} for a strength distribution truncated at the 5% (1%) fractile when the original strength distribution gives $P_f = 10^{-5}$. Other parameters as in Table 4.4.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05			10^{-13} (0.0004)	(10^{-21})
	0.4		(10^{-14})	0.36 (0.69)	0.009 (0.14)
Gumbel	0.4	0.74 (0.90)	0.0006	0.83 (0.95)	0.39 (0.70)

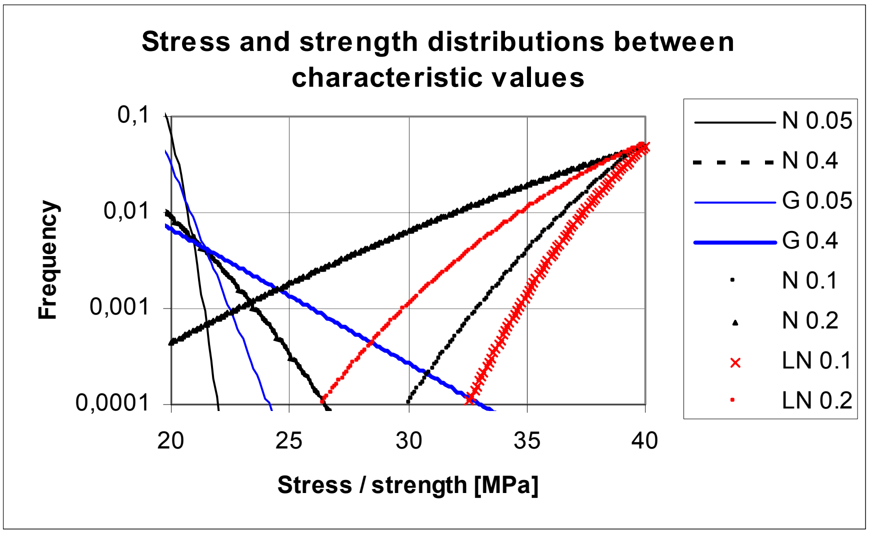


Figure 4.4. Density distributions of stresses above the action of the characteristic load (98% fractile = 20 MPa) and cumulative distributions of strength below the 5% fractile (40 MPa).

4.3.4 Effect of safety factor on reliability

Here we discuss how a 10% change in safety factor will influence the failure probability at different levels of safety and when using various distributions for load and strength as specified in Paragraph 4.3.1. Results are shown in Tables 4.10 to 4.12. It can be seen that for cases typical of dead loads (normal distribution, COV = 5%) and a lognormal strength distribution, the probability of failure is increased by a factor of 10 to 100 when the safety factor is decreased by 10%. When the load corresponds to variable loads (Gumbel, COV = 40%), the effect is smaller: the failure probability is increased by 3 to 5 times.

Table 4.10. Probability of failure ×10^{-6} for a safety factor decreased by 10% of the original value giving $P_f = 10^{-6}$. Other parameters as in Table 4.2.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05			73.5	
	0.4				
Gumbel	0.05				10.2
	0.4	4.48		5.09	3.78

Table 4.11. Probability of failure ×10^{-5} for a safety factor decreased by 10% of the original value giving $P_f = 10^{-5}$. Other parameters as in Table 4.4.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05			46.9	
	0.4				
Gumbel	0.05				8.24
	0.4	3.72	1.59	4.00	3.22

Table 4.12. Probability of failure ×10^{-4} for a safety factor decreased by 10% of the original value giving $P_f = 10^{-4}$. Other parameters as in Table 4.5.

Load distribution		Strength distribution			
		Normal		Lognormal	
	COV	0.1	0.2	0.1	0.2
Normal	0.05			28.5	
	0.4				
Gumbel	0.05				6.42
	0.4	3.05	1.92	3.19	2.72

4.4 Analysis with real material data

4.4.1 Spruce sawn timber

The effect of different distributions fitted to material strength data is analysed when the load distribution is Gumbel (COV=0.4). In the first example, the data given in Table 2.2 (Series S-1), spruce with a depth of 150 mm, machine graded to C30, is used. The fitted distributions to be analysed are:

- Normal distribution fitted to all data (N all), normal fitted to the lowest 5% of the data (N tail) and a combination of the two: different normal distributions below and above the 5%-tile (N+N).

- Lognormal distribution fitted to all data (LN all), lognormal fitted to the lowest 5% of the data (LN tail), and a combination of a lognormal distribution below and a normal distribution above the 5%-tile (LN+N).

- Non-parametric distribution based on direct application of test data, where two alternatives are analysed: test data only indicating that there are no weaker specimens than the lowest value tested (test), and the combination of a non-parametric distribution and a lognormal distribution fitted to the lowest 5 percentile so that a lognormal distribution is used below the lowest test value (LN+test).

The results indicate that a normal distribution fitted to all of the test data gives the lowest safety level (β=3.64) and a lognormal distribution fitted to all of the data the highest (β=4.34) the other combinations being fairly close to each other (β=4.1 to 4.2). The combined distributions fit very closely to the test data (see Fig. 2.2) and are obviously the most correct ones. We can also conclude that the distributions fitted to the lowest 5 percentile of the test data give practically the same reliability as combined distributions. Results are shown in Tables 4.13 in terms of failure probability and in Table 4.14 in terms of beta index.

Table 4.13. Probability of failure $\times 10^{-6}$ when load distribution is Gumbel or normal with a COV=0.4, total safety factor 1,3x1,5=1,95 and material is C30 as given in Table 2.2 (S-1).

Load distribution	Strength distribution							
	N all	N tail	N+N	LN all	LN tail	LN+N	test	LN+test
Normal	135	5.5	5.5	0.12	0.92	0.92	0.14	0.95
Gumbel	137	20	19	7	15	14	13	15

Table 4.14. Beta index when load distribution is Gumbel or normal with a COV=0.4, total safety factor 1,3x1,5=1,95 and material is C30 as given in Table 2.2 (S-1).

Load distribution	Strength distribution							
	N all	N tail	N+N	LN all	LN tail	LN+N	test	LN+test
Normal	3.64	4.41	4.40	5.16	4.77	4.77	5.14	4.76
Gumbel	3.64	4.11	4.12	4.34	4.18	4.18	4.21	4.17

4.4.2 Combined spruce and pine

Data of 1327 machine–graded, sawn timber specimens containing both spruce and pine are given in Section 2.2.4 including fitted models in Table 2.10. The material is graded to strength class C30, and has a mean strength of 47.8 N/mm^2 and a COV of 22.3%. Normal, lognormal and two- and three-parameter Weibull distributions fitted separately to all data points and to the lowest 10% are used in reliability analysis. In analyses with different material models the load has the same characteristic values corresponding to total safety factors 1.4, 1.6, 1.8, 2.0 and 2.2 when the characteristic strength is $f_{0.05}$ = 30 N/mm^2. Load distribution is assumed to be normal with a COV = 5% and the characteristic value is defined as the 50 or 95th percentile, as used for permanent loads. Both normal and Gumbel distributions with COV = 40% are used for loads simulating variable loads, the characteristic value being the 98% fractile.

Results are shown in Table 4.15 in terms of failure probability multiplied by 10^6. Large variation is observed in the P_f values. When the most optimistic strength distribution gives $P_f = 10^{-5}$, the most pessimistic distribution results in 10^{-4} to 10^{-3}, although all the functions have been properly fitted to the same test data. If the target value of P_f is 10^{-5} for Gumbel distributed variable loads, the use of a three-parameter Weibull distribution fitted to all data (cut-off value 16) would result in γ = 2.1, while the use of a three-parameter Weibull or a lognormal distribution fitted to tail data would give γ = 2.2 whereas other distributions would lead to higher γ values not shown in the table. In all loading cases P_f values obtained for a lognormal distribution fitted to tail data and a three-parameter Weibull also fitted to tail data (cut-off value ε = 9.7 N/mm^2) are nearly identical. Sometimes a lognormal distribution is also used for loads. A comparison was made of normal and Gumbel loads when strength was lognormal as in table 4.15 (tail-fitted). When the COV of the load was 5%, a lognormal load resulted in 3% higher P_f values than a normal distribution. When the COV of the load was 40%, a lognormal load resulted in a P_f about 30% higher than that in the case of a Gumbel load.

Table 4.15. Calculated $P_f \times 10^6$ with spruce and pine combined data taken from Table 2.10 corresponding to different safety coefficients.

Load		Strength distribution function							
	γ	N all	N tail	LN all	LN tail	W2 all	W2 tail	W3 all	W3 tail
Normal	1.4	7020	4440	649	4040	8330	4290	2460	4370
COV 5%,	1.6	3350	1820	80.6	1200	4010	1670	329	1430
50 %-tile	1.8	1810	861	10.0	365	2100	724	17.1	464
	2.0	1180	457	1.28	115	1180	343	0.128	146
	2.2	692	266	0.170	37.3	698	176	$5\ 10^{-5}$	42.3
Normal	1.4	4500	2600	194	2030	5380	2450	831	2280
COV 5%	1.6	2200	1090	20.2	555	2590	949	54.9	680
95%-tile	1.8	1220	532	2.15	157	1360	412	0.58	197
	2.0	743	290	0.239	46.4	760	195	$2\ 10^{-4}$	52.7
	2.2	488	173	0.028	3.91	450	99.3	10^{-9}	12.3
Normal	1.4	884	430	37.9	255	916	364	115	286
COV 40%	1.6	442	190	5.51	71.4	440	141	18.6	84.3
98%-tile	1.8	262	97.9	0.860	21.4	231	61.4	2.74	25.3
	2.0	171	56.5	0.138	6.71	129	29.1	0.343	7.65
	2.2	119	35.7	0.024	2.20	76,6	14.8	0.036	2.29
Gumbel	1.4	882	563	223	436	918	520	354	461
COV 40%	1.6	450	244	65.1	162	445	207	116	172
98%-tile	1.8	259	119	20.0	63.3	234	91.0	39.7	68.0
	2.0	164	64.8	6.46	26.0	131	43.4	14.1	28.3
	2.2	112	38.8	2.17	11.2	77.9	22.1	5.17	12.3

4.4.3 LVL

The data of nearly 2000 internal quality control specimens of Kerto LVL is analysed in Section 2.3.3 and the parameters of fitted distributions are given in Table 2.13. All seven of these functions are used as material data to show how much of a difference the choice of statistical function makes to failure probability when the load level is such that a lognormal distribution fitted to all of the data gives a probability of failure of 10^{-6}. A lognormal distribution is selected as a base because it fits well to both edgewise and flatwise bending data in the tail area (see Table 2.14). In edgewise bending, a normal distribution had the best fit when fitted to tail data. COV calculated from test data was 9.6% in edgewise bending and 14.4% in flatwise bending.

Table 4.16. Probability of failure ×10⁻⁶ when the load distribution function is the same, Gumbel or normal, and LVL material data for edgewise bending is taken from Table 2.14. The actual total safety factor is also given.

Load	COV		N all	N tail	**LN all**	LN tail	W2 all	W2 tail	W3 all
Normal	0.05	P_f	37	**2.1**	**1**	**0.62**	621	**11.3**	10^{-6}
		γ	1.27	**1.29**	**1.29**	**1.29**	1.27	**1.29**	1.29
Normal	0.4	P_f	2.8	**0.99**	**1**	**0.89**	18	**1.3**	0.46
		γ	1.49	**1.51**	**1.51**	**1.51**	1.49	**1.51**	1.51
Gumbel	0.4	P_f	1.24	**1.06**	**1**	**0.98**	2.08	**1.39**	0.92
		γ	2.29	**2.32**	**2.32**	**2.32**	2.28	**2.32**	2.32
Gumbel	0.4	P_f	11.5	**10.8**	**10**	**9.94**	15.1	**14.1**	9.53
		γ	1.95	**1.97**	**1.97**	**1.98**	1.94	**1.98**	1.97

Table 4.17. Probability of failure ×10⁻⁶ when load distribution function is the same in all rows, calibrated to give $P_f = 10^{-6}$ or 10^{-5} for lognormal strength distribution, and LVL material data for flatwise bending is taken from Table 2.14. The actual total safety factor is also given.

Load	COV		N all	N tail	**LN all**	LN tail	W2 all	W2 tail	W3 all
Normal	0.05	P_f	209	**1.01**	**1**	0.094	795	**5.95**	10^{-15}
		γ	1.43	**1.47**	**1.47**	1.47	1.43	**1.47**	1.47
Normal	0.4	P_f	17	**0.64**	**1**	0.46	62	**1.10**	0.13
		γ	1.55	**1.60**	**1.59**	1.60	1.56	**1.60**	1.59
Gumbel	0.4	P_f	2.81	**0.95**	**1**	0.87	6.62	**1.19**	0.74
		γ	2.26	**2.33**	**2.32**	2.33	2.27	**2.33**	2.32
Gumbel	0.4	P_f	19.2	**10.4**	**10**	9.4	32	**13.1**	8.4
		γ	1.90	**1.96**	**1.95**	1.96	1.91	**1.96**	1.95

In Tables 4.16 and 4.17 the columns in bold are those which correspond to the best fitting curves in the lower tail area up to the 0.1 percentile. Among these well fitting functions, two-parameter Weibull fitted to the tail results in higher P_f values than the others, the difference, however, being insignificant except in the case of a dead load simulation with a normal distribution with a COV = 0.05. A normal distribution fitted to the tail gives similar P_f values to those given by a lognormal distribution fitted to all of the data. A lognormal distribution fitted to the tail gives slightly lower P_f values. The other distributions may result in incorrect P_f values: normal and two-parameter Weibull distributions fitted to all of the data give unreasonably high values, and a three-parametric Weibull distribution gives low values. The problem with a three-parametric Weibull distribution is the determination of the third parameter which is the lower limit of strength in the material. If this cut-off value is high, the results of the P_f analysis are very favourable.

The safety factors, γ, needed to achieve desired P_f are also given in Tables 4.16 and 4.17. In each loading case, differences of safety factors directly reflect differences of 5% fractiles of strength distributions. The values also confirm earlier observations, how different P_f values are produced by the use of normal and Gumbel distributions when COV is 40%.

5. Applications of reliability analysis

5.1 Calibration of safety factors

In the examples of previous chapters, the relations of safety factor and failure probability have been studied, when we have a single load. These results can be used to analyse what the ratio of safety factors should be for dead loads and variable loads in order to achieve the same reliability level in the extreme cases of dead load only or variable load only. This ratio depends on types of distribution functions and on the variability of material and load values.

Here loading combinations have been analysed. The material safety factor, needed for a given reliability level, has been calculated for given combinations of distribution functions, COVs and safety factors for loads.

Calculations are made for two target safety levels, $P_f = 10^{-5}$ and 10^{-6}. A normal distribution is adopted with a COV = 0.05 for dead loads, the characteristic value being the 95th or 50th percentile. A Gumbel distribution is used for variable loads (COV = 0.4). A lognormal distribution is used with a COV = 0.05, 0.1 and 0.2 for material strength.

Results are shown in Figure 5.1. When the partial safety factors for loads were $\gamma_G = 1.2$ and $\gamma_Q = 1.5$, we obtained lower values for γ_M than those obtained in the case of a single variable load. In the case of having a COV=0.2 for material strength, the γ_M value needed for a certain safety level seems to have a minimum when load ratio is about α = 0.4. When the material COV is smaller, the minimum for γ_M is shifted towards a lower variable load ratio. In typical timber constructions α = 0.6 to 0.8 and in this range, the COV of material strength has only a minor influence, γ_M =1.2 is needed for $P_f = 10^{-5}$.

The same example (mean value of permanent load being characteristic, $P_f = 10^{-5}$) was calculated with the IRELAN software developed by the University of British Columbia, and the results obtained are almost identical to Figure 5.1 (Foschi 2001). Foschi also calculated similar examples on a lower target safety level, β = 3 for a 50-year reference period. The results are shown in Figure 5.2. Here, also, a higher partial safety factor, γ_Q = 1.6, is used in order to obtain a more constant value for the required γ_M. We can also conclude that a lower target safety results in less variability in γ_M values.

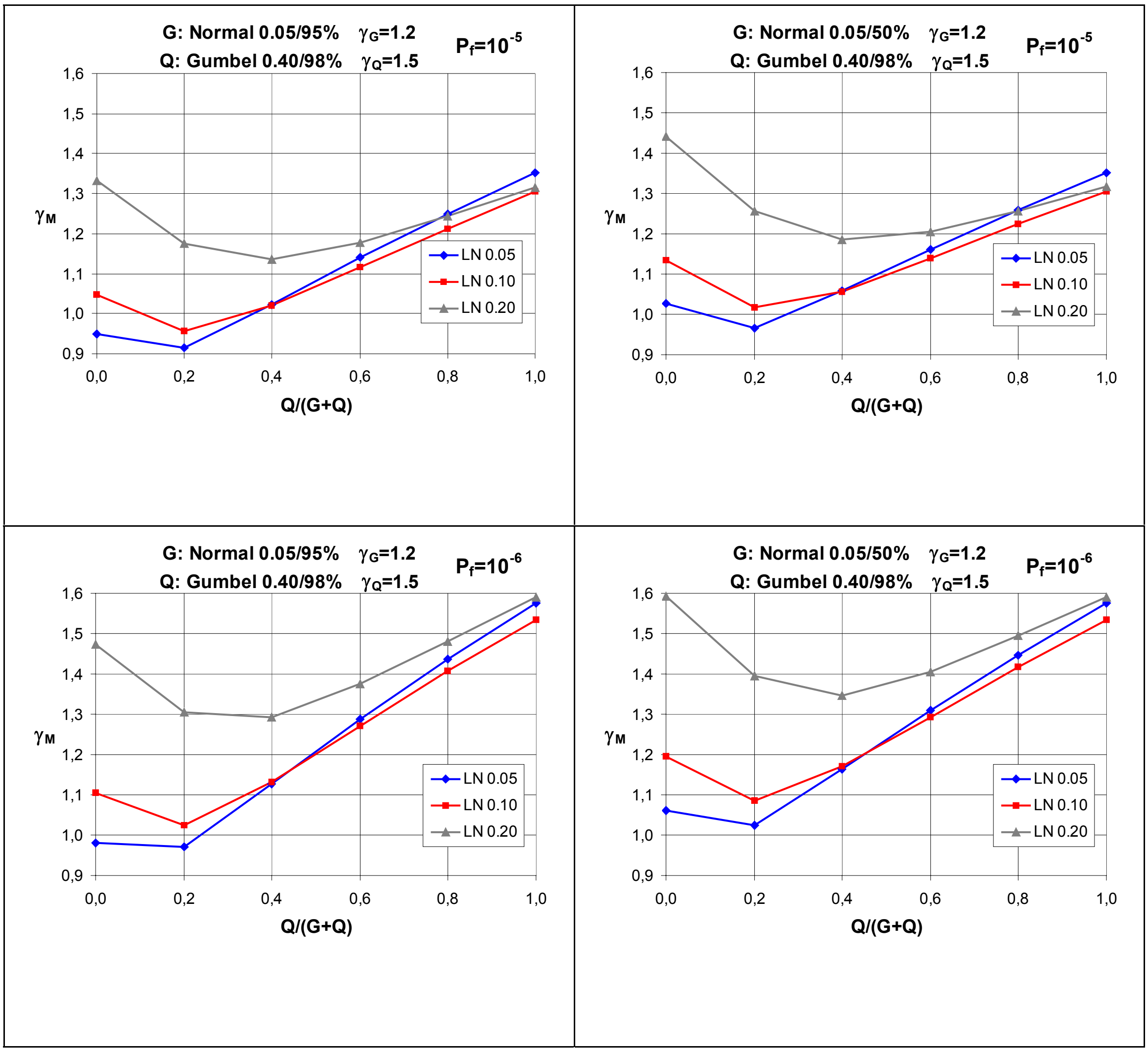
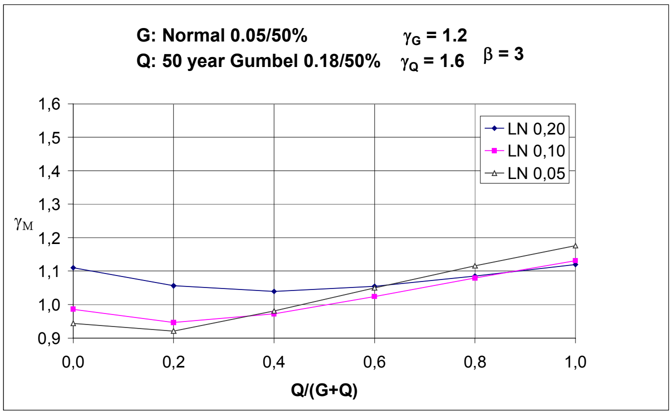
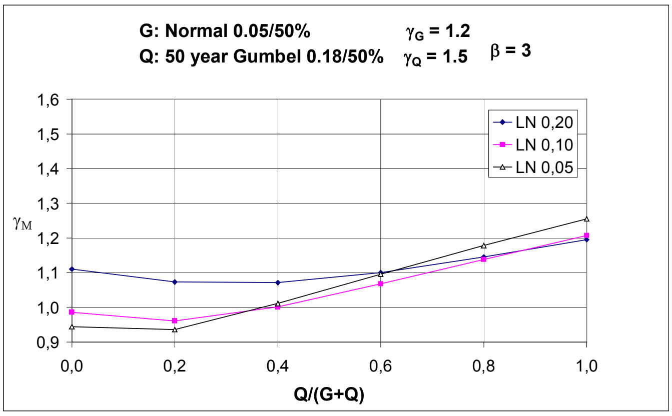


Figure 5.1. Material safety factors, γ_M, needed for a target failure probability of P_f = 10^{-5} or 10^{-6}, when the permanent load, G, follows a normal distribution with a COV = 0.05 and the characteristic value is the 50 or the 95% fractile and the variable load, Q, follows a Gumbel distribution with a COV = 0.4. Partial factors for loads are γ_G = 1.2 and γ_Q = 1.5. Strength follows a lognormal distribution having a COV = 0.05, 0.1 or 0.2. Q/(G+Q) is the ratio of characteristic values.





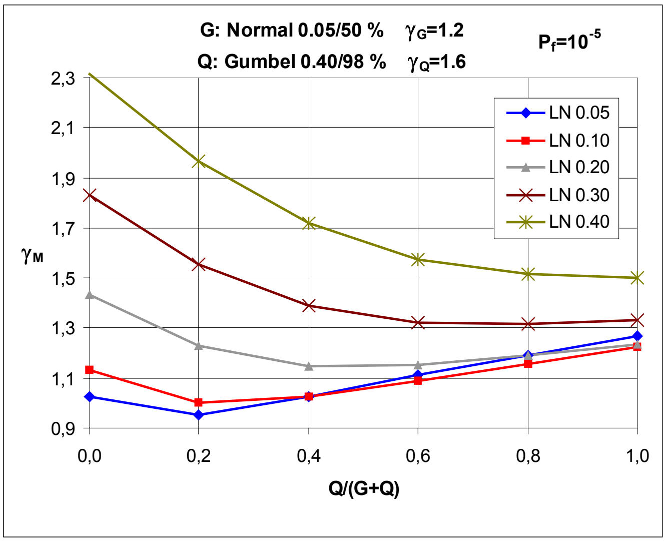


Figure 5.2. Material safety factors, γ_M, needed for a target failure probability of $P_f = 10^{-5}$, when the permanent load, G, follows a normal distribution with a COV = 0.05 and the characteristic value is the 50% fractile and the variable load, Q, follows a Gumbel distribution with a COV = 0.4. Partial factors for loads are γ_G = 1.2 and γ_Q = 1.5 or 1.6. Strength follows a lognormal distribution having a COV = 0.05 to 0,4 (Foschi 2001).

5.2 System effects

Parallel load-bearing wood members behave in a manner that makes system analysis a necessary approach. Sheathed timber member systems obtain their behavioural characteristics from two separate effects: a) composite action between the sheathing and the member, this composite member is simply a member in the system from the reliability standpoint, and b) the load distribution and redistribution characteristics of the system. In this case the load is distributed to the members depending on their moduli of elasticity and depending on the stiffness of the sheathing in the direction perpendicular to the span of the member. The correlation between the strength and stiffness enables weaker specimens not to be loaded as highly as the members with higher strength. This gives a system effect. At higher load levels, the load may be further redistributed to neighbouring members after failure or yielding occurs in a certain member.

In the present study, only the initial load distribution effect between members, where the differences of moduli of elasticity and their correlation to strength, is considered. The secondary structure above the members is considered to be perfectly stiff, that is, the members are loaded with an equal deformation.

Three different approaches to defining the system effects are considered in the following. The system effect is introduced through a load-sharing factor, which may be applied in a single member design by multiplying the resistance value with a factor higher than one due to the additional reliability caused by the system effect. The multiplication factors, ϕ, for resistance are used to obtain an equal probability of failure in all cases.

<u>Method A</u>, where the system failure is defined by the first failure of any member in the system. This is strictly a series system, in which the load distribution due to the correlation between modulus of elasticity and strength is taken into account.

$$k_A = \phi_{member} / \phi_{system,moe,n} \tag{5.1}$$

<u>Method B</u>, where the increase of resistance effect of a single member is considered by its insertion into the system. The correlation between modulus of elasticity and strength is taken into account.

$$k_B = \phi_{member} / \phi_{member,moe,n} \tag{5.2}$$

<u>Method C</u>, where only the increase of resistance effect due to the correlation between the modulus of elasticity and strength is considered:

$k_C = \phi_{system,n} / \phi_{system,moe,n}$ (5.3)

The probability of failure of a single member and of the system are as below. All the four ϕ factors are adjusted to obtain an equal target probability of failure of $P_f = 10^{-6}$. The calculation was carried out using the Strurel Comrel software package. The terms R and E below signify the resistance and the action effect, respectively.

$$P_{member}(\phi_{member}) = P_f(g = \phi_{member}R - E \leq 0) = 10^{-6} \quad (5.4)$$

The probability of failure of a series system of n single members is:

$$P_{system,n}(\phi_{system,n}) = 1 - [1 - P_{member}(\phi_{system,n})]^n = 10^{-6} \quad (5.5)$$

The probability of failure of any single member, j, in the system of size n, where the modulus of elasticity and correlation to strength is considered. This correlation is assumed to be r = 0.7.

$$P_{member,moe,n}(\phi_{member,moe,n}) = P_f(g = \phi_{member,moe,n}R_j - E\frac{n\,Moe_j}{\sum_1^n Moe_i} \leq 0) = 10^{-6} \quad (5.6)$$

The probability of failure of a series system of n single members with modulus of elasticity adjustment:

$$P_{system,moe,n}(\phi_{system,moe,n}) = 1 - [1 - P_{member,moe,n}(\phi_{system,moe,n})]^n = 10^{-6} \quad (5.7)$$

Unfortunately, the real system effect is none of the above as floor or other structural systems are neither series nor parallel systems. In most cases, structural systems do not fail from the first member failure as assumed in a series system. This failed member might still resist some load and have a non-linear ductile failure later. In this case, the load is redistributed to adjacent members. In some earlier studies, systems having members with non-linear force deflection relations and having a system failure criteria of failures in two adjacent members, showed good results compared to experiments. Such an analysis has to be carried out by a Monte Carlo simulation. The real system effect should be in-between the values given by methods A and B, probably closer to method B. In the present study, the variability of strength and load is studied on the system effect as described by these three methods.

The following distribution and base parameters were used in this study: - system size, n, = 1,2,3,6 and 10, - resistance ~lognormal (mean: 56.48, COV 20%), - action effect ~normal (mean: 10.0, COV 40%), - modulus of elasticity ~normal (mean: 12000, COV

10%) and correlation between resistance and elasticity is r = 0.7. The coefficient of variation of the resistance and of the action effect were later varied.

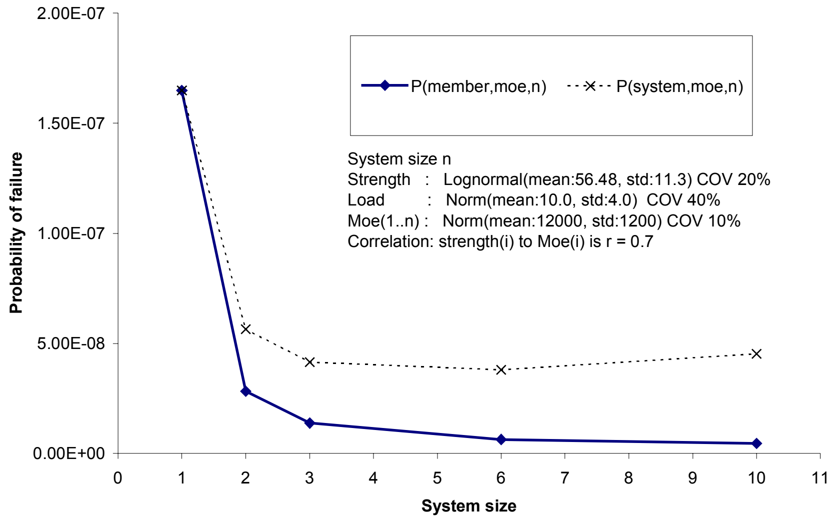


Figure 5.3. Probability of failure $P_{member,moe,n}(\phi=0.9)$ and $P_{system,moe,n}(\phi=0.9\ )$ as a function of the system size.

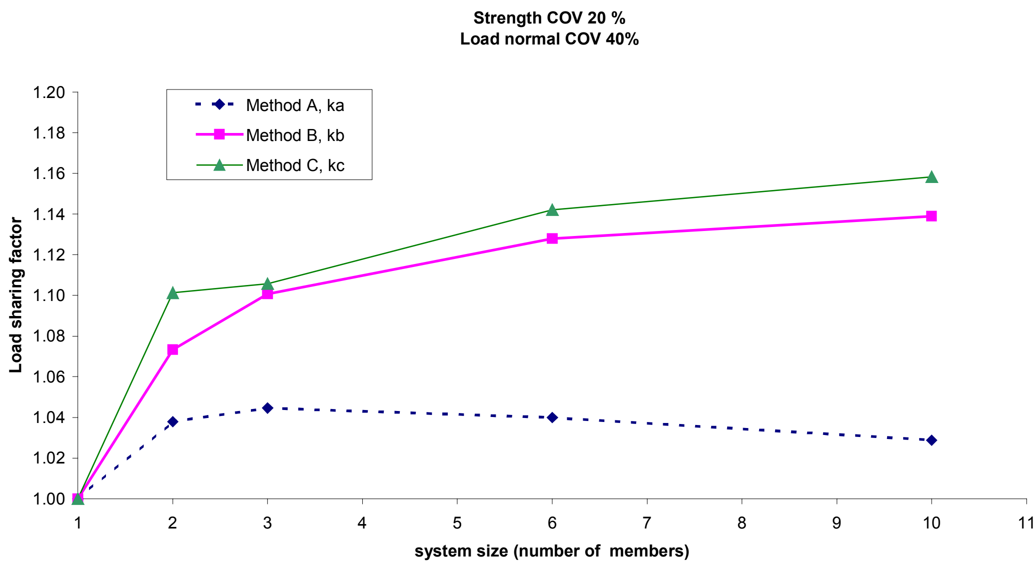


Figure 5.4. Load sharing factors for the different methods, k_A, k_B and k_C for the case of the upper figure as a function of the system size.

It may be noted from Figures 5.3 and 5.4 that the probability of failure decreases highly even with a system size of two members as compared to a single member. After this, changes are rather small. Similar findings were reported by Foschi et al. (1989) on built-up lumber beams in bending using similar methods to A and B in this study. The load-sharing factor calculated by method A, a series system of MOE-adjusted member loads, decreases after a maximum between a system size of 3 and 6. The load-sharing factors calculated using methods B and C are very similar and these increase continuously with the system size. Method B gives the increase of resistance of a single member when it is inserted into the system. Method C gives the increase of resistance effect of the whole system due to the correlation between the modulus of elasticity and strength only.

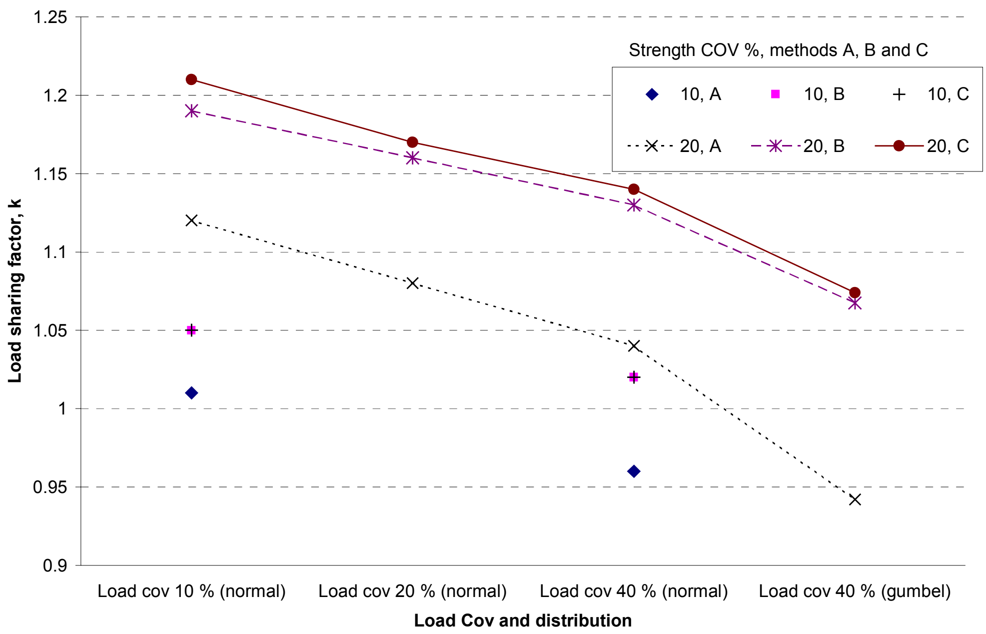


Figure 5.5. Load-sharing factors for the different methods, k_A, k_B and k_C for the case of the upper figure as a function of the system size.

Figure 5.5 shows the effect of the strength and load variabilities on the load-sharing factor. The following may be concluded from these results:

- If the strength COV is 10%, as it is for many engineered wood products, the load-sharing factor is very close to one and thus the system effect may be disregarded.
- If the strength COV is 20%, as for sawn timber, the load-sharing factor of 1.1 may be a good value assuming there are both permanent and variable loads and assuming methods B and C are close to the real behaviour.
- The load COV and distribution type also seem to have a significant effect on the load-sharing factor.

6. Summary

Strength data of sawn timber, LVL, plywood and round timber were analysed. Normal, lognormal, two- and three-parametric Weibull distributions were fitted to data, separately to all data points and to the lowest 10% of the values. The ability of distributions to describe strength data was judged in two respects:

1. How well the 5^{th} percentile is predicted, and
2. How good the fit is to the lowest values.

All functions used accurately predict the 5^{th} percentile when fitted to the lower tail data (10%). When fitted to the whole data set, the ability to predict the 5^{th} percentile is different:

- Normal distribution underestimates the 5^{th} percentile compared to non-parametric data from 0 to 5 percent for graded sawn timber, and 0 to 3 percent for LVL and plywood. A normal distribution gave an exact value for bending for small-diameter round timber, but a 9% underestimation for compression.

- Lognormal distribution generally gives good or optimistic predictions: 1 to 5% optimistic for graded sawn timber, fairly precise for LVL and plywood, 4% too high for bending of round timber but precise for compression.

- Two-parameter Weibull distribution normally gives pessimistic predictions: from 0 to 3% too low for graded sawn timber, 2% too low for LVL and thick veneer plywood, and 1 to 3% too low for round timber.

- Three-parameter Weibull distribution gives a good prediction: deviation from –2 to +2% for sawn timber, exact for LVL, from –2 to +2% for plywood, and from 0 to +1% for round timber.

When fitted functions are used in reliability analysis, it is essential that the fit is good in the lower tail area, the lowest values being most important. When fitted to the same data, two-parameter Weibull gives the most pessimistic prediction for the tail, with a normal distribution being next to it and a lognormal distribution being the most optimistic. Three-parameter Weibull can be extremely optimistic because the third parameter is the cut-off value below which $P_f = 0$. When fitted to our two samples with more than 1000 specimens, LVL and combined spruce and pine sawn timber, the following conclusions could be drawn:

- All functions fitted to the lowest 10% of values give a better fit in the tail area than the best function fitted to all data points. The only exception is a lognormal distribution fitted to all LVL data, which is nearly a perfect fit in the lower tail area.

- Three-parameter Weibull gives nice fits to the data. However, one should be careful with using it with small sample sizes.

Combined machine–graded, sawn timber data (N = 1327) based distribution functions were used in reliability analysis together with load distributions corresponding to variable loads (Gumbel or normal, COV = 0.4) and dead loads (normal, COV = 0.05). The P_f values obtained were 10 times higher for variable loads in the case of a two-parametric Weibull fitting than in the case of a three-parametric Weibull (good fit to tail data) when using a Gumbel load distribution. An even larger difference is obtained when a normal distribution is used for variable loads (10^2) and dead loads (10^3).

In a similar study with different distributions fitted to LVL data, it was concluded that most distributions gave fairly similar P_f values with the following exceptions: a normal distribution fitted to all data gave P_f values 40 times higher in the case of a dead load. Two-parameter Weibull fitted to all data gave a 2 to 600 times higher P_f value depending on load distribution, and when fitted to tail, P_f was up to 10 times higher. Three-parameter Weibull fitted to all data resulted in P_f values of several magnitudes lower when analysing with dead loads.

P_f values are sensitive to the load and strength distributions used. A Gumbel distribution for load results in more than 10 times higher failure probabilities than a normal distribution (COV = 0.4), but these values are less sensitive to material data: form of distribution or COV. The target P_f values should be selected accordingly.

When strength distributions are used in reliability analysis the distributions used should give the correct 5% fractile value and fit well to the lowest test values. It was noticed that COVs corresponding to the tail-fitted distributions can differ remarkably from the COV of test data. In Table 6.1, the COV values corresponding to the results in Chapter 2 are presented. In some cases, the COV of the tail data fitted function is close to the test COV which indicates that the material in question follows that type of distribution. This would lead to the conclusion that bending strength of sawn and round timber follows a normal distribution, and when a lognormal distribution is used a COV = 35% has to be used instead of 22. However, a reliability calculation using a lognormal distribution and a COV = 35% gives a lower failure probability than when using a normal distribution with a COV = 20% as shown in Table 4.15.

LVL and plywood seem to follow a lognormal distribution so that the same COV as that observed in tests can be used in reliability analysis. The tail of compression data of round timber is different from all others so that even a lognormal distribution with a smaller COV than in test data can be used.

Table 6.1. Summary of COV [%] of distribution functions fitted to tail data of bending strength, round timber and also for compression.

	COV of all strength data [%]	Normal	Lognormal	Two-parameter Weibull	Three-parameter Weibull
Sawn timber spruce & pine	22	20	35	17	20
LVL edgewise	10	8	9	5	
LVL flatwise	14	10	12	6	
Plywood data combined	18	14	20	10	
Round timber bending	21	20	34	16	18
Round timber compression	23	13	18	9	18

Reliability analysis with a permanent load and a variable load gives an interesting result: constant reliability level can be obtained by the same value of material safety factor γ_M , when partial load factors are $\gamma_G = 1.2$ and $\gamma_Q = 1.6$ and COV of lognormally distributed strength is not more than 20%.

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Appendix A: Statistical basics

A1. Equations of distributions

Definitions: m = mean value of observations, σ is standard deviation of observations

Normal distribution

$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$	$F(x)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}e^{\frac{-(x-\mu)^2}{2\sigma^2}}dx$
m = μ	

Lognormal distribution

$f(x)=\frac{1}{x\sigma_{\ln}\sqrt{2\pi}}e^{\frac{-(\ln(x)-\mu_{\ln})^2}{2\sigma_{\ln}^2}}$	$F(x)=\frac{1}{x\sigma_{\ln}\sqrt{2\pi}}\int_{-\infty}^{x}e^{\frac{-(\ln(x)-\mu_{\ln})^2}{2\sigma_{\ln}{}^2}}dx$
$m=e^{\mu_{\ln}+\frac{1}{2}\sigma_{\ln}{}^2}$	$\sigma=e^{\left(\mu_{\ln}+\frac{1}{2}\sigma_{\ln}^2\right)}\sqrt{e^{\sigma_{\ln}^2}-1}$
$\mu_{\ln}=\ln(m)-\frac{1}{2}\ln\left(1+\left(\frac{\sigma}{m}\right)^2\right)$	$\sigma_{\ln}=\sqrt{\ln\left(1+\left(\frac{\sigma}{m}\right)^2\right)}$

Gumbel distribution

$f(x)=\alpha e^{\left\{-\alpha(x-u)-e^{[-\alpha(x-u)]}\right\}}$	$F(x)=e^{\left\{-e^{[-\alpha(x-u)]}\right\}}$ $x=u-\frac{1}{a}\ln(-\ln(F))$
$m=u+\frac{0.577216}{\alpha}$	$\sigma=\frac{\pi}{\sqrt{6}\alpha}$
$\alpha=\frac{\pi}{\sqrt{6}\sigma}$	$u=m-\frac{0.577216}{\alpha}$

Two-parameter Weibull distribution

$f(x) = \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$ $x = \beta\left(\ln\left(\frac{1}{1-F}\right)\right)^{\frac{1}{\alpha}}$
$m = \beta\Gamma\left(1+\frac{1}{\alpha}\right)$	$\sigma = \beta\sqrt{\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma^{2}\left(1+\frac{1}{\alpha}\right)}$ $\Gamma(x) = \int_{0}^{\infty} e^{-u} u^{x-1} du$
$\beta = \frac{m}{\Gamma\left(1+\frac{1}{\alpha}\right)}$	$\alpha \approx V^{-1.075}$ $V = \frac{\sigma}{m}$; when $0{,}05 \le V \le 0{,}3$

Three-parameter Weibull distribution

$f(x) = \frac{\alpha}{(\beta-\varepsilon)^{\alpha}} (x-\varepsilon)^{\alpha-1} e^{-\left(\frac{x-\varepsilon}{\beta-\varepsilon}\right)^{\alpha}}$	$F(x) = 1 - e^{-\left(\frac{x-\varepsilon}{\beta-\varepsilon}\right)^{\alpha}}$ $x = (\beta-\varepsilon)\left(\ln\left(\frac{1}{1-F}\right)\right)^{\frac{1}{\alpha}+\varepsilon}$
$m = \varepsilon + (\beta-\varepsilon)\Gamma\left(1+\frac{1}{\alpha}\right)$	$\sigma = (\beta-\varepsilon)\sqrt{\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma^{2}\left(1+\frac{1}{\alpha}\right)}$
$\beta = \frac{m-\varepsilon}{\Gamma\left(1+\frac{1}{\alpha}\right)} + \varepsilon$	$\alpha \approx \left(\frac{V}{1-\frac{\varepsilon}{\beta}}\right)^{-1.075}$ $V = \frac{\sigma}{m}$; when $0{,}05 \le V \le 0{,}3$

A2 Multiplication and addition of distributions

Multiplication of variable

When a statistical distribution is shifted by multiplying all values of a variable by a coefficient, γ, the mean and the standard deviation change by the same factor and the COV remains unchanged. The effect of multiplication on the parameters of different types of distributions is given below.

Normal distribution:	$\mu_{new} = \gamma\, \mu$	$\sigma_{new} = \gamma\, \sigma$	
Lognormal distribution:	$\mu_{\ln,new} = \ln(\gamma) + \mu_{\ln}$	$\sigma_{\ln,new} = \sigma_{\ln}$	
Gumbel distribution:	$\alpha_{new} = \alpha / \gamma$	$u_{new} = \gamma u$	
Weibull distributions	$\alpha_{new} = \alpha$	$\beta_{new} = \gamma\, \beta$	$\varepsilon_{new} = \gamma\, \varepsilon$

Sum of two distributions

Summation of two distributions is needed when two loads, say permanent and variable, are combined. Let's denote the combined function:

$$F_{12}(x, y) = P\{\xi + \eta \le z\} = \int_{x+y\le z} f(x, y)dxdy$$

This is obtained from two independent distributions as follows:

$$F_{12}(x) = \int_{x=-\infty}^{+\infty} f_1(r)F_2(x-r)dr$$

When discrete distributions are used, it is written as

$$F_{12,i} = \sum_{k=1}^{i} f_{1,k}\, F_{2,i-k+1}\, \Delta$$

For mean and variance of a combined distribution, we obtain

$$m = m_{\xi} + m_{\eta}$$
$$\sigma^2 = \sigma_{\xi}^2 + \sigma_{\eta}^2$$

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Title

Reliability analysis of timber structures

Abstract

As part of the European harmonisation of building codes, the determination of design values for loads and materials is important and is the motivation for this research. This report begins with a summary of the probabilistic basis of Eurocodes, analyses the strength distributions of wooden materials, demonstrates the effects of different distribution functions on the calculated safety level and shows some results of the applications of reliability analysis.

When the number of experiments allows, determination of the 5% fractile of strength should be based on the function fitting on the lower tail of the strength values, for instance 10%. All smooth functions fitted to tail data gave good estimates of the 5% fractile. When the 5% fractile was determined from a function fitted to all data, up to 5% error occurred (in one case 9%) when compared to a non-parametric estimate. Three-parameter Weibull distribution gave, in all calculated cases, the 5% fractile within an accuracy of ±3%.

The result of structural reliability analysis depends strongly on the load and strength distribution types used. When fitted functions are used in reliability analysis, it is essential that the fit is good in the lower tail area, the lowest values being most important. When fitted to the same data, a two-parametric Weibull distribution gives the most pessimistic prediction for the tail, with a normal distribution being next, and lognormal and three-parameter Weibull being the most optimistic. In an example, a two-parameter Weibull gave a failure probability 10 times higher than that of a three-parameter Weibull.

The analysis suggests that $\gamma_M = 1.2$ to 1.3 is reasonable for timber structures when $\gamma_G = 1.2$ and $\gamma_Q = 1.5$.

Keywords
wooden structures, reliability, construction, timber construction, strength, building code, design, plywood, Eurocode, failure

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