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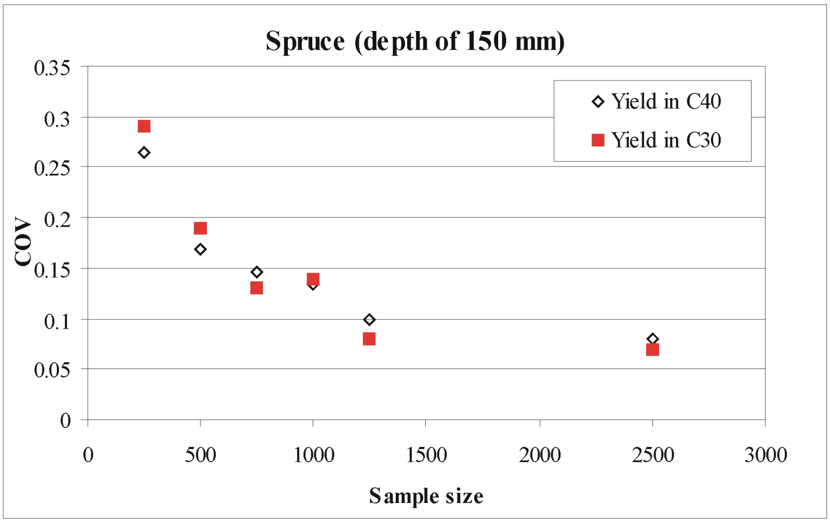

Goran Turk & Alpo Ranta-Maunus

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Abstract

Numerical simulation is used to analyse machine strength grading of sawn timber. Starting from correlations between grade determining properties and measured grading properties, sets of values of properties are generated and machine grading simulated including the determination of settings and application of the settings to another generated sample.

Results are obtained concerning the statistical effect of sample size to the accuracy of grading and yield to various grades. Also the lower tail of strength distribution of various grades was analysed with the conclusion that the highest grade has relatively better strength distribution than the lower grades.

Preface

This report is based on numerical calculations made by Dr. Goran Turk during his visit at VTT 1.4.–30.6.2003. The intention of the publication is to document results of various numerical exercises, also in cases when the applicability of results to practical grading situations is not obvious. Due to the limited duration of the stay in Finland, most of the practical applications follow afterwards and will be published separately.

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Contents

1. Introduction

Basic problem in understanding and development of strength grading of wood is related to the statistical nature of the strength of timber: there is no method which could predict the strength of a certain piece of wood. However, there are known correlations between strength and measurable physical properties. The consequence is that thousands of specimens need to be tested to establish settings for grading machines, and after having done it, there is still an uncertainty about the strength distribution of produced strength graded timber as well as on the yield of timber to different grades.

This report concerns grading of timber according to the forthcoming European standard [1]. Based on earlier experiments, correlations between grade determining properties, strength, modulus of elasticity and density, will be assumed, and numerical simulation of the standard grading procedure will be applied to larger populations than can be afforded in testing. Numerical procedure is needed because requirements concerning all three grade determining properties has to be fulfilled.

This method is expected to be useful in analysis of issues as follows:

- sensitivity of grading result to initial distributions of properties and to the effectiveness of the grading method

- sensitivity of grading result to sample size and to statistical methods used when settings of machine are determined

- yield to different grades when graded simultaneously to one or more grades

- form of lower tail of strength distribution of different grades.

2. Random generation of sample and population

It is assumed that grade determining parameters: strength f, modulus of elasticity E and density ρ, and observed grading parameter f_{model} are all normally distributed random variables. The characteristics of this random vector

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} f \\ E \\ \rho \\ f_{\text{model}} \end{bmatrix} \tag{1}$$

are presented by its mean value vector $\mathbf{m_Y}$ and its covariance matrix $\mathbf{\Sigma_Y}$

$$\mathbf{m_Y} = E[\mathbf{Y}] = \begin{bmatrix} m_{Y_1} \\ m_{Y_2} \\ m_{Y_3} \\ m_{Y_4} \end{bmatrix}$$
$$\mathbf{\Sigma_Y} = E\left[(\mathbf{y} - \mathbf{m_Y})(\mathbf{y} - \mathbf{m_Y})^T\right] = \begin{bmatrix} \sigma_{Y_1}^2 & \sigma_{Y_1,Y_2} & \sigma_{Y_1,Y_3} & \sigma_{Y_1,Y_4} \\ \sigma_{Y_1,Y_2} & \sigma_{Y_2}^2 & \sigma_{Y_2,Y_3} & \sigma_{Y_2,Y_4} \\ \sigma_{Y_1,Y_3} & \sigma_{Y_2,Y_3} & \sigma_{Y_3}^2 & \sigma_{Y_3,Y_4} \\ \sigma_{Y_1,Y_4} & \sigma_{Y_2,Y_4} & \sigma_{Y_3,Y_4} & \sigma_{Y_4}^2 \end{bmatrix} \tag{2}$$

The mean value vector as well as covariance matrix are estimates from a sample of real experimental results $y_{ik}, i = 1,\ldots,4, k = 1,\ldots,n$ (n is the sample size) by the following equations

$$\hat{m}_{Y_i} = \frac{\sum_{k=1}^{n} y_{ik}}{n}$$
$$\hat{\sigma}_{Y_i}^2 = \frac{\sum_{k=1}^{n} \left(y_{ik} - m_{Y_i}\right)^2}{n-1} \tag{3}$$
$$\hat{\sigma}_{Y_iY_j} = \frac{\sum_{k=1}^{n} \left(y_{ik} - m_{Y_i}\right)\left(y_{jk} - m_{Y_j}\right)}{n-1}$$

Two types of sample generation will be used later on: (1) generation of independent random samples, (2) generation of random samples from a given finite size population. The techniques to generate a finite size population and independent random sample are equal.

2.1 Generating a sample of normally distributed random vector

The basic idea for generating a sample of dependent normally distributed random variables is to generate a sample of independent normally distributed random variables and then use a linear transformation to obtain a sample of dependent random variables [2].

2.1.1 Linear transformation

The linear transformation of random vector $\mathbf{X}$, which is taken to be a set of independent random variables with zero mean and unit variances, is defined as

$$\mathbf{Y} = \mathbf{H}\,\mathbf{X} \tag{4}$$

where **H** is transformation matrix, which deforms the coordinate system.

A very important linear transformations are rotations which correspond to orthonormal transformation matrix **H**.

The variance-covariance matrix of random vector **X** is an identity matrix.

An important property of normally distributed random variables is that the linear transformation of normally distributed random variable is also normally distributed.

2.1.2 Generating a random vector with a given covariance matrix

Lets assume that the random vector **Y** has zero mean values. In this case the variance-covariance matrix is defined as follows

$$\mathbf{\Sigma} = E\left[\mathbf{Y}\,\mathbf{Y}^T\right], \tag{5}$$

where $E[.]$ denotes expected values. If we use Eqn. (4) in Eqn. (5), we obtain a useful relationship

$$\mathbf{\Sigma} = E\left[\mathbf{Y}\,\mathbf{Y}^T\right] = E\left[\mathbf{H}\,\mathbf{X}\,\mathbf{X}^T\mathbf{H}^T\right] = \mathbf{H}\,E\left[\mathbf{X}\,\mathbf{X}^T\right]\mathbf{H}^T, \tag{6}$$

Since the variance-covariance matrix of **X** is identity matrix $E\left[\mathbf{X}\,\mathbf{X}^T\right] = \mathbf{I}$, the Eqn. (6) reduces to

$$\mathbf{\Sigma} = \mathbf{H}\,\mathbf{H}^T, \tag{7}$$

We have to find such matrix **H**, for which the Eqn. (7) holds. Since the variance-covariance matrix is nonsingular symmetric matrix, the problem can easily be solved by Cholesky decomposition (see e.g. [3]).

2.1.3 Generating procedure

The generation of normally distributed random vector with known variance-covariance matrix is carried out by the following procedure:

1. Perform a Cholesky decomposition of variance-covariance matrix **Σ** of random vector **X** (determination of **H**).

2. Generate a set of independent random variables with standardized normal distribution (generation of random vector **X**).

3. Perform a transformation which transform independent variables **X** into randomvector **Y** with known variance-covariance matrix (use of equation **Y** = **HX**).

4. Perform an additional transformation to obtain a random vector **Z** with non-zero mean values (use of equation **Z** = **Y** + *E*[**Z**]).

3. Optimum grading

When settings of a grading machine are determined according to EN 14081, both non-destructive testing by the use of grading machine, and destructive testing for determination of the real grade determining properties are made for a representative sample. The first step in the analysis of results is the determination of the “real” grade of each specimen based on information of destructive testing: bending strength, modulus of elasticity and density. This grading is made in such a way that each specimen would be place in as high grade as possible, and is called therefore **optimum grading**. In our case a simultaneous grading to three grades was performed: C40, C30 and C18. The modified requirements for these grades according to prEN 14081 [1] are shown in Table 1.

Table 1. Modified requirements for grades C40, C30 and C18.

Grade	f_r [N/mm^2]	E_r [N/mm^2]	ρ_r [kg/m^3]
C40	40.0	13300	420
C30	26.8	11400	380
C18	16.1	8550	320

During the optimum grading determination we would like to grade as many pieces as possible to higher grades, with the following requirements (constraints):

1. the sample (grade) 5% percentile of the strength $f_{0.05}$ is higher than f_r,
2. the sample (grade) mean (average) of $\overline{E}$ is higher than E_r,
3. the sample (grade) 5% percentile of the density $\rho_{0.05}$ is higher than ρ_r,
4. the sample 5% percentile of the strength $f^a_{0.05}$ is higher than f_r for the sub-sample obtained just by ranking according to the strength,
5. the sample mean of $\overline{E}^a$ is higher than E_r for the sub-sample obtained just by grading according to the modulus of elasticity, and
6. the sample 5% percentile of the density $\rho^a_{0.05}$ is higher than ρ_r for the sub-sample obtained just by ranking according to the density.

A computer code (written in MATHEMATICA [4]) was prepared for automatic determination of optimum grading. In addition to these requirements (constraints) the objective function S was defined as

$$S = \frac{|f_{0.05} - f_r|}{f_r} + \frac{|\overline{E} - E_r|}{E_r} + \frac{|\rho_{0.05} - \rho_r|}{\rho_r} + \frac{|f^a_{0.05} - f_r|}{f_r} + \frac{|\overline{E}^a - E_r|}{E_r} + \frac{|\rho^a_{0.05} - \rho_r|}{\rho_r} \quad (8)$$

It is our goal to find such settings of limits used in optimum grading determination that the function *S* reaches its minimum. These set of limits gives the optimum grading of the sample or population.

The optimization procedure itself utilizes a relatively simple step-by-step bisection-like method, in which the limits for any of grading determining parameters are decreased by a certain step-size if the difference (e.g. $f_{0.05} - f_r$) is positive and increased if it is negative. The increasing is four times faster than decreasing. When all three limits (for *f*, *E* and ρ) are as low as possible with the conditions fulfilled, the step-size is halved and the procedure is repeated. This simple method, which emulates the manual determination of optimal grading, gives very accurate limits for the grade determining parameters. Thus, the obtained grading is almost certainly the optimal one.

3.1 Characteristic value determination

One of the tasks that has to be repeated frequently is the determination of characteristic value (e.g. 5% percentile) from the sample. There are several options of how to deal with this problem.

One of them is to presume the distribution of the random variable which is represented by the sample. We can use either method of moments or maximum likelihood method to determine the characteristic value. In case the sample is very small a special consideration should be put into the fact the moments themselves are not determined exactly, only the estimates based on the sample are known.

Alternatively, the characteristic value can be determined by the use of order statistics. E.g., let's determine the 5% percentile y_k from the sample $y_i = 1,\ldots,n$ where *n* is sample size. The sample is sorted so that $y_i \le y_{i+1}$. The sample element y_j which satisfies the condition

$$j = \lfloor 0.05n \rfloor, \quad (9)$$

is the highest among the elements which are below the characteristic value. The notation $j = \lfloor x \rfloor$ denotes the floor of the number *x*, i.e. the greatest integer less than or equal to *x*. The characteristic value is finally determined by the equation

$$y_k = y_j + (0.05n - j)(y_{j+1} - y_j), \quad (10)$$

which represents the linear interpolation between the two values which embrace the characteristic value. If the sample size is lower than 1/0.05 = 20 the equation (10) can't be used. The reliable determination of characteristic value is in that case difficult. An approximate formula can be used

$$y_k = 0.05n\, y_1$$

In grading procedure a second method based on order statistics, has been used.

4. Machine grading

Basis of machine grading lies on settings which will be given based on comparison of machine readings and real grade determining parameters. Here is a procedure introduced for automatic determination of settings according to prEN 14081-2. The three limits for grades C40, C30 and C18 will be set so that the following conditions are fulfilled:

1. the sample (grade) 5% percentile of the strength $f_{0.05}$ is higher than f_r,
2. the sample (grade) mean (average) of modulus of elasticity $\overline{E}$ is higher than E_r,
3. the sample (grade) 5% percentile of the density $\rho_{0.05}$ is higher than ρ_r,
4. none of the cells in the global cost matrix which indicate wrongly upgraded is grater than 0.2,
5. the number of rejected pieces is grater or equal to 0.5% of the total number of pieces in the sample.

Similarly as for optimum grading determination, an automatic optimization procedure was developed in machine settings determination too. There are several options for the objective function: one can choose to use a sum of all terms in global cost matrix (*GCM*) to be the objective function

$$S = \sum_{i=1}^{m} \sum_{j=1}^{m} GCM(i, j), \tag{11}$$

where *m* is the number of grades used in this particular grading. The minimum value of *S* is sought. In this case we obtained relatively low values for yield in higher classes. The other two options that were tried have greater yields in higher classes:

- Objective function is the sum of all terms in global cost matrix which indicate wrongly downgraded pieces (upper triangular part only)

$$S = \sum_{i=1}^{m} \sum_{j=i+1}^{m} GCM(i, j), \tag{12}$$

The minimum of *S* is sought.

- Objective function is the weighted number of pieces assigned to a particular grade. In our case the following objective function is used:

$$S = 9n_{c40} + 3n_{c30} + n_{c18}\ . \qquad (13)$$

The latter objective function is used for final evaluations.

Since the variance of the assigned values of machine grading parameter f_{model} is relatively high and its correlation to the actual grade determining parameter f and specifically to the density ρ low, the relationship between machine settings limits and objective function is not clear. Therefore we start from random choices of settings, and among the cases that fulfil the requirements the one with the highest objective function is chosen. From that point the machine settings limits were lowered as much as possible so that the requirements are fulfilled and the objective function is maximized. This procedure usually gives relatively high yields in higher grades.

5. Numerical examples

5.1 Old sample generation

Old sample title refers to an earlier paper [5]. Here, the generation of samples has been repeated by using techniques described in this publication, whereas the earlier version was made with Excel and using smaller sample sizes and lesser repetitions of generation. In both cases the grade determining properties are based on an earlier research on spruce (45 x 150) grown in Finland [6].

A random vector of four dependent random variables ($f, E, \rho, f_{\text{model}}$) is considered here. We assume that random variables E, ρ and f_{model} are linear functions of f, which is a normally distributed random variable with mean m_f and standard deviation σ_f

$$\begin{aligned} E &= a + b\,f + X\,, \\ \rho &= c + d\,f + Y\,, \\ f_{\text{model}} &= f + Z\,, \end{aligned} \tag{14}$$

where a, b, c and d are known deterministic constants, and X, Y and Z are normally distributed random variables with zero mean ($m_X = m_Y = m_Z = 0$) and standard deviations σ_X, σ_Y and σ_Z.

In this case we can derive the mean values, variances and covariances in terms of nine parameters $m_f, \sigma_f, a, b, c, d, \sigma_X, \sigma_Y$ and σ_Z.

mean values: $m_E = a + b\,m_f$,

$m_\rho = c + d\,m_f$,

variances: $\sigma_E^2 = b^2\sigma_f^2 + \sigma_X^2$,

$\sigma_\rho^2 = d^2\sigma_f^2 + \sigma_Y^2$,

$\sigma_{f_{\text{model}}}^2 = \sigma_f^2 + \sigma_Z^2$,

covariances: $\sigma_{fE} = b\sigma_f^2$,

$\sigma_{f\rho} = d\sigma_f^2$,

$\sigma_{E\rho} = bd\sigma_f^2$,

$\sigma_{f_{\text{model}}f} = \sigma_f^2$,

$\sigma_{f_{\text{model}}E} = b\sigma_f^2$,

$\sigma_{f_{\text{model}}\rho} = d\sigma_f^2$.

From these relations is easy to see, that correlation coefficients are

$$r_{fE} = \frac{\sigma_{fE}}{\sigma_f \sigma_E} = \frac{b\sigma_f}{\sqrt{b^2\sigma_f^2 + \sigma_X^2}},$$

$$r_{f\rho} = \frac{\sigma_{f\rho}}{\sigma_f \sigma_\rho} = \frac{d\sigma_f}{\sqrt{d^2\sigma_f^2 + \sigma_Y^2}},$$

$$r_{E\rho} = \frac{\sigma_{E\rho}}{\sigma_E \sigma_\rho} = \frac{bd\sigma_f^2}{\sqrt{b^2\sigma_f^2 + \sigma_X^2}\sqrt{d^2\sigma_f^2 + \sigma_Y^2}},$$

$$r_{f_{\text{model}} f} = \frac{\sigma_{f_{\text{model}} f}}{\sigma_{f_{\text{model}}} \sigma_f} = \frac{\sigma_f}{\sqrt{\sigma_f^2 + \sigma_Z^2}},$$

$$r_{f_{\text{model}} E} = \frac{\sigma_{f_{\text{model}} E}}{\sigma_{f_{\text{model}}} \sigma_E} = \frac{b\sigma_f^2}{\sqrt{b^2\sigma_f^2 + \sigma_X^2}\sqrt{\sigma_f^2 + \sigma_Z^2}},$$

$$r_{f_{\text{model}} \rho} = \frac{\sigma_{f_{\text{model}} \rho}}{\sigma_{f_{\text{model}}} \sigma_\rho} = \frac{d\sigma_f^2}{\sqrt{d^2\sigma_f^2 + \sigma_Y^2}\sqrt{\sigma_f^2 + \sigma_Z^2}}.$$

The following values were chosen (or estimated from experimental data) to be:

$$m_f = 45.2, \quad \sigma_f = 11.4, \quad a = 4820, \quad b = 165, \quad \sigma_X = 1400,$$
$$c = 362, \quad d = 2.03, \quad \sigma_Y = 35, \quad \sigma_Z = 15,\ 8.5 \text{ or } 5.5.$$

Standard deviation of random variable Z (σ_Z) depends of the quality of grading. The lowest value of 5.5 corresponds to excellent grading, whereas the highest value of 15 corresponds to poor grading.

The mean values vector and variance-covariance matrix in the case of poor quality grading (σ_Z = 15) are

$$\mathbf{m}_\mathrm{Y} = \begin{bmatrix} 45.2 \\ 12278 \\ 453.756 \\ 45.2 \end{bmatrix} \qquad \boldsymbol{\Sigma}_\mathrm{Y} = \begin{bmatrix} 129.96 & 21443.4 & 263.819 & 129.96 \\ 21443.4 & 5.49816\cdot 10^6 & 43530.1 & 21443.4 \\ 263.819 & 43530.1 & 1760.55 & 263.819 \\ 129.96 & 21443.4 & 263.819 & 354.96 \end{bmatrix}$$

In the cases of good and excellent grading, only the variance of the fourth random variable changes

poor grading	$\text{var}\left[f_{\text{model}}\right] = 354.96$,
good grading	$\text{var}\left[f_{\text{model}}\right] = 202.21$,
excellent grading	$\text{var}\left[f_{\text{model}}\right] = 160.21$.

The corresponding correlation matrix for all three qualities of grading are:

poor grading $$\mathbf{R} = \begin{bmatrix} 1 & 0.802195 & 0.551539 & 0.605083 \\ 0.802195 & 1 & 0.442442 & 0.485395 \\ 0.551539 & 0.442442 & 1 & 0.333727 \\ 0.605083 & 0.485395 & 0.333727 & 1 \end{bmatrix},$$

good grading $$\mathbf{R} = \begin{bmatrix} 1 & 0.802195 & 0.551539 & 0.801685 \\ 0.802195 & 1 & 0.442442 & 0.643108 \\ 0.551539 & 0.442442 & 1 & 0.442161 \\ 0.801685 & 0.643108 & 0.442161 & 1 \end{bmatrix},$$

excellent grading $$\mathbf{R} = \begin{bmatrix} 1 & 0.802195 & 0.551539 & 0.900658 \\ 0.802195 & 1 & 0.442442 & 0.722504 \\ 0.551539 & 0.442442 & 1 & 0.496748 \\ 0.900658 & 0.722504 & 0.496748 & 1 \end{bmatrix}.$$

Different sample size for machine settings were chosen: 250, 500, 750, 1000, 1250 and 2500. Machine settings determination was repeated 100 times with independent samples. The results for mean machine settings parameters are shown in Table 2. Because all generations of values did not allow grading to C40, the number of non-zero generations for which a non-zero yield to C40 was resulted, is given, as also mean values of settings if all zero-yield-to C40-cases are omitted.

Table 2. Mean machine settings parameters for different sample sizes.

Poor grading $\sigma_Z = 15$								
Sample size	Number of nonzero yield in C40		**Include all**			**Include only nonzero yield in C40**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
250	15/100	Mean	93.125	38.100	8.393	66.750	42.267	8.250
		St. dev.	12.597	9.592	0.731	3.432	9.497	0.000
500	29/100	Mean	93.210	38.660	8.275	69.698	41.466	8.250
		St. dev.	16.196	7.280	0.250	4.557	7.655	0.000
750	48/100	Mean	89.553	38.043	8.305	71.271	41.089	8.250
		St. dev.	18.524	7.539	0.550	5.226	6.489	0.000
1000	57/100	Mean	87.068	37.555	8.250	72.303	40.039	8.250
		St. dev.	17.888	6.501	0.000	4.334	6.147	0.000
1250	56/100	Mean	87.978	38.263	8.250	73.929	39.821	8.250
		St. dev.	16.799	5.740	0.000	5.457	5.583	0.000
2500	62/100	Mean	90.158	37.205	8.250	78.250	37.798	8.250
		St. dev.	16.277	3.667	0.000	5.726	3.679	0.000

Good grading $\sigma_Z = 8.5$								
Sample size	Number of nonzero yield in C40		**Include all**			**Include only nonzero yield in C40**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
250	61/100	Mean	67.510	37.273	10.498	56.648	39.557	10.135
		St. dev.	14.686	6.251	2.137	4.987	6.148	1.948
500	76/100	Mean	66.073	36.588	9.728	59.013	37.447	9.576
		St. dev.	13.544	5.395	1.656	5.031	5.449	1.630
750	87/100	Mean	64.708	36.135	9.823	60.649	36.670	9.782
		St. dev.	11.664	4.479	1.664	4.725	4.402	1.591
1000	90/100	Mean	64.370	35.153	9.953	61.472	35.472	9.894
		St. dev.	9.757	4.408	1.597	4.415	4.364	1.598
1250	92/100	Mean	63.713	35.400	9.700	61.272	35.599	9.720
		St. dev.	9.315	3.838	1.586	4.342	3.908	1.593
2500	99/100	Mean	62.618	35.480	8.983	62.242	35.533	8.990
		St. dev.	5.779	2.545	0.855	4.419	2.502	0.856

Several things could be concluded from the results presented in Table 2:

- The number of samples which give non-zero yield in the highest class C40 is greater if the sample size is higher. However, in the case of poor grading the sample size

1000, 1250 and 2500 have approx. same number of non-zero yield in C40 samples, and in the case of good grading, the sample size of 750 have almost the same number of non-zero yield in C40 samples.

- If we take only results for non-zero yield in C40, we obtain somehow surprising result: the average limit for C40 is lower for larger samples, which will result in lower yield in the highest class C40. This result is due to the fact, that in the case of smaller samples there is more chance to have a zero yield in C40, but the remaining samples which have a yield in C40 have better chance to be "good" samples, which will result in a larger (and too optimistic) yield in C40. Obviously, it is not right to take only those samples with non-zero yield in consideration, since the result is clearly biased.

- The limit for class C30 is lower if the sample size is larger. As a result we may expect higher combined yield in C40 and C30. However, the sample sizes larger than 1000 do not contribute a lot to the result.

The values of grading settings obtained by 100 repetitions were applied on the same population of 2000 and 10000 pieces. The results are shown in Appendix Table 1.

With this calculation the yields in individual classes are determined. The ratio between 5% and 0.5% percentile of strength is determined for all three classes. In the case there were not enough pieces in a particular grade, 0.5% percentile has not been determined.

5.2 Sample generation based on descriptive statistics

Based on the available data of 589 pieces of spruce with the depth of 150 mm, 1508 pieces of spruce with various depth and 1995 pieces of spruce and pine with variable depth [6], the descriptive statistics were determined. It has to be noted, that in descriptive statistics E_{mach} related to bending type of grading machine is used.

Descriptive statistics may be summarized in the mean vector **m** and covariance matrix Σ

- spruce (150 mm depth)

$$\mathbf{m} = \begin{bmatrix} 45.247 \\ 13003.17 \\ 448.00 \\ 9495.9 \end{bmatrix} \qquad \mathbf{\Sigma} = \begin{bmatrix} 129.906 & 21759.6 & 263.13 & 10740.7 \\ 21759.6 & 5.98507 \cdot 10^6 & 66481.3 & 2.47506 \cdot 10^6 \\ 263.13 & 66481.3 & 1630.76 & 37881.2 \\ 10740.7 & 2.47506 \cdot 10^6 & 37881.2 & 1.48582 \cdot 10^6 \end{bmatrix}$$

- spruce (all depths)

$$\mathbf{m} = \begin{bmatrix} 43.103 \\ 12430.4 \\ 451.33 \\ 8854.92 \end{bmatrix} \qquad \mathbf{\Sigma} = \begin{bmatrix} 136.537 & 20877. & 233.366 & 14475 \\ 20877. & 6.06694 \cdot 10^6 & 61648.3 & 3.29938 \cdot 10^6 \\ 233.366 & 61648.3 & 1923.18 & 49505.5 \\ 14475 & 3.29938 \cdot 10^6 & 49505.5 & 4.35707 \cdot 10^6 \end{bmatrix}$$

- spruce and pines

$$\mathbf{m} = \begin{bmatrix} 42.887 \\ 12081.08 \\ 460.10 \\ 8867.82 \end{bmatrix} \qquad \mathbf{\Sigma} = \begin{bmatrix} 150.823 & 25321.1 & 296.318 & 14943.1 \\ 25321.1 & 7.04315 \cdot 10^6 & 65768.3 & 3.43771 \cdot 10^6 \\ 296.318 & 657683 & 2316.02 & 51722.1 \\ 14943.1 & 3.43771 \cdot 10^6 & 51722.1 & 3.8163 \cdot 10^6 \end{bmatrix}$$

Correlations between these variables are more evident in correlation matrices **R**:

spruce (150 mm depth)
$$\mathbf{R} = \begin{bmatrix} 1 & 0.78037 & 0.57169 & 0.77310 \\ 0.78037 & 1 & 0.67293 & 0.82998 \\ 0.57169 & 0.67293 & 1 & 0.76957 \\ 0.77310 & 0.82998 & 0.76957 & 1 \end{bmatrix}$$

spruce (all depths)
$$\mathbf{R} = \begin{bmatrix} 1 & 0.72537 & 0.45541 & 0.59347 \\ 0.72537 & 1 & 0.57072 & 0.64173 \\ 0.45541 & 0.57072 & 1 & 0.54081 \\ 0.59347 & 0.64173 & 0.54081 & 1 \end{bmatrix}$$

spruce and pine
$$\mathbf{R} = \begin{bmatrix} 1 & 0.77690 & 0.50136 & 0.62285 \\ 0.77690 & 1 & 0.51495 & 0.66308 \\ 0.50136 & 0.51495 & 1 & 0.55015 \\ 0.62285 & 0.66308 & 0.55015 & 1 \end{bmatrix}$$

If we compare these correlation matrices with correlation matrices used in "Old sample generation" we may observe that correlation between density and modulus of elasticity is higher than one assumed before, whereas the correlation between density and strength depends on which sample we are observing. It is higher if only spruce with 150 mm depth is considered and it is lower if all spruce specimens or all spruce and pine specimens are considered. The correlation between strength and modulus of elasticity is similar to the one used in "old sample generation".

The correlations between strength, modulus of elasticity, density and f_{model} can't be directly compared to the correlation between strength, modulus of elasticity, density and

E_{mach}. However we may observe that the correlation between f_{model} and modulus of elasticity as well as f_{model} and density were probably underestimated. This is particularly true in the case of "poor grading". The latter conclusions should be verified by computing f_{model} from complete data set and recalculating descriptive statistics.

In further analysis only the data for spruce with 150 mm depth and all spruce data are used. The covariance matrix in the case of combined spruce and pine data is very similar to the covariance matrix of all spruce data. Therefore similar results could be expected.

Different sample size for machine settings were chosen: 125, 250, 500, 750, 1000, 1250 and 2500. Machine settings determination was repeated 100 times with independent samples.

The results for mean machine settings parameters are shown in Table 3.

Table 3. Mean machine settings for different sample sizes.

Spruce with depth of 150 mm

Samle size	Number of nonzero yield in C40		**Include all**			**Include only nonzero yield in C40**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
125	61/100	Mean	10923.0	8986.1	7830.6	9897.0	8940.9	7753.4
		St. dev.	1660.7	962.9	1066.2	1293.3	1195.3	1222.5
250	100/100	Mean	9944.4	8211.6	6767.4			
		St. dev.	316.6	403.8	593.5			
500	100/100	Mean	9904.8	7886.7	6478.9			
		St. dev.	205.5	260.2	263.1			
750	100/100	Mean	9884.9	7821.0	6467.9			
		St. dev.	169.2	224.1	299.4			
1000	100/100	Mean	9871.8	7919.5	6883.3			
		St. dev.	158.7	304.2	454.4			
1250	100/100	Mean	9892.1	7734.9	6463.0			
		St. dev.	115.5	194.2	235.9			
2500	100/100	Mean	9857.6	7688.1	6478.6			
		St. dev.	96.0	140.4	210.6			

All spruce

Samle size	Number of nonzero yield in C40		Include all			Include only nonzero yield in C40		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
125	4/100	Mean	14126.2	8700.4	6122.8	10770.3	8439.5	3887.8
		St. dev.	1043.1	917.1	1998.3	236.0	929.0	1056.6
250	35/100	Mean	13468.6	7412.4	4102.0	11072.2	7704.7	3932.3
		St. dev.	1854.0	646.1	989.3	443.7	622.1	1004.0
500	60/100	Mean	12862.1	7143.8	3977.2	11560.1	7250.4	3992.2
		St. dev.	1700.0	488.2	777.0	501.9	473.4	794.9
750	67/100	Mean	12855.6	6976.8	3858.4	11717.6	7065.4	3863.6
		St. dev.	1728.6	388.7	743.0	496.0	380.6	711.7
1000	82/100	Mean	12459.0	6906.5	3902.1	11823.6	6987.2	3885.6
		St. dev.	1453.3	427.4	598.7	538.9	411.3	613.6
1250	79/100	Mean	12645.7	6886.6	3774.0	11812.8	6955.9	3725.7
		St. dev.	1731.9	398.7	526.2	553.8	402.1	497.2
2500	88/100	Mean	12344.5	6839.4	3739.8	11868.1	6873.0	3720.9
		St. dev.	1383.1	274.4	527.7	465.7	254.1	532.2

From Table 3 we can see the importance of separate grading of pieces of 150 mm. In that case the nonzero-yield in the highest grade C40 was 100% for all sample sizes but the smallest one (125 pieces only). The values of machine settings do not differ much for different sample size from 250 to 2500 pieces.

On the other hand if the data from different depth of the pieces were combined lower correlations between grading parameters and machine value results in lower yield in the highest class. This is specifically true for smaller sample size. Therefore, in this case it is beneficiary to have a larger sample size.

The values of grading settings obtained by 100 repetitions were applied on the same population of 10000 pieces. The results are shown in Appendix Table 2.

With this calculation the yields in individual classes are determined. The ratio between 5% and 0.5% percentile of strength is determined for all three classes. In the case there were not enough pieces in a particular grade, 0.5% percentile has not been determined.

The results clearly show the difference between the effectiveness of machine grading the spruce with the depth of 150 mm and all spruce data.

In the case of 150 mm deep spruce specimens the machine grading was quite successful. The yield in C40 was approx. 36% and in C30 between 50 and 55% (compared to optimum grade yield of 62% and 29% in C40 and C30, respectively). The results slightly improve if the sample size is increased from 250 to 2500 (see Fig. 1).

However there is an evident improvement in COV of yield which decrease form above 0.25 in the case of n = 250 to below 0.1 in the case of n = 1250 (see Fig. 2). This reduction in COV results in much narrower confidence interval for the cases when the machine grading were determined from only few samples.

Other parameters don't change for different sample sizes, e.g. the ratio $f_{0.005}/f_{0.05}$ is virtually constant for all sample sizes.

In the case of all spruce data the improvement can be seen in a number of samples that have non-zero yield in C40 (see Fig. 3). Due to low correlation between grade determining properties and grading parameter it may easily happen that the machine settings can't be set so that a yield in C40 is obtained. If the case of smaller samples the possibility for this is even higher as it was clearly shown by this example. All other parameters were not effected by the sample size.

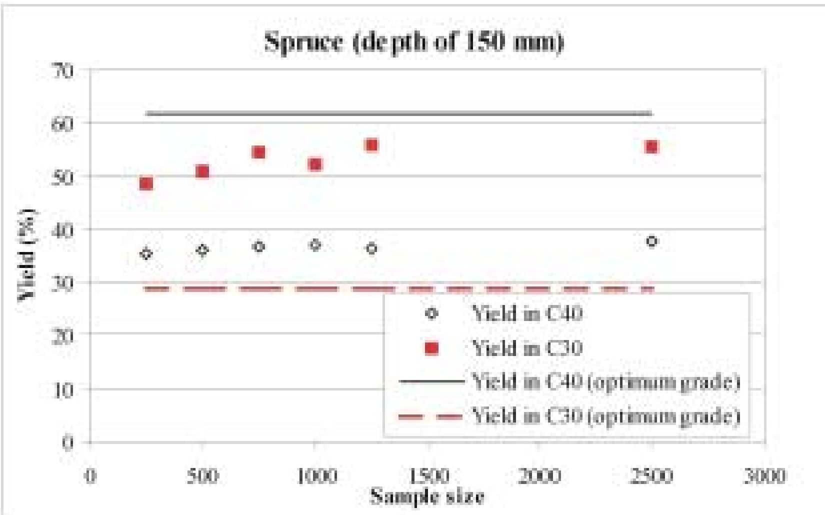

Figure 1. Average yield in C40 and C30 for the generated samples of spruce with 150 mm depth.

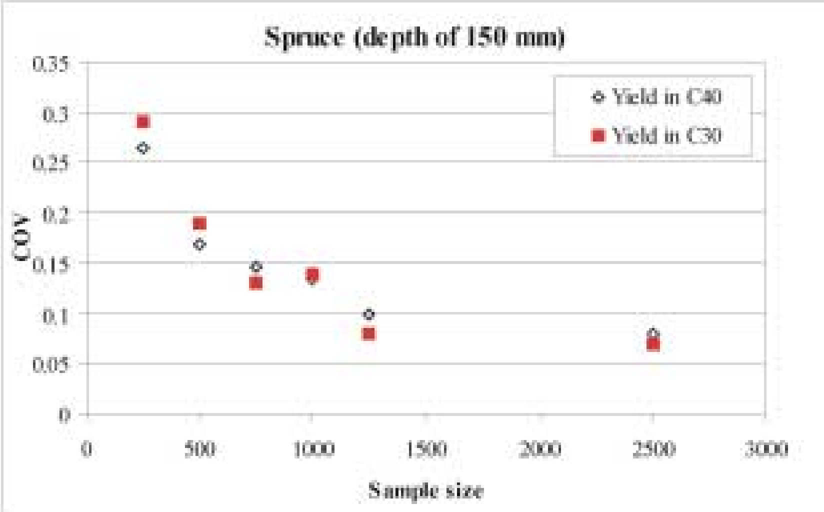

Figure 2. Coefficient of variation for yield in C40 and C30 for the generated samples of spruce with 150 mm depth.

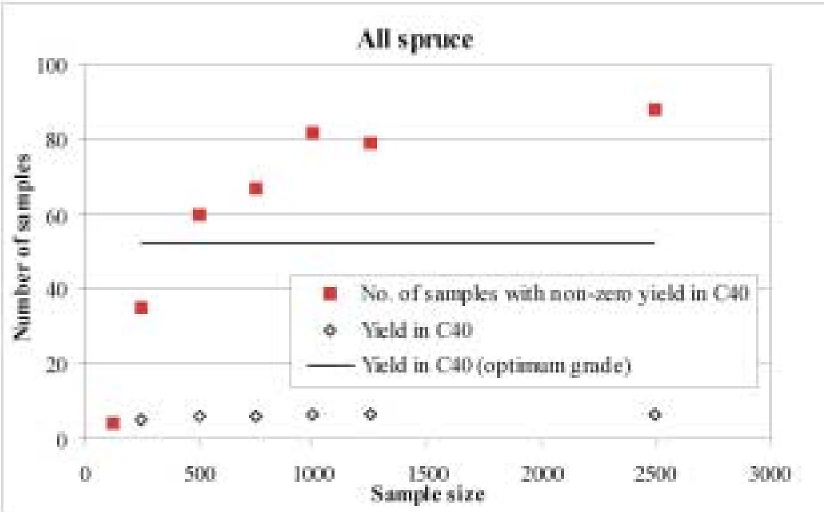

Figure 3. Number of samples with non-zero yield in C40 and average yield in C40 and C30 for the generated samples of all spruce.

5.3 Combined grading

Our final objective is to improve the machine grading. One of the procedures which may improve the machine grading is to include additional indicating property in machine grading.

Therefore, we assumed that in addition to modulus of elasticity the density of the beams are estimated by the machine. Since we don't have any data about the covariances between the density evaluated by machine ρ_m and others parameters we assumed, that ρ_m depends on actual density ρ and can be generated by the following equation

$$\rho_m = \rho + W \, , \tag{15}$$

where W is normally distributed random variable with zero mean and standard deviation ρ_W and represent the error in machine evaluation of the density. As a result we obtain a new mean vector and covariance matrix in which the moments of a new parameter is added. In the case of spruce specimens of various depth, we have

- spruce (150 mm depth)

$$\mathbf{m} = \begin{bmatrix} 45.247 \\ 13003.17 \\ 448.00 \\ 9495.9 \\ 448.00 \end{bmatrix}$$

$$\mathbf{\Sigma} = \begin{bmatrix} 129.906 & 21760 & 263.13 & 10741 & 263.13 \\ 21760 & 5.98507 \cdot 10^6 & 66481 & 2.47506 \cdot 10^6 & 66481 \\ 263.13 & 66481 & 1630.76 & 37881 & 1630.76 \\ 10741 & 2.47506 \cdot 10^6 & 37881 & 1.48582 \cdot 10^6 & 37881 \\ 263.13 & 66481 & 1630.76 & 37881 & 1630.76 + \sigma_W^2 \end{bmatrix}$$

Correlations between these variables are more evident in correlation matrices **R**:

$$\mathbf{R} = \begin{bmatrix} 1 & 0.78037 & 0.57169 & 0.77310 & 0.55493 \\ 0.78037 & 1 & 0.67293 & 0.82998 & 0.65320 \\ 0.57169 & 0.67293 & 1 & 0.76957 & 0.97068 \\ 0.77310 & 0.82998 & 0.76957 & 1 & 0.74700 \\ 0.55493 & 0.65320 & 0.97068 & 0.74700 & 1 \end{bmatrix}$$

- spruce (all depths)

$$\mathbf{m} = \begin{bmatrix} 43.103 \\ 12430.4 \\ 451.33 \\ 8854.92 \\ 451.33 \end{bmatrix}$$

$$\mathbf{\Sigma}=\begin{bmatrix} 136.537 & 20877. & 233.366 & 14475. & 233.366 \\ 20877. & 6.06694\cdot 10^{6} & 61648.3 & 3.29938\cdot 10^{6} & 61648.3 \\ 233.366 & 61648.3 & 1923.18 & 49505.5 & 1923.18 \\ 14475. & 3.29938\cdot 10^{6} & 49505.5 & 4.35707\cdot 10^{6} & 49505.5 \\ 233.366 & 61648.3 & 1923.18 & 49505.5 & 1923.18+\sigma_W^2 \end{bmatrix}$$

Correlations between these variables are more evident in correlation matrices **R**:

$$\mathbf{R}=\begin{bmatrix} 1 & 0.72537 & 0.45541 & 0.59347 & 0.44401 \\ 0.72537 & 1 & 0.57072 & 0.64173 & 0.55644 \\ 0.45541 & 0.57072 & 1 & 0.54081 & 0.97497 \\ 0.59347 & 0.64173 & 0.54081 & 1 & 0.52728 \\ 0.44401 & 0.55644 & 0.97497 & 0.52728 & 1 \end{bmatrix}$$

In last equations the following relationships between the moments are used (they can easily be derived)

$$m_{\rho_m}=m_{\rho},$$
$$[\rho]+\mathrm{var}[W],$$
$$\mathrm{cov}[\rho_m,F]=\mathrm{cov}[\rho,F],$$
$$\mathrm{cov}[\rho_m,E]=\mathrm{cov}[\rho,E],$$
$$\mathrm{cov}[\rho_m,\rho]=\mathrm{var}[\rho],$$
$$\mathrm{cov}[\rho_m,E_{\mathrm{mach}}]=\mathrm{cov}[\rho,E_{\mathrm{mach}}].$$

In this analysis the data for spruce of the depth of 150 mm and all spruce data are used. Different sample size for machine settings were chosen: 125, 250, 500, 750, 1000, 1250 and 2500. Machine settings determination was repeated 100 times with independent samples. The results for mean machine settings parameters are shown in Table 4 and 5.

Optimum grading was obtained by the same procedure as in the previous cases. In machine settings determination, besides the limits for E_{mach} a set of limits for ρ_m was sought. Thus, for each grade two limits are given, a piece is selected if both modulus of elasticity and density equal or exceed the appropriate limits. The search procedure is similar as in the previous cases - a combination of random search and final lowering the limits so that the required conditions are fulfilled.

From the data we may conclude that the additional indicating property in machine grading is beneficiary for the case of all spruce data. In this case the number of non-zero yield in the highest class C40 increases considerably. This is specifically true if small sample size is chosen (125 to 500 pieces) (see Fig. 4). From the Appendix A Table 4 we can see that yield in C40 increases from about 6% in ordinary grading to above 8% in combined grading. Both values are quite low with respect to the value of optimum grading (54.4%). Also we can see that the overall yield in grades C40 and C30 does not change if the combined grading is used: it is approx. 82%. This value is relatively close to the optimum grading value (86.7%). (Appendix Table 3)

In the case of spruce specimens with the depth of 150 mm the differences between ordinary grading and combined grading are negligible.

Table 4. Mean machine settings parameters for different sample sizes (spruce - depth of 150 mm).

Combined grading (modulus of elasticity and density)								
Spruce (150 mm))								
Sample size	Number of nonzero yield in C40		**Include all**					
			Machine modulus of elasticity			**Machine density**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
125	67/100	Mean	10768	8949	7881	402.8	350.3	183.6
		St. dev.	1365	483	610	52.8	46.2	56.8
250	98/100	Mean	9900	8145	6724	395.6	333.5	162.5
		St. dev.	547	500	553	45.0	41.3	71.5
500	100/100	Mean	9844	7818	6545	399.5	326.8	170.9
		St. dev.	234	315	375	39.5	38.7	68.9
750	100/100	Mean	9875	7722	6493	393.4	322.8	166.7
		St. dev.	168	342	338	33.4	36.5	59.4
1000	100/100	Mean	9864	7676	6573	392.5	324.6	157.6
		St. dev.	146	234	272	37.2	34.0	64.6
1250	100/100	Mean	9856	7663	6540	395.6	322.6	166.4
		St. dev.	147	243	237	35.8	36.9	62.6
2500	100/100	Mean	9855	7636	6474	393.4	325.3	161.3
		St. dev.	112	231	228	35.4	32.1	66.8

Grading to modulus of elasticity only								
Spruce (150 mm)								
Sample size	Number of nonzero yield in C40		**Include all**					
			Machine modulus of elasticity			**Machine density**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
125	60/100	Mean	11035	9054	7867			
		St. dev.	1337	410	616			
250	99/100	Mean	9978	8218	6778			
		St. dev.	410	382	588			
500	100/100	Mean	9913	7911	6530			
		St. dev.	216	262	362			
750	100/100	Mean	9895	7800	6467			
		St. dev.	179	257	315			
1000	100/100	Mean	9888	7725	6582			
		St. dev.	160	203	274			
1250	100/100	Mean	9874	7718	6536			
		St. dev.	140	197	248			
2500	100/100	Mean	9868	7695	6494			
		St. dev.	107	150	240			

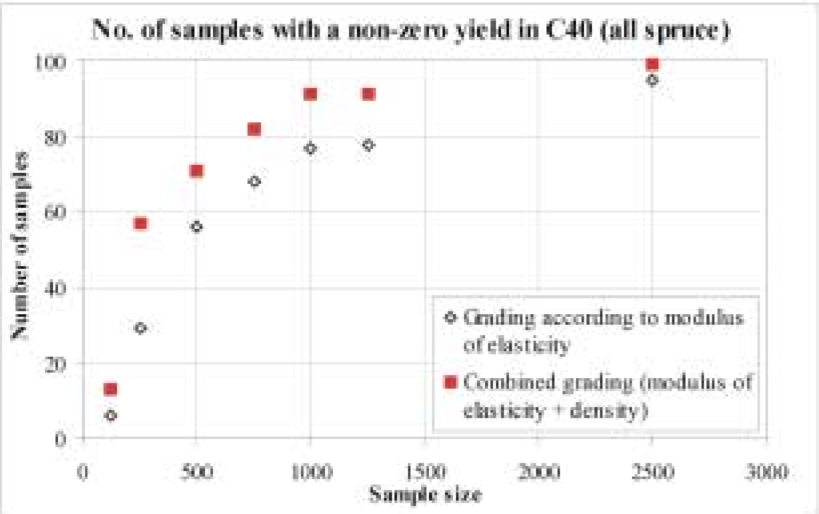

Figure 4. Number of samples with non-zero yield in C40 for two types of grading.

Table 5. Mean machine settings parameters for different sample sizes.

Combined grading (modulus of elasticity and density)
All spruce

Sample Size	Number of nonzero yield in C40		**Include all**					
			Machine modulus of elasticity			**Machine density**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
125	13/100	Mean	13592	8358	5714	357.1	353.6	169.2
		St. dev.	1598	1049	2067	83.6	49.6	72.8
250	57/100	Mean	12368	7373	4098	398.1	337.4	131.5
		St. dev.	2104	616	957	74.2	44.1	76.0
500	71/100	Mean	12324	7049	3828	405.2	322.0	138.2
		St. dev.	1967	493	735	58.5	42.3	75.1
750	82/100	Mean	11960	6975	3867	420.8	318.6	138.3
		St. dev.	1753	501	753	57.9	36.6	73.6
1000	91/100	Mean	11789	6961	3907	413.5	320.7	155.0
		St. dev.	1416	373	613	54.1	38.6	76.9
1250	91/100	Mean	11827	6940	3816	412.5	323.9	141.9
		St. dev.	1348	347	581	55.2	32.8	74.9
2500	99/100	Mean	11696	6847	3656	413.4	324.0	146.1
		St. dev.	752	258	392	53.2	34.2	69.1

Grading to modulus of elasticity only
All spruce

Sample Size	Number of nonzero yield in C40		**Include all**					
			Machine modulus of elasticity			**Machine density**		
		Grades	**C40**	**C30**	**C18**	**C40**	**C30**	**C18**
125	6/100	Mean	13964	8565	5740			
		St. dev.	1189	1041	2096			
250	29/100	Mean	13679	7354	4187			
		St. dev.	1720	604	956			
500	56/100	Mean	13204	7068	3928			
		St. dev.	1918	510	867			
750	68/100	Mean	12867	6926	3905			
		St. dev.	1718	478	757			
1000	77/100	Mean	12626	6936	3902			
		St. dev.	1640	378	584			
1250	78/100	Mean	12653	6932	3846			
		St. dev.	1678	355	555			
2500	95/100	Mean	12170	6850	3673			
		St. dev.	1004	276	414			

5.4 The influence of dependent sampling

In real machine settings determination we will not be able to have a large number of independent samples. In fact we may have only a relatively large population (e.g. N = 100) of all available measurements from which we may draw several samples. These samples are obviously not independent since the population from which we take samples is not infinite.

In this numerical example we illustrate the effect of dependent sampling, i.e. sampling from a finite population. First we generate the population of 1000 pieces (N = 100). Then we randomly select samples from this population. In this case it does not make any sense to analyse larger samples, thus only samples of size 125, 250, 500 and 750 have been analysed.

Table 6. Mean machine settings parameters for different sample sizes.

Spruce with depth of 150 mm

Samle size	Number of nonzero yield in C40		**Independent sampling**		
		Grades	**C40**	**C30**	**C18**
125	61/100	Mean	10923.0	8986.1	7830.6
		St. dev.	1660.7	962.9	1066.2
250	100/100	Mean	9944.4	8211.6	6767.4
		St. dev.	316.6	403.8	593.5
500	100/100	Mean	9904.8	7886.7	6478.9
		St. dev.	205.5	260.2	263.1
750	100/100	Mean	9884.9	7821.0	6467.9
		St. dev.	169.2	224.1	299.4

Samle size	Number of nonzero yield in C40		**Sampling from finite population**		
		Grades	**C40**	**C30**	**C18**
125	67/100	Mean	10651.1	9142.0	7918.1
		St. dev.	1110.5	379.8	666.4
250	100/100	Mean	9957.520	8225.000	6849.780
		St. dev.	258.982	369.046	519.585
500	100/100	Mean	9907.200	7928.700	6662.520
		St. dev.	124.319	232.933	291.063
750	100/100	Mean	9930.760	7866.220	6592.080
		St. dev.	84.515	185.847	170.062

All spruce

Sample size	Number of nonzero yield in C40	Grades	Independent sampling		
			C40	**C30**	**C18**
125	4/100	Mean	14126.2	8700.4	6122.8
		St. dev.	1043.1	917.1	1998.3
250	35/100	Mean	13468.6	7412.4	4102.0
		St. dev.	1854.0	646.1	989.3
500	60/100	Mean	12862.1	7143.8	3977.2
		St. dev.	1700.0	488.2	777.0
750	67/100	Mean	12855.6	6976.8	3858.4
		St. dev.	1728.6	388.7	743.0

Sample size	Number of nonzero yield in C40	Grades	Sampling from finite population		
			C40	**C30**	**C18**
125	8/100	Mean	13791.6	8168.4	5409.2
		St. dev.	1227.9	837.4	1769.5
250	44/100	Mean	12960.6	7230.6	4478.5
		St. dev.	1939.1	627.0	871.5
500	62/100	Mean	12675.0	7024.4	4234.1
		St. dev.	1988.4	422.6	604.6
750	85/100	Mean	12020.5	6874.8	4077.3
		St. dev.	1487.4	207.6	637.6

In the case of spruce with the depth of 150 mm we can see that the effect from sampling from the population of 1000 is not very evident. We can see a slight reduction in variance which is clearly illustrated by the Fig. 5.

In the case of all spruce data the effect of sampling from finite population of 1000 depends from the population of 1000 pieces itself. For all spruce data the variation is so high and correlation is so low that the variation of the results obtained from the sample of 1000 is still to high to consider this a representative sample which may act as a population from which the samples may be drawn. In the example shown in Table 6 we can see that the number of non-zero yield in C40 increased considerably. However if we repeated the generation of population of 1000 and than repeat sampling from that population we might observe decreasing of the same parameter. We should conclude that in such cases where the variation is high and correlations low the sampling from a population of 1000 is not very reliable.

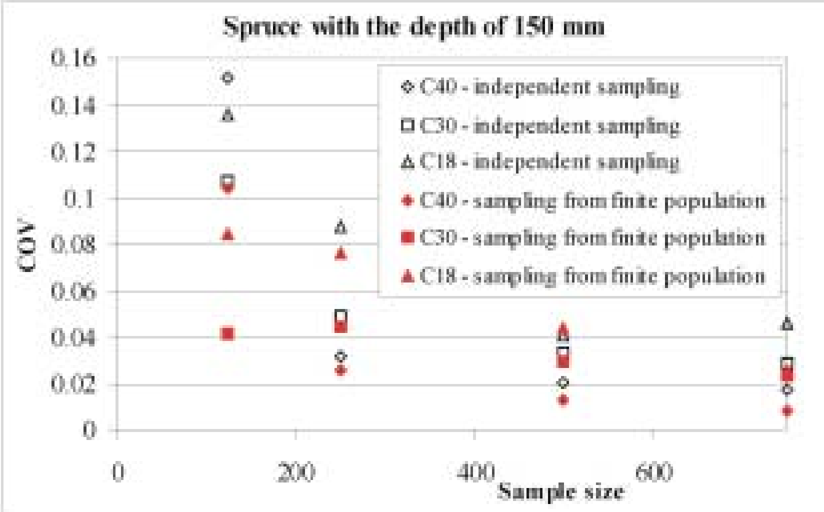

Figure 5. Coefficients of variation of machine settings for the cases of independent sampling and sampling from the finite population of 1000 pieces.

6. Applications

6.1 Effect of sampling size to quality of grading

The sample size used for determination of settings of grading machine may have effect on the grading result when the machine is used. First we analyse how the yield to different grades is influenced by the random factors, and how they can be counteracted by increasing the sample size.

Based on the analysis of the most homogenous sample, spruce 150 mm, described in Appendix Table 2, we observe that when sample size is increased, yield to higher grades increases and COV of yield decreases (see Fig. 6). Results are summarised in Table 7 indicating that when settings are determined by the use of “standard” sampling, the machine gives on average 35% yield to C40, but it may give on a yield from 26 to 44% within confidence limits of 95%. When four times larger sample is used for determination of settings, we obtain mean yield 37% and confidence limits are from 32 to 42%.

Table 7. Confidence intervals for yield to different strength classes [%]. It is assumed that independent sampling is repeated 4 times.

sample size	yield C40		yield C30		yield C18	
250	35.3±9.3 1.96/sqrt(4)		48.6±14.1 1.96/sqrt(4)		13.6±7.9 1.96/sqrt(4)	
	26.2	44.4	34.8	62.4	0.2	15.6
500	35.9±6.1 1.96/sqrt(4)		50.8±9.7 1.96/sqrt(4)		11.0±6.4 1.96/sqrt(4)	
	30.0	41.8	41.3	60.3	0.1	12.6
750	36.7±5.4 1.96/sqrt(4)		54.4±7.1 1.96/sqrt(4)		8.1±3.1 1.96/sqrt(4)	
	31.4	42.0	47.5	61.3	0.1	6.1
1000	37.1±5.0 1.96/sqrt(4)		52.3±7.3 1.96/sqrt(4)		8.4±4.8 1.96/sqrt(4)	
	32.2	42.0	45.1	59.5	0.1	9.4
1250	36.4±3.6 1.96/sqrt(4)		55.9±4.5 1.96/sqrt(4)		7.0±2.4 1.96/sqrt(4)	
	32.8	40.0	51.5	60.3	0.0	4.7
2500	37.5±3.0 1.96/sqrt(4)		55.4±3.9 1.96/sqrt(4)		6.4±1.6 1.96/sqrt(4)	
	34.6	40.4	51.6	59.2	0.0	3.1

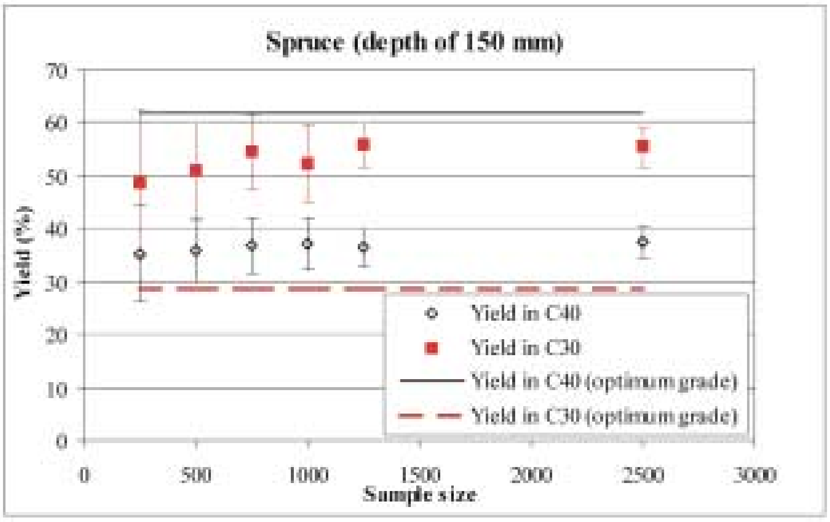

Figure 6. Yield in C40 and C30 - 95 % confidence intervals (4 repetitions of sampling).

The ratios between 0.5% and 5% percentile of the strength are 0.828, 0.753, and 0.605 in classes C40, C30, and C18, respectively. The average values of this ratio don't dependent on the sample size. However, the variance of the ratio is in general lower if larger samples were taken into account (see Fig. 7).

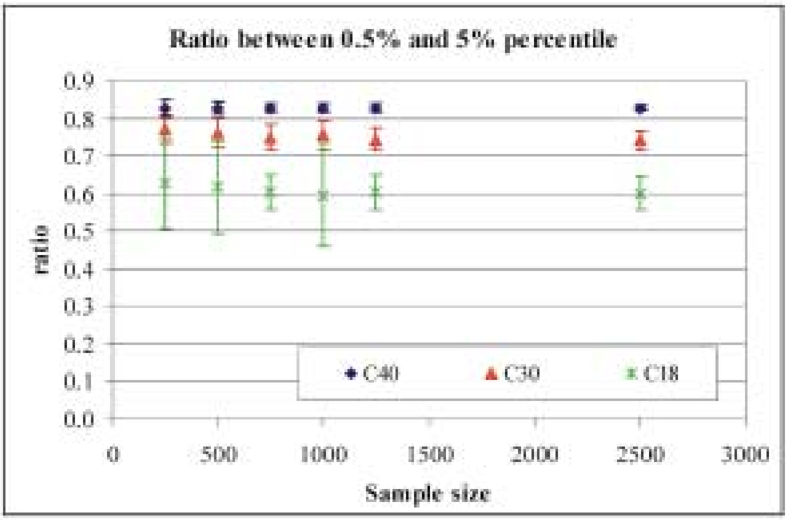

Figure 7. Ratio between 0.5% and 5% percentile of the strength.

Also the form of strength distribution of different grades has been studied. Based on data from 150 mm spruce as above, we have analysed the strength distributions of 10000 specimens when graded simultaneously to C40, C30 and C18 with settings based on mean values of 100 simulations of independent samples. We learned that result is quite independent of the sample size, and is illustrated in Figure 8, where the 5% percentile strength of each grade is denoted by 1 and cumulative distribution of the relative strength is shown in lower tail area. The curves are lognormal distributions which are fitted to simulated results in two points: $f_{0.005}$ and $f_{0.05}$. COV-parameter of the fitted lognormal distribution is indicated in Figure 8, 0.2 for C40 and 0.6 for C18. We observe that C40 is qualitatively much better material than lower grades.

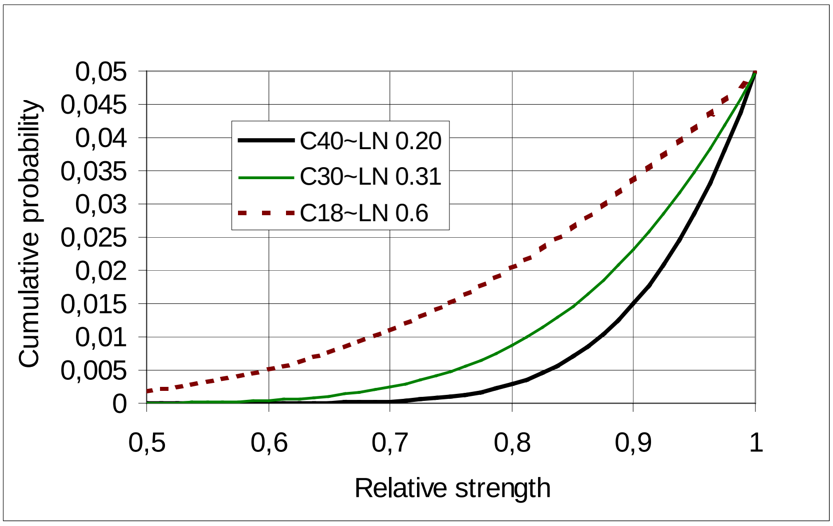

Figure 8. Relative strength of timber graded simultaneously to C40, C30 and C18.

7. Conclusions

- It was important to use actual data to determine random vector characteristics (mean and variance-covariance matrix) used in sample generation.

- The data for spruce pieces with the depth of 150 mm showed quite high correlation between grade determining parameters and grading parameters. As a result the machine grading proved to be quite successful.

- The correlations between the parameters for all spruce data and combined data for spruce and pine were lower. Therefore, the machine grading is less effective. Probable reasons may be: (1) the procedure to adjust values to the values valid for 150 mm deep spruce pieces was not adequate or at least not accurate enough, (2) the values for pieces of different size may have been obtained from different laboratories, (3) the pieces may have been taken from the timber from different region, etc.

- Sampling from finite size population results in reducing the variances in machine settings, but this reduction is not due to better procedure but due to dependent sampling, which may lead to biased results. However, at least for the case of spruce of 150 mm the population of 1000 gives satisfactory results.

- Adding additional grading parameter slightly improved the machine grading in the case of spruce of 150 mm (the yield in C40 increased for approx. 1%).

- It seems that a grading procedure for the data with as low correlations as in the case of all spruce specimens has to be changed, revised, i.e. improved. One of the possibilities is the use of artificial neural networks.

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Appendix A: Generated grading results

Table 1 : Grading results of population of 2000, settings are based on 100 samples ($n_{\text{sample}} = 250$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	84.25/36./8.75	0.2	0.299		72.6	1.135	0.809	26.5	1.179	0.694
2	49.25/34.75/8.25	38.2	1.051	0.805	37.9	1.017	0.847	23.2	1.147	0.658
3	89.5/38.75/8.25	0.1	0.209		67.0	1.209	0.799	32.2	1.225	0.725
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	89.5/37.5/14.	0.1	0.209		69.5	1.167	0.817	29.0	1.218	0.712
100	57.75/55./8.25	18.4	1.154	0.758	6.1	1.417		74.8	1.470	0.666
Mean	**67.51/37.27/10.50**	**5.4**	**1.257**		**64.5**	**1.149**	**0.809**	**29.2**	**1.192**	**0.731**
	mean	13.54	0.916	0.808	55.6	1.133	0.805	30	1.207	0.621
	COV	0.992	0.445	0.053	0.41	0.088	0.020	0.49	0.097	0.199

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	49.25/34.75/8.25	38.2	1.051	0.805	37.9	1.017	0.847	23.2	1.147	0.658
2	50.25/45.75/8.25	35.9	1.066	0.792	12.9	1.194	0.825	50.5	1.355	0.692
3	60./38./11.75	14.4	1.188	0.847	54.3	1.139	0.791	30.2	1.220	0.727
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60	63./38.75/8.25	9.9	1.217		57.3	1.176	0.817	32.2	1.225	0.725
61	57.75/55./8.25	18.4	1.154	0.758	6.1	1.417		74.8	1.470	0.666
Mean	**56.65/39.56/10.14**	**20.6**	**1.137**	**0.780**	**44.3**	**1.165**	**0.823**	**34.3**	**1.257**	**0.707**
	mean	21.9	1.143	0.808	42	1.152	0.808	35.2	1.250	0.661
	COV	0.49	0.057	0.053	0.43	0.094	0.024	0.42	0.092	0.119

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	95.5/53/8.25	0.3	0.448		32.9	1.215	0.762	64.1	1.469	0.652
2	104.25/43.5/8.25	0.1	0.110		52.5	1.135	0.784	44.8	1.363	0.670
3	69.75/39.75/8.25	9.2	1.102		52.3	1.047	0.778	35.8	1.320	0.673
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	92.5/41.75/8.25	0.5	0.610		55.9	1.127	0.772	40.9	1.352	0.661
100	99.5/26.25/10.25	0.1	0.149		82.2	0.999	0.675	14.2	1.165	0.453
Mean	**93.13/38.1/8.39**	**0.4**	**0.523**		**64.8**	**1.051**	**0.780**	**32.1**	**1.294**	**0.686**
	mean	2.18	0.50	0.73	60.17	1.08	0.75	34.91	1.29	0.63
	COV	2.12	0.75	0.02	0.32	0.08	0.06	0.50	0.09	0.14

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	69.75/39.75/8.25	9.2	1.102		52.3	1.047	0.778	35.8	1.320	0.673
2	65/19.5/8.25	14.4	1.064	0.731	76.5	0.930	0.637	6.4	1.079	0.556
3	73.75/44.25/8.25	6.3	1.062		44.9	1.130	0.788	46.1	1.369	0.672
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	65.25/54/8.25	14.0	1.069	0.724	17.0	1.169	0.749	66.3	1.473	0.656
15	63.5/47.5/8.25	16.3	1.031	0.743	28.0	1.137	0.782	53.0	1.421	0.662
Mean	**66.75/42.27/8.25**	**12.3**	**1.073**	**0.703**	**42.7**	**1.062**	**0.803**	**42.3**	**1.355**	**0.665**
	mean	12.65	1.07	0.73	41.35	1.10	0.76	43.32	1.35	0.64
	COV	0.29	0.02	0.02	0.45	0.07	0.06	0.39	0.09	0.11

Table 1 (cont): Grading results of population of 2000, settings are based on 100 samples ($n_{\text{sample}} = 500$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	61./27.25/9.	13.1	1.206	0.829	75.8	1.000	0.807	10.5	1.035	0.281
2	58./39.75/11.75	17.8	1.155	0.833	46.4	1.179	0.819	34.7	1.293	0.687
3	61./36.5/11.	13.1	1.206	0.829	58.6	1.130	0.803	27.4	1.186	0.705
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	52./48./8.25	31.0	1.086	0.816	11.2	1.270	0.826	57.1	1.395	0.676
100	63.75/35./8.25	8.8	1.217		66.8	1.092	0.811	23.7	1.156	0.662
Mean	**66.07/36.59/9.73**	**6.3**	**1.264**		**65.1**	**1.136**	**0.803**	**27.9**	**1.187**	**0.712**
	mean	13.31	0.949	0.825	57.5	1.125	0.803	28.4	1.194	0.637
	COV	0.808	0.435	0.054	0.35	0.072	0.017	0.43	0.088	0.175

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	61./27.25/9.	13.1	1.206	0.829	75.8	1.000	0.807	10.5	1.035	0.281
2	58./39.75/11.75	17.8	1.155	0.833	46.4	1.179	0.819	34.7	1.293	0.687
3	61./36.5/11.	13.1	1.206	0.829	58.6	1.130	0.803	27.4	1.186	0.705
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
75	52./48./8.25	31.0	1.086	0.816	11.2	1.270	0.826	57.1	1.395	0.676
76	63.75/35./8.25	8.8	1.217		66.8	1.092	0.811	23.7	1.156	0.662
Mean	**59.01/37.45/9.58**	**16.1**	**1.184**	**0.858**	**53.6**	**1.132**	**0.795**	**29.6**	**1.198**	**0.734**
	mean	17.47	1.172	0.825	51.4	1.131	0.805	30.4	1.211	0.650
	COV	0.512	0.052	0.054	0.37	0.074	0.018	0.41	0.088	0.155

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	103.25/33.25/8.25	0.1	0.110		73.3	1.015	0.794	23.9	1.189	0.647
2	108.75/36.5/8.25	0.1	0.110		67.8	1.051	0.769	29.4	1.286	0.680
3	113/47.5/8.25	0.0	0.000		44.3	1.195	0.777	53.0	1.421	0.662
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	99.5/42.5/8.25	0.1	0.149		54.4	1.132	0.782	42.8	1.355	0.667
100	65.5/42.75/8.25	13.8	1.079	0.715	40.4	1.064	0.813	43.1	1.355	0.667
Mean	**93.21/38.66/8.275**	**0.4**	**0.523**		**63.6**	**1.053**	**0.775**	**33.3**	**1.303**	**0.681**
	mean	3.016	0.445	0.721	59.2	1.076	0.767	35.1	1.296	0.658
	COV	1.682	0.951	0.022	0.27	0.062	0.042	0.39	0.074	0.077

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	69.5/45/8.25	9.3	1.091		39.7	1.130	0.787	48.4	1.386	0.671
2	72.25/42.75/8.25	7.4	1.093		46.8	1.127	0.779	43.1	1.355	0.667
3	74.25/32.5/8.25	6.0	1.050		68.6	1.002	0.777	22.8	1.188	0.629
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28	68.75/36.5/8.25	9.9	1.089		58.0	1.038	0.772	29.4	1.286	0.680
29	65.5/42.75/8.25	13.8	1.079	0.715	40.4	1.064	0.813	43.1	1.355	0.667
Mean	**69.70/41.47/8.25**	**9.2**	**1.102**		**47.9**	**1.062**	**0.810**	**40.2**	**1.349**	**0.660**
	mean	10.04	1.069	0.721	46.6	1.085	0.770	40.7	1.332	0.668
	COV	0.435	0.022	0.022	0.36	0.065	0.041	0.36	0.073	0.053

Table 1 (cont): Grading results of population of 2000, settings are based on 100 samples ($n_{\text{sample}} = 750$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	61.75/39./10.5	11.6	1.219	0.897	54.9	1.176	0.816	32.6	1.225	0.725
2	61.5/31.5/10.	12.1	1.218	0.899	70.2	1.040	0.805	16.9	1.075	0.524
3	64./36.75/8.25	8.6	1.219		62.7	1.133	0.803	28.0	1.187	0.715
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	65.5/37./8.25	6.7	1.251		63.9	1.139	0.800	28.7	1.188	0.725
100	94.75/30./9.5	0.1	0.110		85.2	1.042	0.802	14.0	1.067	0.413
Mean	**64.708/36.135/9.823**	**7.6**	**1.219**		**65.0**	**1.129**	**0.808**	**26.7**	**1.181**	**0.696**
	mean	12.75	1.065	0.828	59.4	1.115	0.806	27	1.184	0.626
	COV	0.643	0.326	0.054	0.27	0.063	0.014	0.38	0.079	0.171

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	61.75/39./10.5	11.6	1.219	0.897	54.9	1.176	0.816	32.6	1.225	0.725
2	61.5/31.5/10.	12.1	1.218	0.899	70.2	1.040	0.805	16.9	1.075	0.524
3	64./36.75/8.25	8.6	1.219		62.7	1.133	0.803	28.0	1.187	0.715
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
86	64./42./8.25	8.6	1.219		49.6	1.221	0.796	41.1	1.313	0.681
87	65.5/37./8.25	6.7	1.251		63.9	1.139	0.800	28.7	1.188	0.725
Mean	**60.649/36.67/9.782**	**13.5**	**1.199**	**0.836**	**57.8**	**1.129**	**0.802**	**28.0**	**1.187**	**0.714**
	mean	14.63	1.188	0.828	56.4	1.119	0.807	28.2	1.194	0.639
	COV	0.484	0.043	0.054	0.26	0.064	0.014	0.36	0.078	0.148

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	61.25/48.5/8.25	19.0	1.035	0.754	23.3	1.144	0.776	55.1	1.427	0.660
2	100.5/42.5/8.25	0.1	0.149		54.4	1.132	0.782	42.8	1.355	0.667
3	67/36.25/8.25	12.1	1.089	0.691	56.2	1.032	0.774	29.0	1.283	0.677
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	118.25/30/8.25	0.0	0.000		78.3	1.007	0.716	18.9	1.172	0.552
100	65.5/41.25/8.25	13.8	1.079	0.715	43.7	1.050	0.813	39.8	1.344	0.662
Mean	**89.5525/38.0425/8.305**	**0.9**	**1.036**		**64.3**	**1.050**	**0.779**	**32.1**	**1.294**	**0.686**
	mean	4.305	0.586	0.708	58.8	1.068	0.767	34.2	1.287	0.645
	COV	1.246	0.833	0.036	0.29	0.062	0.050	0.41	0.077	0.092

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	61.25/48.5/8.25	19.0	1.035	0.754	23.3	1.144	0.776	55.1	1.427	0.660
2	67/36.25/8.25	12.1	1.089	0.691	56.2	1.032	0.774	29.0	1.283	0.677
3	66/55/8.25	13.0	1.081	0.705	16.6	1.176	0.743	67.7	1.480	0.656
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
47	68.75/41.25/8.25	9.9	1.089		47.5	1.062	0.810	39.8	1.344	0.662
48	65.5/41.25/8.25	13.8	1.079	0.715	43.7	1.050	0.813	39.8	1.344	0.662
Mean	**71.27/41.09/8.25**	**8.0**	**1.099**		**50.0**	**1.067**	**0.810**	**39.3**	**1.334**	**0.667**
	mean	8.863	1.088	0.708	48.8	1.082	0.780	39.7	1.327	0.658
	COV	0.502	0.039	0.036	0.31	0.059	0.033	0.33	0.065	0.056

Table 1 (cont): Grading results of population of 2000, settings are based on 100 samples ($n_{\text{sample}} = 1000$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	60.75/35.25/9.25	13.4	1.197	0.837	61.7	1.077	0.824	24.3	1.164	0.667
2	69.25/32./10.5	4.6	1.250		77.2	1.057	0.800	17.3	1.078	0.538
3	92.75/25.75/9.75	0.1	0.110		91.0	1.007	0.774	8.3	1.004	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	72.25/34.25/10.	3.0	1.265		74.3	1.095	0.808	21.9	1.138	0.642
100	90.25/34.5/9.75	0.1	0.209		76.8	1.109	0.799	22.4	1.141	0.648
Mean	**64.37/35.15/9.95**	**8.1**	**1.219**		**67.1**	**1.092**	**0.811**	**24.0**	**1.160**	**0.664**
	mean	11.89	1.098	0.832	62.5	1.101	0.806	24.8	1.162	0.630
	COV	0.586	0.284	0.055	0.23	0.060	0.018	0.38	0.075	0.167

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	60.75/35.25/9.25	13.4	1.197	0.837	61.7	1.077	0.824	24.3	1.164	0.667
2	69.25/32./10.5	4.6	1.250		77.2	1.057	0.800	17.3	1.078	0.538
3	65.75/26.75/9.	6.6	1.264		83.0	1.007	0.801	9.8	1.024	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
89	58.75/31.75/9.5	16.3	1.178	0.863	65.8	1.038	0.805	17.2	1.077	0.534
90	72.25/34.25/10.	3.0	1.265		74.3	1.095	0.808	21.9	1.138	0.642
Mean	**61.47/35.47/9.89**	**12.2**	**1.210**	**0.875**	**62.2**	**1.091**	**0.814**	**24.9**	**1.172**	**0.673**
	mean	13.19	1.200	0.832	60.5	1.104	0.807	25.5	1.169	0.631
	COV	0.459	0.041	0.055	0.23	0.061	0.018	0.37	0.075	0.169

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	62/52/8.25	17.9	1.035	0.749	17.4	1.126	0.779	62.0	1.455	0.654
2	79/36/8.25	3.8	1.154		65.0	1.047	0.771	28.5	1.254	0.684
3	67/36/8.25	12.1	1.089	0.691	56.7	1.030	0.776	28.5	1.254	0.684
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	107.75/33.25/8.25	0.1	0.110		73.3	1.015	0.794	23.9	1.189	0.647
100	68/39.5/8.25	11.1	1.053	0.703	50.8	1.043	0.778	35.4	1.319	0.674
Mean	**87.07/37.56/8.25**	**1.3**	**1.219**		**64.8**	**1.051**	**0.780**	**31.2**	**1.292**	**0.687**
	mean	4.49	0.68	0.71	60.07	1.06	0.77	32.76	1.28	0.66
	COV	1.00	0.72	0.03	0.25	0.05	0.04	0.37	0.07	0.07

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	62/52/8.25	17.9	1.035	0.749	17.4	1.126	0.779	62.0	1.455	0.654
2	79/36/8.25	3.8	1.154		65.0	1.047	0.771	28.5	1.254	0.684
3	67/36/8.25	12.1	1.089	0.691	56.7	1.030	0.776	28.5	1.254	0.684
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
56	71.25/43.5/8.25	8.0	1.099		44.5	1.128	0.787	44.8	1.363	0.670
57	68/39.5/8.25	11.1	1.053	0.703	50.8	1.043	0.778	35.4	1.319	0.674
Mean	**72.30/40.04/8.25**	**7.4**	**1.093**		**53.5**	**1.048**	**0.800**	**36.4**	**1.319**	**0.674**
	mean	7.80	1.10	0.71	52.04	1.07	0.78	37.48	1.31	0.66
	COV	0.40	0.04	0.03	0.26	0.06	0.03	0.33	0.06	0.04

Table 1 (cont): Grading results of population of 2000, settings are based on 100 samples ($n_{\text{sample}} = 1250$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	55/43.25/8.5	24.5	1.111	0.773	31.0	1.209	0.801	43.9	1.328	0.684
2	65.5/37/11.25	6.7	1.251		63.9	1.139	0.800	28.4	1.188	0.721
3	57/40/10.5	19.5	1.150	0.765	43.7	1.182	0.820	35.9	1.291	0.689
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	67.5/29.75/8.25	5.4	1.257		80.0	1.027	0.808	13.8	1.066	0.406
100	69.25/32.5/10.5	4.6	1.250		76.1	1.067	0.791	18.5	1.089	0.577
Mean	**63.713/35.4/9.7**	**8.9**	**1.217**		**65.7**	**1.096**	**0.811**	**24.7**	**1.170**	**0.671**
	mean	12.37	1.116	0.829	61.6	1.104	0.806	25.2	1.168	0.622
	COV	0.587	0.258	0.055	0.23	0.053	0.013	0.34	0.073	0.162

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	55/43.25/8.5	24.5	1.111	0.773	31.0	1.209	0.801	43.9	1.328	0.684
2	65.5/37/11.25	6.7	1.251		63.9	1.139	0.800	28.4	1.188	0.721
3	57/40/10.5	19.5	1.150	0.765	43.7	1.182	0.820	35.9	1.291	0.689
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
91	67.5/29.75/8.25	5.4	1.257		80.0	1.027	0.808	13.8	1.066	0.406
92	69.25/32.5/10.5	4.6	1.250		76.1	1.067	0.791	18.5	1.089	0.577
Mean	**61.272/35.595/9.72**	**12.3**	**1.211**	**0.875**	**62.0**	**1.092**	**0.813**	**25.1**	**1.174**	**0.674**
	mean	13.43	1.199	0.829	60.1	1.105	0.806	25.7	1.172	0.626
	COV	0.488	0.041	0.055	0.23	0.055	0.013	0.34	0.074	0.161

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	111.25/26.75/8.25	0.1	0.110		81.8	1.001	0.713	15.5	1.136	0.534
2	78/38.25/8.25	4.0	1.156		60.8	1.046	0.775	32.5	1.295	0.685
3	65/44.25/8.25	14.4	1.064	0.731	36.8	1.074	0.828	46.1	1.369	0.672
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	73/44.75/8.25	6.8	1.081		42.9	1.136	0.784	47.6	1.381	0.671
100	75/42/8.25	5.6	1.044		50.0	1.118	0.773	41.8	1.355	0.663
Mean	**87.9775/38.2625/8.25**	**1.1**	**1.219**		**63.8**	**1.049**	**0.779**	**32.5**	**1.295**	**0.685**
	mean	3.973	0.676	0.706	59.5	1.064	0.776	33.9	1.287	0.659
	COV	1.115	0.727	0.018	0.23	0.052	0.032	0.33	0.063	0.070

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	78/38.25/8.25	4.0	1.156		60.8	1.046	0.775	32.5	1.295	0.685
2	65/44.25/8.25	14.4	1.064	0.731	36.8	1.074	0.828	46.1	1.369	0.672
3	79.75/33.5/8.25	3.4	1.174		69.6	1.010	0.797	24.4	1.199	0.649
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
55	73/44.75/8.25	6.8	1.081		42.9	1.136	0.784	47.6	1.381	0.671
56	75/42/8.25	5.6	1.044		50.0	1.118	0.773	41.8	1.355	0.663
Mean	**73.9286/39.8214/8.25**	**6.2**	**1.057**		**55.3**	**1.050**	**0.783**	**35.8**	**1.320**	**0.673**
	mean	7.013	1.105	0.706	53.5	1.072	0.780	36.8	1.307	0.668
	COV	0.533	0.051	0.018	0.25	0.056	0.027	0.31	0.061	0.043

Table 1 (cont): Grading results of population of 2000, settings are based on 100 samples ($n_{\text{sample}} = 2500$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	63.25/34/8.25	9.5	1.214		68.1	1.076	0.810	21.7	1.138	0.639
2	61/37/9.5	13.1	1.206	0.829	57.6	1.133	0.799	28.7	1.188	0.725
3	62.75/35.5/8.25	10.3	1.219	0.894	64.0	1.093	0.813	25.0	1.173	0.674
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	62/35/8.25	11.3	1.219	0.896	64.3	1.082	0.818	23.7	1.156	0.662
100	61.75/33.75/8.5	11.6	1.219	0.897	66.5	1.066	0.816	21.2	1.135	0.632
Mean	**62.62/35.48/8.98**	**10.5**	**1.213**	**0.899**	**63.9**	**1.093**	**0.813**	**25.0**	**1.173**	**0.673**
	mean	11.89	1.199	0.835	62.2	1.103	0.809	25.2	1.165	0.659
	COV	0.454	0.101	0.054	0.16	0.038	0.012	0.22	0.045	0.106

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	63.25/34/8.25	9.5	1.214		68.1	1.076	0.810	21.7	1.138	0.639
2	61/37/9.5	13.1	1.206	0.829	57.6	1.133	0.799	28.7	1.188	0.725
3	62.75/35.5/8.25	10.3	1.219	0.894	64.0	1.093	0.813	25.0	1.173	0.674
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	62/35/8.25	11.3	1.219	0.896	64.3	1.082	0.818	23.7	1.156	0.662
99	61.75/33.75/8.5	11.6	1.219	0.897	66.5	1.066	0.816	21.2	1.135	0.632
Mean	**62.24/35.53/8.99**	**11.1**	**1.219**	**0.895**	**63.2**	**1.092**	**0.813**	**25.0**	**1.173**	**0.674**
	mean	12.01	1.210	0.835	62	1.104	0.809	25.3	1.166	0.661
	COV	0.441	0.041	0.054	0.16	0.038	0.012	0.22	0.045	0.100

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	73.75/35.5/8.25	6.3	1.062		63.1	1.038	0.776	27.9	1.225	0.691
2	70.75/42.5/8.25	8.5	1.094		46.0	1.120	0.782	42.8	1.355	0.667
3	74.75/41./8.25	5.6	1.045		52.7	1.087	0.797	39.0	1.329	0.670
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	71.5/45./8.25	7.8	1.096		41.2	1.130	0.787	48.4	1.386	0.671
100	120.75/38./8.25	0.0	0.000		65.2	1.051	0.780	32.1	1.294	0.686
Mean	**90.16/37.21/8.25**	**0.7**	**0.854**		**66.0**	**1.049**	**0.770**	**30.6**	**1.293**	**0.686**
	mean	2.851	0.742		63	1.055	0.779	31.4	1.274	0.666
	COV	1.061	0.697		0.14	0.036	0.017	0.23	0.047	0.041

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**59.6**	**1.014**	**0.959**	**23.6**	**0.927**	**0.902**	**16.8**	**0.991**	**0.315**
1	73.75/35.5/8.25	6.3	1.062		63.1	1.038	0.776	27.9	1.225	0.691
2	70.75/42.5/8.25	8.5	1.094		46.0	1.120	0.782	42.8	1.355	0.667
3	74.75/41./8.25	5.6	1.045		52.7	1.087	0.797	39.0	1.329	0.670
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
61	69./43.25/8.25	9.8	1.089		43.1	1.121	0.778	44.4	1.359	0.670
62	71.5/45./8.25	7.8	1.096		41.2	1.130	0.787	48.4	1.386	0.671
Mean	**78.25/37.8/8.25**	**4.0**	**1.156**		**61.5**	**1.047**	**0.776**	**31.8**	**1.294**	**0.686**
	mean	4.552	1.141		60.2	1.057	0.778	32.6	1.285	0.668
	COV	0.586	0.064		0.14	0.036	0.019	0.22	0.045	0.041

Table 1 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 250$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	84.25/36./8.75	0.3	1.524		73.0	1.144	0.811	26.2	1.171	0.627
2	49.25/34.75/8.25	38.3	1.016	0.828	38.1	1.045	0.805	23.2	1.136	0.600
3	89.5/38.75/8.25	0.1	0.691		66.8	1.186	0.797	32.7	1.224	0.616
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	89.5/37.5/14.	0.1	0.691		69.8	1.167	0.801	28.9	1.246	0.702
100	57.75/55./8.25	18.4	1.120	0.862	5.6	1.338	0.779	75.6	1.414	0.663
Mean	**67.51/37.273/10.498**	**5.9**	**1.266**	**0.872**	**64.4**	**1.151**	**0.806**	**29.0**	**1.217**	**0.660**
	mean	13.51	1.157	0.848	55.6	1.134	0.794	30.2	1.202	0.639
	COV	0.984	0.259	0.023	0.42	0.069	0.025	0.5	0.082	0.070

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	49.25/34.75/8.25	38.3	1.016	0.828	38.1	1.045	0.805	23.2	1.136	0.600
2	50.25/45.75/8.25	35.6	1.029	0.835	12.3	1.176	0.832	51.8	1.326	0.668
3	60./38./11.75	14.4	1.155	0.840	54.1	1.144	0.808	30.7	1.244	0.696
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60	63./38.75/8.25	10.0	1.210	0.884	56.9	1.163	0.806	32.7	1.224	0.616
61	57.75/55./8.25	18.4	1.120	0.862	5.6	1.338	0.779	75.6	1.414	0.663
Mean	**56.648/39.557/10.135**	**20.5**	**1.109**	**0.857**	**44.4**	**1.147**	**0.810**	**34.5**	**1.254**	**0.691**
	mean	21.82	1.113	0.848	42	1.148	0.801	35.6	1.237	0.652
	COV	0.483	0.062	0.024	0.44	0.074	0.022	0.43	0.079	0.064

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	95.5/53/8.25	0.3	1.192		32.9	1.218	0.770	64.4	1.416	0.680
2	104.25/43.5/8.25	0.1	0.858		52.8	1.142	0.759	44.6	1.331	0.667
3	69.75/39.75/8.25	9.4	1.036	0.776	51.3	1.084	0.732	36.8	1.305	0.666
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	92.5/41.75/8.25	0.5	1.203		56.1	1.131	0.753	41.0	1.321	0.667
100	99.5/26.25/10.25	0.2	1.215		83.7	1.001	0.712	13.1	1.146	0.544
Mean	**93.125/38.1/8.3925**	**0.5**	**1.228**		**63.5**	**1.089**	**0.733**	**33.6**	**1.287**	**0.670**
	mean	2.20	1.047	0.777	60.42	1.089	0.735	34.89	1.283	0.637
	COV	2.10	0.249	0.016	0.32	0.069	0.039	0.50	0.067	0.068

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	69.75/39.75/8.25	9.4	1.036	0.776	51.3	1.084	0.732	36.8	1.305	0.666
2	65/19.5/8.25	14.4	1.002	0.787	76.7	0.928	0.672	6.5	1.172	0.529
3	73.75/44.25/8.25	6.3	1.042	0.740	45.2	1.133	0.758	46.0	1.342	0.665
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
14	65.25/54/8.25	14.1	1.003	0.786	17.2	1.153	0.723	66.3	1.423	0.687
15	63.5/47.5/8.25	16.1	0.987	0.766	28.8	1.128	0.764	52.7	1.372	0.674
Mean	**66.75/42.267/8.25**	**12.4**	**1.015**	**0.799**	**43.3**	**1.105**	**0.755**	**41.9**	**1.326**	**0.667**
	mean	12.66	1.010	0.777	41.67	1.097	0.739	43.25	1.328	0.655
	COV	0.28	0.023	0.016	0.44	0.062	0.035	0.39	0.059	0.061

Table 1 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 500$)

Good grading quality $r^2 = 0.64$ **All samples are included**

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	61./27.25/9.	12.8	1.169	0.834	76.4	1.005	0.771	10.3	1.019	0.579
2	58./39.75/11.75	17.9	1.127	0.857	46.5	1.157	0.808	34.8	1.275	0.685
3	61./36.5/11.	12.8	1.169	0.834	59.5	1.132	0.808	27.0	1.195	0.631
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	52./48./8.25	31.1	1.053	0.842	10.7	1.229	0.822	57.9	1.348	0.669
100	63.75/35./8.25	9.2	1.219	0.880	66.7	1.115	0.794	23.7	1.139	0.604
Mean	**66.073/36.588/9.728**	**7.0**	**1.243**	**0.884**	**65.1**	**1.140**	**0.813**	**27.4**	**1.192**	**0.634**
	mean	13.39	1.124	0.853	57.6	1.126	0.796	28.5	1.186	0.632
	COV	0.803	0.237	0.022	0.36	0.055	0.025	0.44	0.079	0.067

Good grading quality $r^2 = 0.64$ **Only non-zero yield in C40 is included**

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	61./27.25/9.	12.8	1.169	0.834	76.4	1.005	0.771	10.3	1.019	0.579
2	58./39.75/11.75	17.9	1.127	0.857	46.5	1.157	0.808	34.8	1.275	0.685
3	61./36.5/11.	12.8	1.169	0.834	59.5	1.132	0.808	27.0	1.195	0.631
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
75	52./48./8.25	31.1	1.053	0.842	10.7	1.229	0.822	57.9	1.348	0.669
76	63.75/35./8.25	9.2	1.219	0.880	66.7	1.115	0.794	23.7	1.139	0.604
Mean	**59.013/37.447/9.576**	**15.9**	**1.139**	**0.851**	**54.0**	**1.136**	**0.813**	**29.5**	**1.211**	**0.680**
	mean	17.55	1.146	0.853	51.4	1.129	0.799	30.5	1.201	0.636
	COV	0.507	0.062	0.022	0.37	0.055	0.025	0.42	0.078	0.066

Poor grading quality $r^2 = 0.36$ **All samples are included**

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	103.25/33.25/8.25	0.1	0.858		73.1	1.050	0.737	24.4	1.232	0.610
2	108.75/36.5/8.25	0.0	0.092		67.3	1.073	0.740	30.3	1.275	0.606
3	113/47.5/8.25	0.0	0.092		44.8	1.169	0.770	52.7	1.372	0.674
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	99.5/42.5/8.25	0.2	1.215		55.1	1.134	0.756	42.3	1.326	0.668
100	65.5/42.75/8.25	13.8	1.002	0.786	40.9	1.107	0.750	42.8	1.328	0.669
Mean	**93.21/38.66/8.275**	**0.5**	**1.225**		**62.4**	**1.095**	**0.730**	**34.7**	**1.290**	**0.671**
	mean	3.038	0.889	0.765	59.1	1.089	0.739	35.4	1.289	0.641
	COV	1.675	0.429	0.027	0.27	0.051	0.028	0.39	0.053	0.061

Poor grading quality $r^2 = 0.36$ **Only non-zero yield in C40 is included**

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	69.5/45/8.25	9.6	1.035	0.779	40.2	1.131	0.753	47.7	1.349	0.668
2	72.25/42.75/8.25	7.5	1.040	0.751	47.3	1.127	0.750	42.8	1.328	0.669
3	74.25/32.5/8.25	6.0	1.046	0.730	68.5	1.034	0.735	23.1	1.226	0.616
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28	68.75/36.5/8.25	10.3	1.027	0.790	57.0	1.044	0.745	30.3	1.275	0.606
29	65.5/42.75/8.25	13.8	1.002	0.786	40.9	1.107	0.750	42.8	1.328	0.669
Mean	**69.698/41.466/8.25**	**9.4**	**1.037**	**0.776**	**47.6**	**1.103**	**0.741**	**40.5**	**1.319**	**0.666**
	mean	10.08	1.028	0.765	46.6	1.094	0.741	40.9	1.315	0.655
	COV	0.434	0.029	0.027	0.36	0.050	0.028	0.36	0.052	0.048

Table 1 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 750$)

Good grading quality $r^2 = 0.64$ All samples are included

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	61.75/39./10.5	11.7	1.193	0.871	54.6	1.163	0.807	33.1	1.250	0.688
2	61.5/31.5/10.	12.0	1.187	0.869	71.1	1.060	0.786	16.3	1.088	0.585
3	64./36.75/8.25	8.9	1.222	0.886	62.8	1.138	0.814	27.9	1.184	0.618
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	65.5/37./8.25	7.6	1.222	0.891	63.6	1.144	0.810	28.5	1.186	0.619
100	94.75/30./9.5	0.0	0.368		85.4	1.063	0.762	14.1	1.073	0.574
Mean	**64.708/36.135/9.823**	**8.3**	**1.222**	**0.896**	**64.8**	**1.134**	**0.813**	**26.4**	**1.186**	**0.634**
	mean	12.8	1.105	0.859	59.6	1.121	0.795	27.1	1.180	0.629
	COV	0.637	0.254	0.023	0.27	0.048	0.021	0.39	0.070	0.068

Good grading quality $r^2 = 0.64$ Only non-zero yield in C40 is included

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	61.75/39./10.5	11.7	1.193	0.871	54.6	1.163	0.807	33.1	1.250	0.688
2	61.5/31.5/10.	12.0	1.187	0.869	71.1	1.060	0.786	16.3	1.088	0.585
3	64./36.75/8.25	8.9	1.222	0.886	62.8	1.138	0.814	27.9	1.184	0.618
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
86	64./42./8.25	8.9	1.222	0.886	49.4	1.218	0.806	41.4	1.277	0.678
87	65.5/37./8.25	7.6	1.222	0.891	63.6	1.144	0.810	28.5	1.186	0.619
Mean	**60.649/36.67/9.782**	**13.4**	**1.160**	**0.839**	**58.5**	**1.133**	**0.816**	**27.6**	**1.193**	**0.635**
	mean	14.69	1.168	0.859	56.5	1.125	0.797	28.3	1.189	0.634
	COV	0.476	0.058	0.023	0.26	0.048	0.019	0.37	0.069	0.068

Poor grading quality $r^2 = 0.36$ All samples are included

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	61.25/48.5/8.25	19.2	0.967	0.766	23.4	1.132	0.752	55.0	1.385	0.673
2	100.5/42.5/8.25	0.2	1.072		55.2	1.134	0.756	42.3	1.326	0.668
3	67/36.25/8.25	12.1	1.015	0.799	55.7	1.035	0.750	29.8	1.271	0.596
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	118.25/30/8.25	0.0	0.092		78.5	1.030	0.730	19.1	1.188	0.602
100	65.5/41.25/8.25	13.8	1.002	0.786	43.6	1.095	0.736	40.1	1.316	0.666
Mean	**89.5525/38.0425/8.305**	**0.9**	**1.180**		**63.3**	**1.085**	**0.734**	**33.4**	**1.287**	**0.670**
	mean	4.33	0.810	0.759	58.9	1.082	0.738	34.3	1.282	0.637
	COV	1.252	0.470	0.037	0.29	0.051	0.030	0.41	0.056	0.059

Poor grading quality $r^2 = 0.36$ Only non-zero yield in C40 is included

		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	61.25/48.5/8.25	19.2	0.967	0.766	23.4	1.132	0.752	55.0	1.385	0.673
2	67/36.25/8.25	12.1	1.015	0.799	55.7	1.035	0.750	29.8	1.271	0.596
3	66/55/8.25	13.3	1.002	0.784	16.0	1.168	0.721	68.2	1.426	0.687
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
47	68.75/41.25/8.25	10.3	1.027	0.790	47.1	1.100	0.738	40.1	1.316	0.666
48	65.5/41.25/8.25	13.8	1.002	0.786	43.6	1.095	0.736	40.1	1.316	0.666
Mean	**71.271/41.089/8.25**	**8.1**	**1.040**	**0.759**	**49.7**	**1.101**	**0.739**	**39.7**	**1.315**	**0.666**
	mean	8.933	1.044	0.759	48.7	1.095	0.744	40	1.312	0.649
	COV	0.504	0.043	0.037	0.31	0.043	0.020	0.32	0.048	0.052

Table 1 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1000$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	60.75/35.25/9.25	13.2	1.164	0.835	62.1	1.108	0.798	24.2	1.159	0.604
2	69.25/32./10.5	4.5	1.288	0.900	77.6	1.081	0.782	17.3	1.115	0.586
3	92.75/25.75/9.75	0.0	0.368		90.9	1.010	0.766	8.5	1.003	0.589
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	72.25/34.25/10.	2.7	1.344	0.876	74.8	1.114	0.791	21.9	1.138	0.610
100	90.25/34.5/9.75	0.1	0.605		76.9	1.123	0.790	22.4	1.141	0.616
Mean	**64.37/35.153/9.953**	**8.6**	**1.222**	**0.884**	**67.0**	**1.119**	**0.791**	**23.9**	**1.159**	**0.630**
	mean	11.94	1.132	0.862	62.6	1.111	0.793	24.9	1.163	0.623
	COV	0.576	0.187	0.022	0.23	0.048	0.021	0.39	0.069	0.066

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	60.75/35.25/9.25	13.2	1.164	0.835	62.1	1.108	0.798	24.2	1.159	0.604
2	69.25/32./10.5	4.5	1.288	0.900	77.6	1.081	0.782	17.3	1.115	0.586
3	65.75/26.75/9.	7.3	1.233	0.892	82.5	1.009	0.768	9.7	1.011	0.582
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
89	58.75/31.75/9.5	16.5	1.137	0.852	66.1	1.055	0.780	16.9	1.092	0.591
90	72.25/34.25/10.	2.7	1.344	0.876	74.8	1.114	0.791	21.9	1.138	0.610
Mean	**61.472/35.472/9.894**	**12.1**	**1.188**	**0.869**	**62.7**	**1.122**	**0.790**	**24.7**	**1.170**	**0.631**
	mean	13.26	1.181	0.862	60.6	1.112	0.794	25.6	1.169	0.625
	COV	0.448	0.056	0.022	0.23	0.048	0.021	0.38	0.068	0.067

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	62/52/8.25	18.1	0.970	0.760	17.1	1.144	0.747	62.4	1.410	0.679
2	79/36/8.25	3.7	1.153	0.732	64.6	1.058	0.739	29.2	1.267	0.596
3	67/36/8.25	12.1	1.015	0.799	56.3	1.034	0.749	29.2	1.267	0.596
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	107.75/33.25/8.25	0.0	0.286		73.1	1.050	0.737	24.4	1.232	0.610
100	68/39.5/8.25	11.0	1.025	0.781	50.3	1.078	0.734	36.3	1.298	0.668
Mean	**87.0675/37.555/8.25**	**1.2**	**1.163**		**63.9**	**1.081**	**0.737**	**32.5**	**1.285**	**0.666**
	mean	4.52	0.843	0.757	59.95	1.076	0.739	33.09	1.279	0.636
	COV	1.01	0.447	0.030	0.25	0.044	0.025	0.37	0.048	0.055

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	62/52/8.25	18.1	0.970	0.760	17.1	1.144	0.747	62.4	1.410	0.679
2	79/36/8.25	3.7	1.153	0.732	64.6	1.058	0.739	29.2	1.267	0.596
3	67/36/8.25	12.1	1.015	0.799	56.3	1.034	0.749	29.2	1.267	0.596
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
56	71.25/43.5/8.25	8.1	1.041	0.759	44.8	1.131	0.754	44.6	1.331	0.667
57	68/39.5/8.25	11.0	1.025	0.781	50.3	1.078	0.734	36.3	1.298	0.668
Mean	**72.3026/40.0395/8.25**	**7.4**	**1.040**	**0.751**	**52.8**	**1.095**	**0.728**	**37.3**	**1.308**	**0.665**
	mean	7.87	1.053	0.757	51.91	1.088	0.745	37.77	1.301	0.647
	COV	0.41	0.039	0.030	0.26	0.044	0.017	0.33	0.046	0.049

Table 1 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1250$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	55/43.25/8.5	24.0	1.083	0.830	30.9	1.189	0.801	44.7	1.290	0.677
2	65.5/37/11.25	7.6	1.222	0.891	63.6	1.144	0.810	28.2	1.222	0.627
3	57/40/10.5	19.8	1.117	0.859	43.9	1.155	0.810	35.6	1.264	0.685
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	67.5/29.75/8.25	5.9	1.266	0.872	79.8	1.052	0.765	13.9	1.047	0.582
100	69.25/32.5/10.5	4.5	1.288	0.900	76.8	1.087	0.781	18.1	1.118	0.596
Mean	**63.713/35.4/9.7**	**9.2**	**1.219**	**0.880**	**65.7**	**1.124**	**0.790**	**24.5**	**1.169**	**0.631**
	mean	12.51	1.118	0.861	61.7	1.112	0.793	25.2	1.166	0.621
	COV	0.573	0.188	0.023	0.23	0.039	0.018	0.35	0.064	0.069

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	55/43.25/8.5	24.0	1.083	0.830	30.9	1.189	0.801	44.7	1.290	0.677
2	65.5/37/11.25	7.6	1.222	0.891	63.6	1.144	0.810	28.2	1.222	0.627
3	57/40/10.5	19.8	1.117	0.859	43.9	1.155	0.810	35.6	1.264	0.685
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
91	67.5/29.75/8.25	5.9	1.266	0.872	79.8	1.052	0.765	13.9	1.047	0.582
92	69.25/32.5/10.5	4.5	1.288	0.900	76.8	1.087	0.781	18.1	1.118	0.596
Mean	**61.27/35.6/9.72**	**12.4**	**1.178**	**0.844**	**62.1**	**1.122**	**0.789**	**25.0**	**1.172**	**0.632**
	mean	13.59	1.176	0.861	60.2	1.113	0.794	25.6	1.170	0.624
	COV	0.472	0.053	0.023	0.23	0.040	0.018	0.35	0.065	0.069

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	111.25/26.75/8.25	0.0	0.092		83.1	1.005	0.711	14.5	1.151	0.561
2	78/38.25/8.25	4.1	1.105	0.777	59.6	1.079	0.737	33.9	1.288	0.672
3	65/44.25/8.25	14.4	1.002	0.787	37.2	1.122	0.752	46.0	1.342	0.665
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	73/44.75/8.25	6.9	1.041	0.745	43.4	1.134	0.759	47.2	1.347	0.666
100	75/42/8.25	5.5	1.062	0.717	50.6	1.123	0.752	41.4	1.325	0.666
Mean	**87.9775/38.2625/8.25**	**1.1**	**1.168**		**62.5**	**1.087**	**0.733**	**34.0**	**1.288**	**0.672**
	mean	3.968	0.872	0.752	59.3	1.083	0.740	34.3	1.284	0.642
	COV	1.121	0.403	0.038	0.23	0.041	0.019	0.32	0.045	0.056

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	78/38.25/8.25	4.1	1.105	0.777	59.6	1.079	0.737	33.9	1.288	0.672
2	65/44.25/8.25	14.4	1.002	0.787	37.2	1.122	0.752	46.0	1.342	0.665
3	79.75/33.5/8.25	3.4	1.158	0.716	69.5	1.043	0.740	24.7	1.237	0.609
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
55	73/44.75/8.25	6.9	1.041	0.745	43.4	1.134	0.759	47.2	1.347	0.666
56	75/42/8.25	5.5	1.062	0.717	50.6	1.123	0.752	41.4	1.325	0.666
Mean	**73.9286/39.8214/8.25**	**6.2**	**1.043**	**0.736**	**54.4**	**1.096**	**0.730**	**37.0**	**1.305**	**0.666**
	mean	7.016	1.070	0.752	53.3	1.088	0.744	37.2	1.300	0.647
	COV	0.535	0.051	0.038	0.24	0.041	0.016	0.3	0.041	0.054

Table 1 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 2500$)

Good grading quality $r^2 = 0.64$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	63.25/34/8.25	9.7	1.217	0.882	68.3	1.099	0.785	21.6	1.126	0.588
2	61/37/9.5	12.8	1.169	0.834	58.3	1.135	0.814	28.4	1.195	0.648
3	62.75/35.5/8.25	10.3	1.205	0.883	64.3	1.124	0.789	25.0	1.148	0.604
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	62/35/8.25	11.3	1.197	0.868	64.6	1.108	0.799	23.7	1.139	0.604
100	61.75/33.75/8.5	11.7	1.193	0.871	66.9	1.094	0.785	21.0	1.123	0.584
Mean	**62.62/35.48/8.98**	**10.5**	**1.205**	**0.883**	**64.2**	**1.123**	**0.789**	**24.8**	**1.166**	**0.609**
	mean	12.04	1.181	0.864	62.4	1.116	0.796	25.1	1.161	0.618
	COV	0.443	0.109	0.023	0.17	0.025	0.017	0.23	0.046	0.056

Good grading quality $r^2 = 0.64$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	63.25/34/8.25	9.7	1.217	0.882	68.3	1.099	0.785	21.6	1.126	0.588
2	61/37/9.5	12.8	1.169	0.834	58.3	1.135	0.814	28.4	1.195	0.648
3	62.75/35.5/8.25	10.3	1.205	0.883	64.3	1.124	0.789	25.0	1.148	0.604
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	62/35/8.25	11.3	1.197	0.868	64.6	1.108	0.799	23.7	1.139	0.604
99	61.75/33.75/8.5	11.7	1.193	0.871	66.9	1.094	0.785	21.0	1.123	0.584
Mean	**62.24/35.53/8.99**	**11.0**	**1.204**	**0.864**	**63.6**	**1.123**	**0.789**	**25.0**	**1.168**	**0.611**
	mean	12.16	1.192	0.864	62.2	1.116	0.796	25.2	1.163	0.618
	COV	0.429	0.057	0.023	0.16	0.025	0.016	0.23	0.045	0.056

Poor grading quality $r^2 = 0.36$	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	73.75/35.5/8.25	6.3	1.042	0.740	63.1	1.048	0.742	28.1	1.259	0.596
2	70.75/42.5/8.25	8.6	1.040	0.764	46.7	1.123	0.752	42.3	1.326	0.668
3	74.75/41./8.25	5.7	1.059	0.720	52.3	1.104	0.742	39.6	1.315	0.666
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	71.5/45./8.25	7.9	1.041	0.756	41.9	1.133	0.754	47.7	1.349	0.668
100	120.75/38./8.25	0.0	0.092		64.2	1.087	0.733	33.4	1.287	0.670
Mean	**90.16/37.21/8.25**	**0.8**	**1.177**		**65.1**	**1.079**	**0.738**	**31.7**	**1.282**	**0.655**
	mean	2.838	0.808	0.741	62.8	1.076	0.740	31.9	1.275	0.634
	COV	1.085	0.547	0.038	0.14	0.029	0.010	0.22	0.029	0.050

Poor grading quality $r^2 = 0.36$	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**57.4**	**1.014**	**0.964**	**26.4**	**0.954**	**0.885**	**16.2**	**1.013**	**0.546**
1	73.75/35.5/8.25	6.3	1.042	0.740	63.1	1.048	0.742	28.1	1.259	0.596
2	70.75/42.5/8.25	8.6	1.040	0.764	46.7	1.123	0.752	42.3	1.326	0.668
3	74.75/41./8.25	5.7	1.059	0.720	52.3	1.104	0.742	39.6	1.315	0.666
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
61	69./43.25/8.25	10.0	1.025	0.790	43.5	1.124	0.752	44.0	1.331	0.665
62	71.5/45./8.25	7.9	1.041	0.756	41.9	1.133	0.754	47.7	1.349	0.668
Mean	**78.25/37.8/8.25**	**4.0**	**1.118**	**0.766**	**60.7**	**1.079**	**0.738**	**33.0**	**1.286**	**0.668**
	mean	4.558	1.101	0.741	59.9	1.078	0.741	33.1	1.281	0.639
	COV	0.599	0.049	0.038	0.14	0.030	0.010	0.22	0.029	0.050

Table 2: Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 250$)

Spruce (depth of 150 mm)	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**61.9**	**1.005**	**0.971**	**28.7**	**0.921**	**0.916**	**9.4**	**0.912**	**0.434**
1	10002/8030/6783	32.8	1.017	0.831	55.7	0.966	0.762	10.2	1.075	0.572
2	9753/8424/6019	40.9	0.982	0.826	39.7	0.997	0.781	19.2	1.078	0.613
3	10086/8041/6793	30.4	1.031	0.824	58.0	0.969	0.769	10.3	1.077	0.572
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	10185/7565/6684	27.6	1.046	0.842	66.6	0.922	0.724	4.8	0.970	0.579
100	10202/7926/6792	27.1	1.049	0.839	63.2	0.967	0.757	8.4	1.045	0.576
Mean	**9944.36/8211.57/6767.35**	**34.7**	**1.011**	**0.835**	**50.3**	**0.979**	**0.779**	**13.7**	**1.125**	**0.590**
	mean	35.29	1.011	0.828	48.6	0.983	0.768	13.6	1.118	0.624
	COV	0.264	0.049	0.013	0.29	0.055	0.023	0.58	0.131	0.096

All spruce	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	14791/7735/7023	0.2	1.088		69.1	0.982	0.709	11.3	1.268	0.581
2	14788/6779/3868	0.2	1.088		83.2	0.916	0.673	15.8	0.968	0.481
3	15147/6855/3520	0.1	0.687		82.6	0.920	0.676	16.8	0.945	0.485
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11058/7398/4784	14.5	0.932	0.764	60.4	0.920	0.713	22.6	1.078	0.549
100	11220/7297/4342	12.9	0.937	0.777	63.8	0.914	0.682	21.9	1.043	0.493
Mean	**13468.6/7412.4/4102.**	**1.2**	**1.148**		**73.6**	**0.953**	**0.705**	**24.2**	**1.040**	**0.485**
	mean	5.32	0.906	0.761	68.58	0.945	0.694	24.07	1.056	0.515
	COV	1.40	0.289	0.015	0.20	0.046	0.033	0.43	0.077	0.096

All spruce	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	10730/7425/4554	18.1	0.903	0.755	56.5	0.914	0.705	23.6	1.063	0.495
2	10725/8295/2958	18.2	0.903	0.755	41.7	0.974	0.716	40.0	1.115	0.497
3	10893/7268/4188	16.1	0.916	0.767	61.0	0.905	0.685	21.7	1.039	0.494
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
34	11058/7398/4784	14.5	0.932	0.764	60.4	0.920	0.713	22.6	1.078	0.549
35	11220/7297/4342	12.9	0.937	0.777	63.8	0.914	0.682	21.9	1.043	0.493
Mean	**11072.2/7704.7/3932.3**	**14.3**	**0.933**	**0.78**	**55.5**	**0.944**	**0.725**	**29.3**	**1.075**	**0.487**
	mean	14.63	0.932	0.761	54.81	0.943	0.704	28.95	1.082	0.513
	COV	0.33	0.036	0.015	0.20	0.049	0.028	0.38	0.063	0.090

Table 2 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 500$)

Spruce (depth of 150 mm)	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**61.9**	**1.005**	**0.971**	**28.7**	**0.921**	**0.916**	**9.4**	**0.912**	**0.434**
1	9701/8334/7050	42.5	0.970	0.818	40.0	0.975	0.796	15.3	1.171	0.738
2	9811/8255/7929	39.0	0.989	0.825	45.1	0.973	0.796	6.1	1.323	0.712
3	10321.5/8252/6702	24.1	1.068	0.846	60.1	1.009	0.782	14.8	1.106	0.607
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9631/7670/6747	45.0	0.962	0.821	48.1	0.893	0.744	5.7	1.007	0.584
100	9865.5/8465/7984	37.1	1.000	0.824	42.4	1.006	0.793	9.8	1.375	0.721
Mean	**9912.73/8076.79/6863.42**	**35.6**	**1.006**	**0.836**	**52.1**	**0.963**	**0.763**	**10.8**	**1.095**	**0.568**
	mean	35.91	1.005	0.828	50.8	0.964	0.763	11	1.101	0.616
	COV	0.169	0.030	0.009	0.19	0.042	0.025	0.58	0.098	0.101

All spruce	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11247/7284/3687	12.7	0.936	0.778	64.2	0.914	0.683	22.5	1.018	0.496
2	14932/7000/5064	0.2	0.973		80.7	0.924	0.678	15.5	1.042	0.656
3	14386/7075/4059	0.4	1.140		79.5	0.925	0.678	19.1	1.017	0.489
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11024/7099/3219	14.9	0.930	0.757	64.6	0.896	0.678	20.2	0.974	0.480
100	11344/7535/3460	11.6	0.946	0.771	61.2	0.934	0.711	26.8	1.037	0.488
Mean	**12862.1/7143.8/3977.2**	**2.4**	**1.106**	**0.71**	**76.6**	**0.926**	**0.691**	**20.2**	**1.017**	**0.495**
	mean	6.16	0.927	0.753	72.00	0.924	0.686	20.39	1.016	0.504
	COV	0.96	0.240	0.030	0.14	0.034	0.033	0.36	0.062	0.081

All spruce	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11247/7284/3687	12.7	0.936	0.778	64.2	0.914	0.683	22.5	1.018	0.496
2	12596/7466/4485	3.2	1.071	0.695	70.9	0.951	0.708	24.3	1.064	0.485
3	12120/6318/4153	5.5	1.027	0.729	83.2	0.877	0.653	10.2	0.924	0.491
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
59	11024/7099/3219	14.9	0.930	0.757	64.6	0.896	0.678	20.2	0.974	0.480
60	11344/7535/3460	11.6	0.946	0.771	61.2	0.934	0.711	26.8	1.037	0.488
Mean	**11560.1/7250.4/3992.2**	**9.5**	**0.971**	**0.758**	**67.9**	**0.920**	**0.686**	**21.8**	**1.027**	**0.496**
	mean	10.08	0.973	0.753	66.60	0.922	0.691	21.81	1.029	0.506
	COV	0.44	0.047	0.030	0.13	0.034	0.033	0.34	0.060	0.086

Table 2 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 750$)

Spruce (depth of 150 mm)	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**61.9**	**1.005**	**0.971**	**28.7**	**0.921**	**0.916**	**9.4**	**0.912**	**0.434**
1	9705/7710/6596	42.4	0.972	0.820	50.2	0.905	0.751	6.5	0.986	0.601
2	10041/7669/7116	31.8	1.021	0.825	61.4	0.926	0.732	4.4	1.038	0.529
3	10015/8299/6712	32.5	1.018	0.830	50.7	0.998	0.781	15.6	1.126	0.612
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9877/7536/6119	36.7	1.002	0.827	57.7	0.898	0.719	5.3	0.913	0.625
100	10188/8061/6020	27.5	1.047	0.841	60.5	0.975	0.772	11.8	1.008	0.607
Mean	**9884.93/7820.97/6467.93**	**36.5**	**1.004**	**0.825**	**55.0**	**0.935**	**0.748**	**7.8**	**0.999**	**0.599**
	mean	36.71	1.000	0.828	54.4	0.932	0.748	8.09	1.000	0.604
	COV	0.146	0.026	0.007	0.13	0.026	0.023	0.38	0.047	0.038

All spruce	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11384/7233/5708	11.1	0.947	0.77	66.5	0.915	0.686	15.7	1.138	0.620
2	12264/7599/3184	4.6	1.030	0.715	67.1	0.960	0.714	28.1	1.037	0.486
3	12131/7049/2915	5.4	1.027	0.728	74.8	0.916	0.678	19.6	0.967	0.479
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	15592/7070/4426	0.1	0.457		79.9	0.926	0.678	18.5	1.030	0.503
100	12126/7233/3976	5.4	1.027	0.729	72.2	0.925	0.696	21.5	1.023	0.497
Mean	**12855.6/6976.8/3858.4**	**2.4**	**1.106**	**0.711**	**78.7**	**0.921**	**0.678**	**18.2**	**0.984**	**0.492**
	mean	5.95	0.901	0.750	74.71	0.913	0.680	18.07	0.993	0.498
	COV	0.86	0.286	0.032	0.12	0.023	0.029	0.31	0.057	0.081

All spruce	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11384/7233/5708	11.1	0.947	0.770	66.5	0.915	0.686	15.7	1.138	0.620
2	12264/7599/3184	4.6	1.030	0.715	67.1	0.960	0.714	28.1	1.037	0.486
3	12131/7049/2915	5.4	1.027	0.728	74.8	0.916	0.678	19.6	0.967	0.479
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
66	11334/6981/3523	11.7	0.947	0.771	69.4	0.902	0.670	18.5	0.973	0.483
67	12126/7233/3976	5.4	1.027	0.729	72.2	0.925	0.696	21.5	1.023	0.497
Mean	**11717.6/7065.42/3863.58**	**8.1**	**0.983**	**0.741**	**71.9**	**0.912**	**0.680**	**19.2**	**0.995**	**0.502**
	mean	8.79	0.986	0.750	70.70	0.912	0.684	19.29	1.002	0.500
	COV	0.44	0.049	0.032	0.11	0.024	0.030	0.29	0.052	0.081

Table 2 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1000$)

Spruce (depth of 150 mm)	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**61.9**	**1.005**	**0.971**	**28.7**	**0.921**	**0.916**	**9.4**	**0.912**	**0.434**
1	9949./7745./6735.	34.6	1.011	0.835	57.7	0.929	0.749	6.6	1.006	0.589
2	9685.5/7889./6562.	43.2	0.967	0.822	47.6	0.928	0.750	8.4	1.006	0.598
3	9670./7634./7063.	43.7	0.965	0.821	49.8	0.890	0.730	4.3	1.045	0.523
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9751./7618./7087.5	40.9	0.982	0.826	52.7	0.898	0.732	4.0	1.042	0.513
100	9902./8062./7826.	35.9	1.006	0.825	52.1	0.962	0.764	3.4	1.297	0.716
Mean	**9871.8/7919.5/6883.3**	**36.9**	**1.001**	**0.825**	**53.4**	**0.942**	**0.754**	**8.1**	**1.045**	**0.574**
	mean	37.09	0.999	0.828	52.3	0.943	0.754	8.34	1.070	0.584
	COV	0.134	0.024	0.008	0.14	0.039	0.026	0.57	0.092	0.188

All spruce	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11970/6375/4342	6.4	1.017	0.755	81.7	0.877	0.651	10.5	0.961	0.490
2	11758/6548/3614	7.8	1.000	0.754	78.2	0.884	0.657	13.4	0.934	0.495
3	15473/6101/4137	0.1	0.457		90.5	0.880	0.635	8.4	0.893	0.481
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	15942/6655/3871	0	0.163		84.9	0.905	0.660	14.3	0.958	0.489
100	14787/5845/5148	0.2	1.088		92.2	0.866	0.632	3.7	0.943	0.507
Mean	**12459./6906.45/3902.12**	**3.7**	**1.051**	**0.717**	**78.3**	**0.915**	**0.675**	**17.2**	**0.982**	**0.484**
	mean	6.65	0.915	0.744	74.84	0.908	0.675	17.37	0.984	0.496
	COV	0.71	0.231	0.037	0.12	0.023	0.031	0.33	0.058	0.068

All spruce	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11970/6375/4342	6.4	1.017	0.755	81.7	0.877	0.651	10.5	0.961	0.490
2	11758/6548/3614	7.8	1.000	0.754	78.2	0.884	0.657	13.4	0.934	0.495
3	11929/7032/3601	6.6	1.016	0.759	73.8	0.914	0.679	19.0	0.983	0.482
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
81	10881/7648/4107	16.3	0.917	0.766	54.7	0.935	0.719	28.0	1.072	0.479
82	12634/6400/3695	3.1	1.075	0.692	84.7	0.884	0.656	11.6	0.922	0.484
Mean	**11823.6/6987.16/3885.6**	**7.3**	**1.004**	**0.744**	**73.7**	**0.911**	**0.680**	**18.2**	**0.985**	**0.491**
	mean	8.09	0.997	0.744	72.41	0.909	0.679	18.36	0.992	0.497
	COV	0.49	0.052	0.037	0.11	0.024	0.030	0.31	0.058	0.073

Table 2 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1250$)

Spruce (depth of 150 mm)	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**61.9**	**1.005**	**0.971**	**28.7**	**0.921**	**0.916**	**9.4**	**0.912**	**0.434**
1	9937/7971/6623	34.9	1.009	0.836	54.7	0.953	0.758	9.5	1.027	0.593
2	10020/7820/6616	32.4	1.019	0.828	59.0	0.941	0.752	7.6	1.006	0.594
3	9797/7408/6433	39.5	0.987	0.828	56.3	0.876	0.720	3.6	0.952	0.641
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9701/7899/6532	42.5	0.970	0.818	48.1	0.930	0.749	8.5	1.007	0.598
100	9668/7816/6280	43.7	0.965	0.821	47.8	0.921	0.748	8.0	0.982	0.609
Mean	**9892.07/7734.89/6462.97**	**36.2**	**1.004**	**0.825**	**56.2**	**0.925**	**0.751**	**6.9**	**0.983**	**0.605**
	mean	36.38	1.002	0.828	55.9	0.923	0.744	6.96	0.986	0.604
	COV	0.1	0.018	0.006	0.08	0.024	0.020	0.34	0.036	0.040

All spruce	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	15917/6753/3659	0	0.163		83.8	0.915	0.672	15.6	0.957	0.471
2	11956/7415/3591	6.5	1.017	0.755	68.3	0.939	0.713	24.7	1.023	0.478
3	12505/6701/3182	3.6	1.078	0.696	80.8	0.903	0.667	15.4	0.925	0.487
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	12032/6419/4634	6	1.014	0.747	81.6	0.880	0.657	10.4	0.985	0.525
100	11097/7071/3532	14.1	0.932	0.781	65.8	0.896	0.673	19.6	0.983	0.486
Mean	**12645.7/6886.59/3773.95**	**3.1**	**1.075**	**0.691**	**79.2**	**0.914**	**0.677**	**17.0**	**0.970**	**0.474**
	mean	6.50	0.870	0.736	75.31	0.907	0.675	17.29	0.976	0.489
	COV	0.74	0.326	0.120	0.11	0.023	0.029	0.32	0.049	0.042

All spruce	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	11956/7415/3591	6.5	1.017	0.755	68.3	0.939	0.713	24.7	1.023	0.478
2	12505/6701/3182	3.6	1.078	0.696	80.8	0.903	0.667	15.4	0.925	0.487
3	11397/6262/3486	11.0	0.946	0.770	78.2	0.861	0.643	10.4	0.879	0.488
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
78	12032/6419/4634	6.0	1.014	0.747	81.6	0.880	0.657	10.4	0.985	0.525
79	11097/7071/3532	14.1	0.932	0.781	65.8	0.896	0.673	19.6	0.983	0.486
Mean	**11812.8/6955.86/3725.72**	**7.4**	**1**	**0.748**	**74.0**	**0.909**	**0.677**	**18.0**	**0.979**	**0.477**
	mean	8.21	0.996	0.736	72.71	0.907	0.678	18.26	0.981	0.489
	COV	0.48	0.056	0.120	0.10	0.025	0.030	0.30	0.050	0.040

Table 2 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 2500$)

Spruce (depth of 150 mm)	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**61.9**	**1.005**	**0.971**	**28.7**	**0.921**	**0.916**	**9.4**	**0.912**	**0.434**
1	9946/7674/6410	34.7	1.011	0.835	58.4	0.922	0.736	6.3	0.981	0.603
2	9836/7603/6658	38.1	0.991	0.829	55.7	0.900	0.738	5.3	0.981	0.586
3	9764/7734/6355	40.5	0.982	0.827	52.0	0.918	0.750	7.0	0.981	0.606
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9801/7689/6278	39.4	0.988	0.827	53.6	0.913	0.741	6.6	0.967	0.613
100	10071/7405/6492	30.9	1.029	0.821	64.9	0.896	0.716	3.5	0.953	0.639
Mean	**9857.64/7688.1/6478.63**	**37.4**	**0.996**	**0.826**	**55.6**	**0.918**	**0.738**	**6.4**	**0.983**	**0.603**
	mean	37.45	0.997	0.828	55.4	0.915	0.741	6.35	0.981	0.600
	COV	0.081	0.015	0.005	0.07	0.017	0.018	0.25	0.026	0.038

All spruce	**All samples are included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	12428/6582/4645	3.9	1.054	0.716	81.7	0.894	0.655	12.4	0.993	0.534
2	13685/6840/3297	0.9	1.129		82.0	0.917	0.677	16.8	0.939	0.487
3	11371/6491/3238	11.3	0.947	0.77	75.4	0.876	0.659	13.1	0.894	0.491
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11623/7033/3267	9	0.970	0.756	71.4	0.909	0.681	19.3	0.972	0.490
100	11445/7000/3135	10.5	0.952	0.765	70.4	0.905	0.670	18.9	0.967	0.474
Mean	**12344.5/6839.4/3739.8**	**4.2**	**1.042**	**0.701**	**78.7**	**0.911**	**0.676**	**16.5**	**0.967**	**0.471**
	mean	6.67	0.921	0.747	75.95	0.903	0.671	16.52	0.969	0.494
	COV	0.56	0.262	0.030	0.08	0.015	0.018	0.22	0.042	0.058

All spruce	**Only non-zero yield in C40 is included**									
		C40			C30			C18		
		Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
	Optimum grading	**52.5**	**1.005**	**0.975**	**33.3**	**0.908**	**0.931**	**13.9**	**0.895**	**0.804**
1	12428/6582/4645	3.9	1.054	0.716	81.7	0.894	0.655	12.4	0.993	0.534
2	13685/6840/3297	0.9	1.129		82.0	0.917	0.677	16.8	0.939	0.487
3	11371/6491/3238	11.3	0.947	0.770	75.4	0.876	0.659	13.1	0.894	0.491
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
87	11623/7033/3267	9.0	0.970	0.756	71.4	0.909	0.681	19.3	0.972	0.490
88	11445/7000/3135	10.5	0.952	0.765	70.4	0.905	0.670	18.9	0.967	0.474
Mean	**11868.1/6873.0/3720.9**	**7**	**1.016**	**0.768**	**75.5**	**0.905**	**0.679**	**16.9**	**0.967**	**0.474**
	mean	7.58	1.001	0.747	74.66	0.903	0.672	16.91	0.972	0.494
	COV	0.39	0.045	0.030	0.07	0.014	0.017	0.21	0.040	0.061

Table 3: Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 250$)

Grading according to MOE only	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	10132/8137/6671	0/0/0	30.2	1.044	0.844	57.3	0.993	0.738	11.7	0.995	0.638
2	9670/8499/6337	0/0/0	44.8	0.973	0.810	35.4	1.006	0.766	19.3	1.099	0.589
3	10104/7983/6398	0/0/0	31.1	1.042	0.819	58.7	0.975	0.732	9.7	0.944	0.524
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	10050/7805/6693	0/0/0	32.5	1.030	0.828	59.4	0.954	0.739	7.1	0.933	0.622
100	9407/8265/6238	0/0/0	53.3	0.935	0.783	31.6	0.964	0.758	14.7	1.005	0.619
Mean	**9978/8218/6778**	**0/0/0**	**34.5**	**1.021**	**0.828**	**51.3**	**0.996**	**0.767**	**13.1**	**1.045**	**0.636**
	mean		35.7	1.014	0.821	49.1	0.992	0.752	12.83	1.070	0.606
	COV		0.26	0.045	0.016	0.25	0.048	0.033	0.475	0.166	0.117

Grading according to MOE only	**Spruce (depth of 150 mm)**	**Only samples with non-zero yield in C40**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	10132/8137/6671	0/0/0	30.2	1.044	0.844	57.3	0.993	0.738	11.7	0.995	0.638
2	9670/8499/6337	0/0/0	44.8	0.973	0.810	35.4	1.006	0.766	19.3	1.099	0.589
3	10104/7983/6398	0/0/0	31.1	1.042	0.819	58.7	0.975	0.732	9.7	0.944	0.524
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	10050/7805/6693	0/0/0	32.5	1.030	0.828	59.4	0.954	0.739	7.1	0.933	0.622
99	9407/8265/6238	0/0/0	53.3	0.935	0.783	31.6	0.964	0.758	14.7	1.005	0.619
Mean	**9948/8218/6778**	**0/0/0**	**35.4**	**1.013**	**0.828**	**50.4**	**0.994**	**0.768**	**13.1**	**1.045**	**0.637**
	mean		36.03	1.016	0.821	48.72	0.992	0.752	12.84	1.070	0.605
	COV		0.23	0.043	0.016	0.24	0.048	0.033	0.48	0.167	0.118

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	8614/8123/6678	482.5/361.5/249.5	19.9	0.952	0.772	67.5	1.016	0.763	11.7	0.999	0.650
2	9622/8506/6326	399.5/318/184.5	46	0.968	0.812	34.1	1.003	0.767	19.4	1.100	0.589
3	10104/7983/6398	370.5/301/89	31.1	1.042	0.819	58.7	0.975	0.732	9.7	0.944	0.524
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	10050/7805/6693	350/274.5/108	32.5	1.030	0.828	59.4	0.954	0.739	7.1	0.933	0.622
100	9445/8273/6231	397/351/108.5	51.6	0.939	0.805	33.1	0.967	0.757	14.9	1.007	0.617
Mean	**9900/8145/6724**	**395.6/333.5/162.5**	**36.8**	**1.007**	**0.824**	**50.4**	**0.978**	**0.733**	**11.8**	**1.007**	**0.645**
	mean		35.21	1.008	0.817	48.91	0.991	0.747	13.85	1.077	0.605
	COV		0.27	0.059	0.019	0.26	0.054	0.042	0.55	0.157	0.107

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**Only samples with non-zero yield in C40**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	8614/8123/6678	482.5/361.5/249.5	19.9	0.952	0.772	67.5	1.016	0.763	11.7	0.999	0.650
2	9622/8506/6326	399.5/318/184.5	46.0	0.968	0.812	34.1	1.003	0.767	19.4	1.100	0.589
3	10104/7983/6398	370.5/301/89	31.1	1.042	0.819	58.7	0.975	0.732	9.7	0.944	0.524
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
97	10050/7805/6693	350/274.5/108	32.5	1.030	0.828	59.4	0.954	0.739	7.1	0.933	0.622
98	9445/8273/6231	397/351/108.5	51.6	0.939	0.805	33.1	0.967	0.757	14.9	1.007	0.617
Mean	**9840/8141/6712**	**394.4/333.4/160.9**	**39**	**1**	**0.814**	**48.4**	**0.975**	**0.729**	**11.7**	**1.007**	**0.644**
	mean		35.92	1.006	0.817	48.22	0.989	0.746	13.87	1.074	0.603
	COV		0.23	0.043	0.019	0.24	0.053	0.042	0.56	0.156	0.107

Table 3 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 500$)

Grading according to MOE only	**Spruce (depth of 150 mm)**	**All samples are included**							
			C40			C30			C18
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	10083/8172/6545	0/0/0	31.7	1.036	0.822	55.1	0.994	0.749	12.6	0.999	0.627
2	9974/7923/6556	0/0/0	34.7	1.019	0.830	55.9	0.962	0.736	8.7	0.941	0.539
3	10257/7526/6914	0/0/0	26.6	1.069	0.832	68.0	0.933	0.709	3.8	0.924	0.693
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9753/7961/6617	0/0/0	41.9	0.985	0.814	48.3	0.951	0.722	9.1	0.959	0.600
100	9547/7819/6646	0/0/0	49.0	0.957	0.806	42.8	0.916	0.737	7.3	0.922	0.560
Mean	**9913/7911/6530**	**0/0/0**	**36.5**	**1.008**	**0.828**	**54.3**	**0.956**	**0.730**	**8.5**	**0.940**	**0.538**
	mean		36.9	1.010	0.821	53.3	0.954	0.732	8.85	0.946	0.559
	COV		0.18	0.033	0.013	0.16	0.029	0.026	0.4	0.098	0.150

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	10083/8172/6545	424/287.5/95	30.8	1.036	0.819	55.9	0.995	0.756	12.6	0.999	0.627
2	9942/7908/6593	394.5/326/103.5	35.5	1.012	0.829	55.3	0.957	0.736	8.4	0.941	0.537
3	10257/7476/6914	423.5/349/108	26.1	1.069	0.831	68.8	0.931	0.707	3.6	0.927	0.680
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9777/7958/6613	350.5/323.5/209.5	41.1	0.989	0.811	49.1	0.953	0.723	9.0	0.959	0.599
100	9497/7819/6646	416.5/338/248.5	48.6	0.952	0.809	43.2	0.918	0.736	7.4	0.924	0.559
Mean	**9844/7818/6545**	**399.5/326.8/170.9**	**38.8**	**1**	**0.814**	**53.0**	**0.940**	**0.732**	**7.5**	**0.906**	**0.543**
	mean		36.37	1.004	0.820	54.18	0.949	0.726	8.44	0.947	0.554
	COV		0.19	0.030	0.013	0.16	0.031	0.032	0.42	0.114	0.174

Table 7 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 750$)

Grading according to MOE only	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	10083/7922/7145	0/0/0	31.7	1.036	0.822	59.0	0.968	0.733	6.7	1.049	0.674
2	10024/7623/6556	0/0/0	33.0	1.027	0.830	60.8	0.931	0.726	5.4	0.854	0.487
3	9924/7819/6434	0/0/0	36.1	1.010	0.827	55.7	0.945	0.731	7.7	0.895	0.552
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9608/8049/6345	0/0/0	46.9	0.965	0.809	42.0	0.949	0.723	10.7	0.961	0.529
100	9697/7669/6546	0/0/0	43.8	0.976	0.807	49.6	0.914	0.736	5.9	0.869	0.528
Mean	**9895/7800/6467**	**0/0/0**	**37.1**	**1.006**	**0.821**	**54.8**	**0.941**	**0.734**	**7.5**	**0.895**	**0.548**
	mean		37.4	1.007	0.820	54.2	0.941	0.727	7.61	0.907	0.521
	COV		0.15	0.028	0.011	0.13	0.028	0.024	0.43	0.103	0.211

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	10192/7752/6399	408/402/138.5	28.2	1.051	0.84	55.8	0.978	0.725	15.5	1.045	0.606
2	9976/7557/6550	422/313.5/204.5	33.7	1.019	0.829	60.7	0.923	0.711	5.0	0.852	0.442
3	9924/7819/6434	395/339/189	36	1.010	0.827	55.7	0.946	0.730	7.7	0.896	0.553
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9627/7758/6763	343/313.5/217	46.2	0.968	0.812	46.2	0.918	0.737	6.5	0.921	0.596
100	9697/7669/6546	426.5/330.5/243.5	41.2	0.979	0.813	52.2	0.921	0.735	5.9	0.870	0.529
Mean	**9875/7722/6493**	**393.4/322.8/166.7**	**37.8**	**1.003**	**0.813**	**55.0**	**0.933**	**0.732**	**6.6**	**0.877**	**0.543**
	mean		36.84	1.006	0.820	54.74	0.938	0.721	7.57	0.916	0.527
	COV		0.15	0.025	0.011	0.13	0.030	0.033	0.51	0.109	0.216

Table 3 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1000$)

Grading according to MOE only	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	9959/8106/6401	0/0/0	35.1	1.016	0.830	52.9	0.978	0.730	11.6	0.983	0.527
2	10251/7570/6660	0/0/0	26.8	1.066	0.830	67.5	0.938	0.716	4.9	0.875	0.549
3	9703/8226/6565	0/0/0	43.6	0.977	0.811	42.1	0.976	0.772	13.6	1.018	0.623
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9927/8008/6313	0/0/0	36.0	1.010	0.827	53.5	0.968	0.729	10.1	0.948	0.529
100	9895/7523/6881	0/0/0	37.1	1.006	0.821	57.6	0.910	0.706	3.9	0.912	0.573
Mean	**9888/7725/6582**	**0/0/0**	**37.4**	**1.005**	**0.815**	**55.4**	**0.934**	**0.733**	**6.5**	**0.885**	**0.535**
	mean		37.6	1.006	0.820	55	0.933	0.724	6.49	0.901	0.552
	COV		0.13	0.025	0.011	0.11	0.022	0.020	0.37	0.071	0.192

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	9959/8106/6401	422/284/94	34.2	1.016	0.827	53.8	0.979	0.729	11.6	0.983	0.527
2	9992/7808/6643	397/321/106	34	1.022	0.827	57.9	0.952	0.731	7.2	0.919	0.560
3	9703/8226/6565	319/300.5/150	43.6	0.977	0.811	42.1	0.976	0.772	13.6	1.018	0.623
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9927/8008/6313	358/326/194.5	36	1.010	0.827	53.5	0.968	0.729	10.1	0.948	0.529
100	9895/7523/6881	419.5/313.5/141	36.3	1.006	0.823	58.4	0.912	0.706	3.9	0.913	0.573
Mean	**9864/7676/6573**	**392.5/324.6/157.6**	**38.2**	**1.002**	**0.815**	**55.0**	**0.928**	**0.732**	**6.0**	**0.876**	**0.530**
	mean		37.04	1.004	0.819	55.58	0.931	0.719	6.49	0.906	0.537
	COV		0.12	0.022	0.011	0.10	0.024	0.026	0.37	0.077	0.207

Table 7 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1250$)

Grading according to MOE only	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	9959/7906/6451	0/0/0	35.1	1.016	0.830	55.7	0.958	0.738	8.6	0.926	0.548
2	10024/7823/6606	0/0/0	33.0	1.027	0.830	58.7	0.954	0.734	7.4	0.919	0.533
3	9698/7946/6656	0/0/0	43.8	0.976	0.807	46.5	0.945	0.728	8.9	0.956	0.593
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9988/7424/6550	0/0/0	34.2	1.022	0.827	61.4	0.907	0.691	3.7	0.794	0.314
100	9842/7686/6141	0/0/0	39.0	1.001	0.814	54.1	0.928	0.734	6.7	0.826	0.423
Mean	**9874/7718/6536**	**0/0/0**	**37.9**	**1.004**	**0.813**	**54.9**	**0.932**	**0.734**	**6.5**	**0.877**	**0.539**
	mean		38	1.004	0.820	54.7	0.931	0.723	6.5	0.890	0.520
	COV		0.12	0.021	0.010	0.1	0.023	0.021	0.33	0.080	0.227

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	9959/7906/6451	422/274/96.5	34.2	1.016	0.827	56.7	0.963	0.736	8.6	0.926	0.548
2	9734/7830/6601	444/341/250	35.8	0.993	0.815	55.8	0.953	0.725	7.6	0.924	0.533
3	9608/7828/6646	433/365/49	42.4	0.970	0.8	48.6	0.940	0.731	8.2	0.942	0.565
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	10223/7207/6687	405/358/267.5	27.4	1.056	0.839	68.9	0.907	0.679	2.7	0.797	0.600
100	9780/7839/6143	315.5/302.5/99	41	0.989	0.811	50.7	0.938	0.733	8.1	0.875	0.534
Mean	**9856/7663/6540**	**395.6/322.6/166.4**	**38.4**	**1.002**	**0.815**	**55.1**	**0.924**	**0.725**	**5.9**	**0.875**	**0.520**
	mean		37.22	1.003	0.819	55.34	0.930	0.718	6.65	0.901	0.528
	COV		0.12	0.021	0.011	0.10	0.024	0.027	0.39	0.079	0.187

Table 3 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 2500$)

Grading according to MOE only	**Spruce (depth of 150 mm)**	**All samples are included**							
			C40			C30			C18
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	9959/7856/6451	0/0/0	35.1	1.016	0.830	56.3	0.954	0.732	8.0	0.905	0.553
2	9684/7680/6601	0/0/0	44.2	0.974	0.810	49.0	0.914	0.732	6.0	0.880	0.528
3	9871/7490/6291	0/0/0	38.0	1.003	0.814	57.0	0.907	0.700	4.6	0.801	0.430
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9877/7958/6413	0/0/0	37.9	1.004	0.812	52.3	0.957	0.723	9.4	0.941	0.520
100	9697/7719/6496	0/0/0	43.8	0.976	0.807	49.0	0.921	0.737	6.6	0.876	0.542
Mean	**9868/7695/6494**	**0/0/0**	**38.1**	**1.003**	**0.814**	**54.9**	**0.929**	**0.733**	**6.4**	**0.872**	**0.541**
	mean		38.1	1.003	0.819	54.9	0.929	0.724	6.27	0.881	0.532
	COV		0.09	0.016	0.009	0.08	0.015	0.017	0.28	0.057	0.180

Combined grading according to MOE and density	**Spruce (depth of 150 mm)**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**63.1**	**1.008**	**0.966**	**27.8**	**0.918**	**0.914**	**9.0**	**0.897**	**0.753**
1	9959/7856/6451	422/271.5/96.5	34.2	1.016	0.827	57.3	0.955	0.737	8.0	0.905	0.553
2	9654/7717/6611	427/337/230	42.3	0.974	0.814	50.5	0.925	0.736	6.4	0.891	0.529
3	9874/7519/6284	392.5/324/181.5	37.9	1.003	0.814	56.8	0.910	0.705	4.9	0.814	0.454
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	9877/7958/6413	355.5/323.5/199.5	37.9	1.004	0.812	52.3	0.957	0.723	9.4	0.941	0.520
100	9697/7619/6546	426.5/328/243.5	41.2	0.979	0.813	52.7	0.916	0.721	5.5	0.855	0.490
Mean	**9855/7636/6474**	**393.4/325.3/161.3**	**38.4**	**1.002**	**0.815**	**55.3**	**0.921**	**0.729**	**5.7**	**0.858**	**0.515**
	mean		37.51	1.002	0.818	55.46	0.927	0.718	6.35	0.888	0.508
	COV		0.10	0.017	0.009	0.08	0.019	0.029	0.27	0.059	0.210

Table 4: Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 250$)

Grading according to MOE only — All spruce — All samples are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	15042/7351/3791	0/0/0	0.1	0.515		77.5	0.945	0.704	21.7	1.014	0.468
2	14886/8111/3552	0/0/0	0.2	0.965		64.8	1.000	0.730	34.6	1.089	0.469
3	14380/7392/5766	0/0/0	0.4	1.193		76.4	0.949	0.708	16.6	1.133	0.570
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11861/7983/4024	0/0/0	7.6	1.002	0.787	59.5	0.975	0.714	31.9	1.095	0.465
100	13995/7120/2474	0/0/0	0.6	1.093		80.0	0.937	0.685	19.2	0.912	0.422
Mean	**13679/7354/4187**	**0/0/0**	**0.9**	**1.100**		**76.5**	**0.943**	**0.705**	**21.2**	**1.025**	**0.462**
	mean		4.3	0.871	0.745	72.1	0.946	0.686	21.5	1.024	0.466
	COV		1.54	0.363	0.049	0.16	0.037	0.037	0.44	0.105	0.125

Grading according to MOE only — All spruce — Only samples with non-zero yield in C40 are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	10680/6934/6076	0/0/0	19.2	0.914	0.690	63.7	0.898	0.661	8.5	1.086	0.543
2	10236/7700/4166	0/0/0	25.6	0.885	0.696	46.7	0.923	0.659	26.5	1.080	0.455
3	10721/7739/2765	0/0/0	18.9	0.917	0.683	52.6	0.936	0.668	28.4	1.049	0.455
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28	11089/7838/2642	0/0/0	14.3	0.934	0.746	55.4	0.950	0.708	30.2	1.063	0.456
29	11861/7983/4024	0/0/0	7.6	1.002	0.787	59.5	0.975	0.714	31.9	1.095	0.465
Mean	**11177/7522/3885**	**0/0/0**	**13.5**	**0.941**	**0.765**	**61.5**	**0.935**	**0.677**	**24.2**	**1.053**	**0.452**
	mean		13.96	0.944	0.745	59.70	0.938	0.677	25.01	1.036	0.457
	COV		0.31	0.030	0.049	0.18	0.044	0.041	0.45	0.097	0.124

Combined grading according to MOE and density — All spruce — All samples are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	15061/7278/3785	311/378/82.5	0.1	0.515		76.5	0.950	0.708	22.7	1.026	0.467
2	10469/8541/3581	392.5/369.5/35.5	21.9	0.905	0.711	34.0	0.986	0.705	43.5	1.150	0.491
3	14377/7877/4055	315/394/234	0.4	1.193		65.6	1.006	0.696	32.9	1.113	0.458
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	14585/6863/4001	408/354/100	0.3	1.138		82.8	0.930	0.669	15.9	0.914	0.438
100	10688/7440/2507	443/365/84	16.4	0.941	0.695	58.8	0.933	0.670	24.7	1.014	0.466
Mean	**12368/7373/4098**	**398.1/337.4/131.5**	**4.5**	**1.042**	**0.756**	**72.5**	**0.942**	**0.695**	**21.8**	**1.028**	**0.462**
	mean		8.60	0.914	0.737	66.17	0.944	0.682	23.28	1.036	0.468
	COV		0.93	0.273	0.050	0.23	0.034	0.037	0.44	0.103	0.117

Combined grading according to MOE and density — All spruce — Only samples with non-zero yield in C40 are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	10469/8541/3581	392.5/369.5/35.5	21.9	0.905	0.711	34.0	0.986	0.705	43.5	1.150	0.491
2	10680/6934/6076	433/307/140	17.3	0.922	0.689	65.6	0.902	0.662	8.5	1.086	0.543
3	10110/7939/3911	485.5/392.5/137	12.0	0.947	0.788	53.0	0.981	0.702	34.1	1.117	0.457
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
56	10320/7920/4073	456/281.5/23	17.6	0.918	0.696	50.7	0.949	0.714	30.7	1.091	0.463
57	10688/7440/2507	443/365/84	16.4	0.941	0.695	58.8	0.933	0.670	24.7	1.014	0.466
Mean	**10645/7585/3971**	**439.0/339.4/106.9**	**17.2**	**0.928**	**0.711**	**56.6**	**0.934**	**0.662**	**25.2**	**1.065**	**0.454**
	mean		14.82	0.938	0.737	56.42	0.945	0.684	27.13	1.063	0.467
	COV		0.31	0.032	0.050	0.21	0.038	0.040	0.41	0.093	0.115

Table 4 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 500$)

Grading according to MOE only All spruce All samples are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	15811/6700/3583	0/0/0	0.0	0.179		85.2	0.918	0.649	14.2	0.896	0.402
2	15945/7426/4741	0/0/0	0.0	0.179		76.3	0.951	0.707	21.4	1.060	0.470
3	15529/7477/4184	0/0/0	0.0	0.275		75.5	0.958	0.699	23.1	1.054	0.458
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	15054/7776/3441	0/0/0	0.1	0.515		70.6	0.983	0.700	28.9	1.069	0.455
100	15751/6820/4000	0/0/0	0.0	0.266		84.0	0.927	0.667	14.9	0.901	0.419
Mean	**13204/7068/3928**	**0/0/0**	**1.9**	**1.092**		**79.4**	**0.936**	**0.680**	**17.8**	**0.961**	**0.468**
	mean		5.72	0.814	0.765	74.8	0.928	0.674	18	0.967	0.449
	COV		0.98	0.409	0.034	0.13	0.031	0.036	0.44	0.094	0.102

Grading according to MOE only All spruce Only samples with non-zero yield in C40 are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11446/6267/3808	0/0/0	10.9	0.955	0.778	78.9	0.875	0.627	9.4	0.832	0.385
2	11452/6627/3370	0/0/0	10.8	0.957	0.776	75.1	0.893	0.646	13.7	0.889	0.391
3	12198/7211/4918	0/0/0	5.4	1.018	0.773	74.1	0.934	0.691	17.7	1.026	0.488
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
55	11554/6846/3339	0/0/0	9.9	0.965	0.774	73.9	0.910	0.657	15.9	0.894	0.397
56	11797/7432/3480	0/0/0	8.2	0.992	0.756	68.1	0.936	0.689	23.3	1.021	0.473
Mean	**11592/7101/3964**	**0/0/0**	**9.6**	**0.962**	**0.775**	**71.3**	**0.923**	**0.682**	**18.1**	**0.963**	**0.473**
	mean		10.10	0.972	0.765	70.01	0.922	0.672	18.20	0.974	0.452
	COV		0.35	0.033	0.034	0.13	0.030	0.034	0.47	0.090	0.101

Combined grading according to MOE and density All spruce All samples are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	15811/6700/3583	273.5/289.5/113.5	0	0.179		85.2	0.919	0.649	14.2	0.896	0.402
2	15928/6861/4788	448/379/79	0	0.179		81.2	0.937	0.677	16.4	0.995	0.474
3	15529/7477/4184	324.5/319/130	0	0.275		75.5	0.958	0.699	23.2	1.054	0.458
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	15044/6922/3428	467/334/57.5	0.1	0.515		82.9	0.933	0.679	16.6	0.902	0.411
100	11581/6707/4017	423.5/362/89	9.3	0.967	0.773	74.9	0.910	0.653	14.7	0.901	0.414
Mean	**12324/7049/3828**	**405.2/322.0/138.2**	**4.7**	**1.032**	**0.767**	**76.8**	**0.931**	**0.682**	**17.8**	**0.960**	**0.468**
	mean		8.52	0.857	0.754	71.71	0.925	0.670	18.55	0.964	0.446
	COV		0.77	0.303	0.043	0.14	0.029	0.033	0.39	0.088	0.096

Combined grading according to MOE and density All spruce Only samples with non-zero yield in C40 are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	10005/6722/3804	454/338.5/123	21.2	0.903	0.714	63.6	0.884	0.643	14.4	0.899	0.405
2	11452/6627/3370	308/301.5/127	10.8	0.957	0.776	75.1	0.894	0.646	13.7	0.889	0.391
3	11417/6932/3541	471.5/370.5/93.5	7.8	1.018	0.804	73.5	0.929	0.679	18.2	0.945	0.483
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
70	12038/7173/3866	394.5/395.5/187.5	6.4	1.002	0.777	68.7	0.947	0.675	24.1	1.055	0.463
71	11581/6707/4017	423.5/362/89	9.3	0.967	0.773	74.9	0.910	0.653	14.7	0.901	0.414
Mean	**11140/7095/3840**	**423.9/318.6/136.5**	**13.1**	**0.943**	**0.785**	**67.9**	**0.918**	**0.677**	**18.3**	**0.967**	**0.472**
	mean		11.95	0.960	0.754	67.78	0.921	0.668	19.04	0.972	0.449
	COV		0.37	0.038	0.043	0.13	0.027	0.029	0.38	0.084	0.089

Table 4 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 750$)

Grading according to MOE only	**All spruce**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	12148/7513/3465	0/0/0	6.0	1.026	0.728	68.0	0.942	0.668	25.4	1.015	0.442
2	12047/6803/3826	0/0/0	6.5	1.010	0.753	76.9	0.898	0.643	15.5	0.960	0.377
3	15486/7003/3501	0/0/0	0.1	0.390		80.7	0.926	0.649	18.6	0.971	0.440
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	14920/6837/5280	0/0/0	0.2	0.947		82.8	0.913	0.643	12.5	1.035	0.527
100	17102/6862/4071	0/0/0	0.0	0.091		82.5	0.916	0.640	16.2	0.999	0.372
Mean	**12868/6926/3905**	**0/0/0**	**2.9**	**1.098**	**0.831**	**78.9**	**0.911**	**0.638**	**17.1**	**0.995**	**0.405**
	mean		5.97	0.908	0.753	75.4	0.909	0.648	17.1	0.968	0.427
	COV		0.79	0.299	0.025	0.12	0.029	0.021	0.36	0.078	0.136

Grading according to MOE only	**All spruce**	**Only samples with non-zero yield in C40 are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	12148/7513/3465	0/0/0	6.0	1.026	0.728	68.0	0.942	0.668	25.4	1.015	0.442
2	12047/6803/3826	0/0/0	6.5	1.010	0.753	76.9	0.898	0.643	15.5	0.960	0.377
3	11955/6179/2652	0/0/0	7.0	1.009	0.767	82.7	0.867	0.634	10.1	0.785	0.393
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
67	12113/7096/3833	0/0/0	6.2	1.023	0.734	73.5	0.916	0.647	19.3	0.998	0.437
68	11885/7184/3102	0/0/0	7.4	1.019	0.768	71.2	0.920	0.654	21.1	0.968	0.435
Mean	**11753/7040/3817**	**0/0/0**	**8.3**	**1.017**	**0.742**	**71.9**	**0.909**	**0.649**	**18.8**	**0.996**	**0.436**
	mean		8.70	1.004	0.753	71.38	0.910	0.650	18.60	0.980	0.428
	COV		0.35	0.030	0.025	0.10	0.028	0.020	0.32	0.068	0.111

Combined grading according to MOE and density	**All spruce**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11147/7088/3461	456.5/388.5/42	10.4	0.987	0.785	66.0	0.924	0.651	23.0	1.006	0.434
2	10876/6910/3841	478/324.5/167	9.6	0.976	0.802	72.3	0.900	0.635	17.1	0.988	0.407
3	15486/7003/3501	432/318/132	0.1	0.390		80.7	0.926	0.649	18.6	0.972	0.440
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11099/7206/4549	410/344/143.5	13.3	0.962	0.769	64.8	0.910	0.651	19.9	1.035	0.456
100	17102/6862/4071	448.5/314.5/182	0	0.091		82.5	0.915	0.640	16.2	0.999	0.372
Mean	**11960/6975/3867**	**420.8/318.6/138.3**	**6.8**	**1.021**	**0.752**	**74.3**	**0.908**	**0.642**	**17.9**	**0.991**	**0.432**
	mean		8.62	0.926	0.775	71.75	0.909	0.650	18.17	0.975	0.431
	COV		0.62	0.237	0.033	0.14	0.030	0.025	0.37	0.079	0.117

Combined grading according to MOE and density	**All spruce**	**Only samples with non-zero yield in C40 are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11147/7088/3461	456.5/388.5/42	10.4	0.987	0.785	66.0	0.924	0.651	23.0	1.006	0.434
2	10876/6910/3841	478/324.5/167	9.6	0.976	0.802	72.3	0.900	0.635	17.1	0.988	0.407
3	11066/8086/3601	449/364/174	11.7	0.980	0.785	52.3	0.993	0.686	35.2	1.081	0.494
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
81	11885/7184/3102	394/353.5/154	7.3	1.017	0.768	70.9	0.926	0.652	21.4	0.973	0.434
82	11099/7206/4549	410/344/143.5	13.3	0.962	0.769	64.8	0.910	0.651	19.9	1.035	0.456
Mean	**11217/7056/3846**	**432.2/321.3/132.5**	**11.7**	**0.976**	**0.764**	**68.3**	**0.903**	**0.643**	**19.0**	**0.998**	**0.436**
	mean		10.47	0.986	0.775	68.86	0.910	0.652	19.20	0.987	0.437
	COV		0.37	0.038	0.033	0.12	0.030	0.025	0.34	0.074	0.114

Table 4 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1000$)

Grading according to MOE only   All spruce   All samples are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	12098/7113/4765	0/0/0	6.2	1.012	0.743	73.2	0.916	0.653	17.9	1.023	0.461
2	12086/7234/3926	0/0/0	6.3	1.007	0.749	71.7	0.928	0.653	20.9	1.006	0.442
3	11248/7954/3569	0/0/0	12.4	0.967	0.765	54.4	0.968	0.684	32.5	1.072	0.485
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11628/7018/4669	0/0/0	9.2	1.003	0.765	71.4	0.905	0.649	17.1	1.013	0.464
100	12113/6183/4146	0/0/0	6.2	1.023	0.734	83.5	0.869	0.633	9.0	0.864	0.380
Mean	**12626/6936/3902**	**0./0./0**	**3.7**	**1.092**	**0.767**	**77.9**	**0.910**	**0.638**	**17.3**	**1.001**	**0.408**
	mean		6.61	0.916	0.753	74.8	0.908	0.647	17.23	0.972	0.415
	COV		0.69	0.278	0.027	0.1	0.022	0.016	0.296	0.061	0.101

Grading according to MOE only   All spruce   Only samples with non-zero yield in C40 are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	12098/7113/4765	0/0/0	6.2	1.012	0.743	73.2	0.916	0.653	17.9	1.023	0.461
2	12086/7234/3926	0/0/0	6.3	1.007	0.749	71.7	0.928	0.653	20.9	1.006	0.442
3	11248/7954/3569	0/0/0	12.4	0.967	0.765	54.4	0.968	0.684	32.5	1.072	0.485
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
76	11628/7018/4669	0/0/0	9.2	1.003	0.765	71.4	0.905	0.649	17.1	1.013	0.464
77	12113/6183/4146	0/0/0	6.2	1.023	0.734	83.5	0.869	0.633	9.0	0.864	0.380
Mean	**11781/6993/3887**	**0/0/0**	**8.1**	**1.016**	**0.737**	**72.8**	**0.908**	**0.644**	**18.0**	**0.991**	**0.437**
	mean		8.55	1.006	0.753	72.24	0.907	0.648	17.89	0.981	0.419
	COV		0.37	0.034	0.027	0.09	0.022	0.017	0.29	0.054	0.106

Combined grading according to MOE and density   All spruce   All samples are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11135/7340/4793	433/302/218.5	12.2	0.976	0.765	64.4	0.918	0.655	20.6	1.037	0.520
2	12086/7234/3926	384/324/78.5	6.3	1.007	0.749	71.6	0.928	0.653	21.0	1.006	0.442
3	11246/7977/3590	349.5/319.5/231.5	12.4	0.965	0.767	53.9	0.970	0.687	32.9	1.075	0.486
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	11049/7360/3603	442/287/289.5	12.6	0.977	0.765	63.8	0.918	0.654	22.9	1.006	0.441
100	12113/6183/4146	374.5/235/279	6.2	1.023	0.734	83.5	0.869	0.633	9.0	0.864	0.380
Mean	**11789/6961/3907**	**413.5/320.7/155.0**	**7.9**	**1.023**	**0.729**	**73.3**	**0.904**	**0.640**	**17.7**	**1.001**	**0.422**
	mean		8.78	0.941	0.767	71.97	0.907	0.649	17.86	0.980	0.426
	COV		0.50	0.194	0.036	0.11	0.023	0.018	0.30	0.060	0.129

Combined grading according to MOE and density   All spruce   Only samples with non-zero yield in C40 are included

			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11135/7340/4793	433/302/218.5	12.2	0.976	0.765	64.4	0.918	0.655	20.6	1.037	0.520
2	12086/7234/3926	384/324/78.5	6.3	1.007	0.749	71.6	0.928	0.653	21.0	1.006	0.442
3	11246/7977/3590	349.5/319.5/231.5	12.4	0.965	0.767	53.9	0.970	0.687	32.9	1.075	0.486
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
90	11049/7360/3603	442/287/289.5	12.6	0.977	0.765	63.8	0.918	0.654	22.9	1.006	0.441
91	12113/6183/4146	374.5/235/279	6.2	1.023	0.734	83.5	0.869	0.633	9.0	0.864	0.380
Mean	**11400/6994/3876**	**420.5/323.1/155.4**	**10.5**	**0.977**	**0.757**	**70.3**	**0.903**	**0.643**	**18.1**	**0.993**	**0.437**
	mean		9.64	0.994	0.767	70.69	0.908	0.649	18.38	0.983	0.426
	COV		0.37	0.038	0.036	0.10	0.023	0.019	0.28	0.054	0.112

Table 4 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 1250$)

Grading according to MOE only	**All spruce**	**All samples are included**								
			C40			C30			C18	
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11498/7413/3565	0/0/0	10.3	0.984	0.753	65.3	0.931	0.656	23.8	1.014	0.445
2	15531/7196/3849	0/0/0	0.1	0.334		78.4	0.936	0.662	20.6	1.005	0.439
3	11950/7179/4698	0/0/0	7.0	1.009	0.767	71.5	0.921	0.654	18.9	1.027	0.489
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	15515/6556/3176	0/0/0	0.1	0.334		86.1	0.900	0.644	13.4	0.863	0.383
100	11669/6782/2777	0/0/0	8.8	1.006	0.757	74.8	0.894	0.637	16.1	0.917	0.366
Mean	**12653/6932/3846**	**0/0/0**	**3.7**	**1.090**	**0.766**	**78.1**	**0.910**	**0.638**	**17.3**	**0.993**	**0.411**
	mean		6.61	0.883	0.755	74.9	0.907	0.647	17.27	0.971	0.417
	COV		0.71	0.309	0.031	0.1	0.020	0.016	0.265	0.057	0.101

Grading according to MOE only	**All spruce**	**Only samples with non-zero yield in C40 are included**								
			C40			C30			C18	
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11498/7413/3565	0/0/0	10.3	0.984	0.753	65.3	0.931	0.656	23.8	1.014	0.445
2	11950/7179/4698	0/0/0	7.0	1.009	0.767	71.5	0.921	0.654	18.9	1.027	0.489
3	12548/6421/3513	0/0/0	4.0	1.074	0.695	83.5	0.883	0.636	11.8	0.860	0.387
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
77	11285/6584/3902	0/0/0	12.1	0.968	0.763	73.8	0.879	0.635	13.1	0.924	0.374
78	11669/6782/2777	0/0/0	8.8	1.006	0.757	74.8	0.894	0.637	16.1	0.917	0.366
Mean	**11810/6981/3803**	**0/0/0**	**7.9**	**1.01**	**0.739**	**73.2**	**0.906**	**0.642**	**17.9**	**0.986**	**0.438**
	mean		8.46	1.010	0.755	72.47	0.907	0.648	17.90	0.975	0.424
	COV		0.41	0.038	0.031	0.09	0.020	0.016	0.25	0.056	0.100

Combined grading according to MOE and density	**All spruce**	**All samples are included**								
			C40			C30			C18	
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11197/7038/3561	459/386/47	9.9	0.989	0.781	67.3	0.916	0.654	22.0	1.006	0.434
2	10876/7560/3841	478/357/167	9.6	0.976	0.802	63.3	0.944	0.698	26.1	1.035	0.457
3	11950/6929/4848	405/379.5/166	7	1.009	0.766	72.6	0.913	0.642	17.5	1.035	0.455
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	12929/6481/3209	426/357.5/5.5	2.6	1.090	0.822	83.6	0.898	0.649	13.3	0.861	0.404
100	11719/6782/2777	436/265.5/128.5	7.9	1.027	0.777	75.7	0.895	0.637	16.1	0.917	0.366
Mean	**11827/6940/3816**	**412.5/323.9/141.9**	**7.6**	**1.028**	**0.768**	**73.9**	**0.903**	**0.638**	**17.5**	**1.001**	**0.415**
	mean		8.44	0.956	0.770	72.55	0.907	0.647	17.79	0.974	0.423
	COV		0.49	0.184	0.034	0.10	0.020	0.018	0.27	0.059	0.111

Combined grading according to MOE and density	**All spruce**	**Only samples with non-zero yield in C40 are included**								
			C40			C30			C18	
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11197/7038/3561	459/386/47	9.9	0.989	0.781	67.3	0.916	0.654	22.0	1.006	0.434
2	10876/7560/3841	478/357/167	9.6	0.976	0.802	63.3	0.944	0.698	26.1	1.035	0.457
3	11950/6929/4848	405/379.5/166	7.0	1.009	0.766	72.6	0.913	0.642	17.5	1.035	0.455
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
90	12929/6481/3209	426/357.5/5.5	2.6	1.090	0.822	83.6	0.898	0.649	13.3	0.861	0.404
91	11719/6782/2777	436/265.5/128.5	7.9	1.027	0.777	75.7	0.895	0.637	16.1	0.917	0.366
Mean	**11456/6966/3817**	**419.0/324.5/137.3**	**10.2**	**0.977**	**0.757**	**70.9**	**0.902**	**0.641**	**17.9**	**0.991**	**0.432**
	mean		9.27	0.997	0.770	71.38	0.907	0.647	18.13	0.979	0.425
	COV		0.36	0.036	0.034	0.09	0.020	0.018	0.26	0.055	0.111

Table 4 (cont): Grading results of population of 10000, settings are based on 100 samples ($n_{\text{sample}} = 2500$)

Grading according to MOE only	**All spruce**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	12098/7313/3615	0/0/0	6.1	1.001	0.776	71.9	0.935	0.697	21.4	0.997	0.476
2	12497/6553/4476	0/0/0	4.0	1.060	0.855	82.9	0.901	0.653	11.4	0.890	0.391
3	11500/6679/4348	0/0/0	10.4	0.962	0.770	75.1	0.896	0.651	13.0	0.899	0.421
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	13063/7117/3646	0/0/0	2.2	1.101	0.828	78.6	0.936	0.682	18.7	0.963	0.478
100	11739/6831/3723	0/0/0	8.6	0.972	0.764	75.2	0.912	0.656	15.5	0.903	0.434
Mean	**12170/6850/3673**	**0/0/0**	**5.7**	**1.018**	**0.775**	**78.0**	**0.919**	**0.658**	**15.7**	**0.907**	**0.437**
	mean		7.02	0.956	0.780	76.6	0.914	0.664	15.68	0.915	0.433
	COV		0.47	0.206	0.033	0.07	0.016	0.026	0.213	0.045	0.083

Grading according to MOE only	**All spruce**	**Only samples with non-zero yield in C40 are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	12098/7313/3615	0/0/0	6.1	1.001	0.776	71.9	0.935	0.697	21.4	0.997	0.476
2	12497/6553/4476	0/0/0	4.0	1.060	0.855	82.9	0.901	0.653	11.4	0.890	0.391
3	11500/6679/4348	0/0/0	10.4	0.962	0.770	75.1	0.896	0.651	13.0	0.899	0.421
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
94	13063/7117/3646	0/0/0	2.2	1.101	0.828	78.6	0.936	0.682	18.7	0.963	0.478
95	11739/6831/3723	0/0/0	8.6	0.972	0.764	75.2	0.912	0.656	15.5	0.903	0.434
Mean	**11968/6863/3659**	**0/0/0**	**6.9**	**0.998**	**0.785**	**76.6**	**0.916**	**0.655**	**15.9**	**0.906**	**0.442**
	mean		7.39	1.000	0.780	76.07	0.914	0.664	15.84	0.917	0.435
	COV		0.40	0.042	0.033	0.06	0.016	0.025	0.21	0.045	0.082

Combined grading according to MOE and density	**All spruce**	**All samples are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11197/7088/3661	459/388.5/52	10.3	0.972	0.763	67.0	0.936	0.678	22.1	1.014	0.469
2	12480/6571/4475	350/247.5/175	4.1	1.057	0.78	82.5	0.902	0.652	11.7	0.895	0.392
3	10969/6468/4587	487.5/302.5/148	7.7	0.972	0.76	80.1	0.893	0.637	10.3	0.889	0.416
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	13083/6969/4424	413.5/369/152	2.1	1.097	0.829	79.0	0.937	0.682	17.3	0.984	0.473
100	10771/6783/3748	490.5/332/190.5	8.1	0.967	0.756	76.3	0.911	0.647	15.0	0.909	0.416
Mean	**11696/6847/3656**	**413.4/324.0/146.1**	**8.7**	**0.967**	**0.773**	**75.0**	**0.913**	**0.656**	**15.7**	**0.903**	**0.439**
	mean		8.17	0.983	0.773	75.06	0.915	0.662	16.08	0.921	0.434
	COV		0.39	0.100	0.029	0.07	0.016	0.024	0.21	0.049	0.081

Combined grading according to MOE and density	**All spruce**	**Only samples with non-zero yield in C40 are included**									
			C40			C30			C18		
			Yield	$\frac{f_{0.05}}{40}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{30}$	$\frac{f_{0.005}}{f_{0.05}}$	Yield	$\frac{f_{0.05}}{18}$	$\frac{f_{0.005}}{f_{0.05}}$
		Optimum grading	**54.4**	**1.004**	**0.974**	**31.8**	**0.914**	**0.923**	**13.3**	**0.895**	**0.790**
1	11197/7088/3661	459/388.5/52	10.3	0.972	0.763	67.0	0.936	0.678	22.1	1.014	0.469
2	12480/6571/4475	350/247.5/175	4.1	1.057	0.780	82.5	0.902	0.652	11.7	0.895	0.392
3	10969/6468/4587	487.5/302.5/148	7.7	0.972	0.760	80.1	0.893	0.637	10.3	0.889	0.416
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
98	13083/6969/4424	413.5/369/152	2.1	1.097	0.829	79.0	0.937	0.682	17.3	0.984	0.473
99	10771/6783/3748	490.5/332/190.5	8.1	0.967	0.756	76.3	0.911	0.647	15.0	0.909	0.416
Mean	**11650/6853/3651**	**413.4/324.5/145.1**	**9.1**	**0.965**	**0.774**	**74.6**	**0.913**	**0.656**	**15.8**	**0.903**	**0.441**
	mean		8.25	0.992	0.773	74.92	0.915	0.663	16.15	0.922	0.435
	COV		0.37	0.040	0.029	0.07	0.016	0.023	0.20	0.048	0.081

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Title

Analysis of strength grading of sawn timber based on numerical simulation

Abstract

Numerical simulation is used to analyse machine strength grading of sawn timber. Starting from correlations between grade determining properties and measured grading properties, sets of values of properties are generated and machine grading simulated including the determination of settings and application of the settings to another generated sample.

Results are obtained concerning the statistical effect of sample size to the accuracy of grading and yield to various grades. Also the lower tail of strength distribution of various grades was analysed with the conclusion that the highest grade has relatively better strength distribution than the lower grades.

Keywords
timber, construction timber, sawn timber, spruce, materials properties, strength, strength grading, numerical simulation, accurancy

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