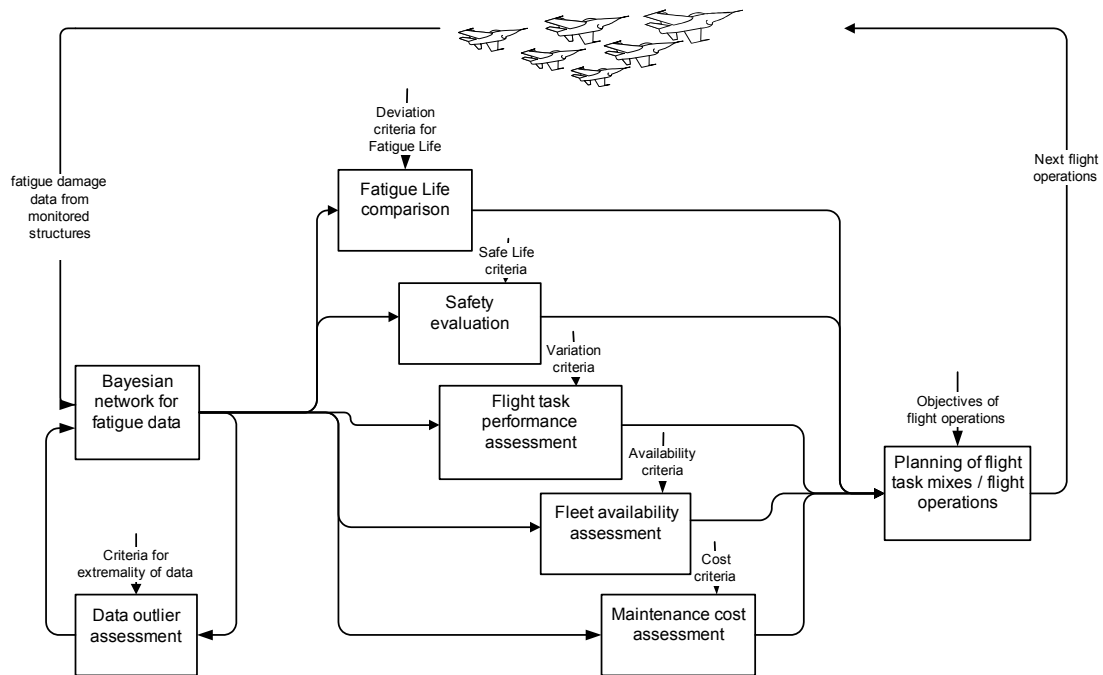


## Fatigue analysis framework to support fleet management



# Fatigue Analysis for Fleet Management Using Bayesian Networks

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VTT Industrial Systems

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Abstract <p>A fatigue analysis framework for estimating and predicting (cumulative) fatigue damage of aircraft structures, based on fatigue damage data from a fleet of aircraft, is described. The data model is defined by a Bayesian Network. Assessments related to safety, maintenance and life cycle cost, flight task performance and fatigue damage distribution between structures, are supported.</p> <p>The fatigue analysis framework guides the flight planner in specifying flight syllabi that maximise operational and training effectiveness while simultaneously meeting safety and cost constraints.</p> <p>The implementation of the fatigue analysis framework requires computerised functions for data sorting, algorithms related to the Bayesian statistical inferences of the fatigue damage data model, and a graphical display of multiple analysis results.</p>		
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## Foreword

The objective of the working report is to outline a framework for fatigue life analysis and prediction to support the fleet management of FAF aircraft. The framework is based on Bayesian Network modelling. The research has been conducted during 2004 in cooperation between Aslak Siljander (Group Manager), Sauli Liukkonen, Mika Bäckström, Keijo Koski and Tony Rosqvist. Research Professor Urho Pulkkinen provided comments and feedback to the development of the basic data model. The author wants express his gratitude to the FAF HQ for financing the research. It is the hope of the author that the working report will raise comments within the community of fleet managers controlling the usage of FAF aircrafts and fatigue experts in general. It is also the hope of the author that the working report will support and guide the preparation of future project proposals aimed at developing fleet and fatigue management of the FAF aircraft.

Espoo 1.6.2005

Tony Rosqvist

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# 1. Introduction

The motivations for the working report are the following statements in the VTT report BVAL33-011139 / AOS (Siljander, 2001):

”Fatigue management covers the organisation and management of any functions required to keep the FAF aircraft fleet flying until the planned Out of Service Date without the need for major modifications or repairs to the aircraft’s load carrying structure, while enabling the operational and training objectives to be met simultaneously precluding any issues relating to structural fatigue to affect [compromise] the safety of flight, aircraft availability or costs of operation.” (section 2, p. 5).

”The goal for the fatigue management of the FAF can currently be formulated as follows: *Adjust the content of training syllabi and the peacetime operational usage such that operational and training effectiveness is maximised while simultaneously meeting a given Out of Service Date*” (ch. 3, p. 5).

The aim of the working report is to formulate more precisely the fatigue management challenges stated above. The scope is limited to problem formulation and a mathematical outline of a fatigue analysis framework supporting fatigue management of a fleet of aircraft. Possible operative constraints in implementing the analysis framework in the currently adopted fatigue management functions are not addressed.

The main challenge for the flight syllabi planner is to specify sets of flight tasks for each aircraft such that the flight syllabi maximises the value of the training for the fleet while complying with safety and cost criteria. A specific challenge is to specify flight syllabi such that the life cycle cost constraint of the fleet is met when the planned Out of Service date is reached.

The aircraft perform different sets of flight tasks i.e. flight syllabi, where a flight task can be viewed as the basic fatigue load unit or ‘building block’ in the syllabi. A flight syllabus is a set of flight tasks with a specific training objective. When a fleet performs a given flight training program, the participating planes usually perform different flight syllabi in order for the fleet to fulfil the overall objective of the training program. The structures of the aircraft are therefore subject to different numbers of flight tasks, each with its own fatigue load and damage characteristic.

From the point of view of fleet management it is desirable to get a prediction whether any structure of the aircraft will exceed a predefined level of critical fatigue damage with some probability during the next flight syllabus (set of flight tasks). The criticality

can be evaluated with respect to safety, NDT – testing requirement, replacement need of structure, maintenance cost of flight syllabi, etc.

The fatigue damage related to a flight task is not exactly estimable / predictable. For similar flight tasks, the fatigue load and damage on a structure will vary due to variability in pilot performance and external conditions. The uncertainty related to the fatigue damage of a structure, can, however, be quantified by collecting data in the form of {flight task id, fatigue damage measurement value} and making statistical inference on fatigue damage parameters of a data model.

The probabilistic inference on data model parameters is based on strain gauge- i.e. OLM-measurements, and acceleration sensor- i.e. *g*-measurements depicting *fatigue load* of a structure. These measurement scales are functionally related to a fatigue index, such as *FI* or *FLE*, depicting *fatigue damage*. It is generally considered that OLM-measurements better capture the structural damage caused by the load, compared to *g*-measurements, for some structures. OLM measurement systems are, however, more expensive and usually have to be retrofitted to the structures.

OLM-measurement values are obtained from a subpopulation of structures whereas *g*-measurement values are obtained from all structures in the fleet. Obviously, it is preferable to use all information (evidence) that is available for the inference. An inferential procedure that links different types of evidence is provided by Bayesian statistical theory (Gelman et al., 2004). The inferential structure is depicted by Bayesian Networks. The Bayesian approach is therefore adopted in the development the fatigue analysis framework introduced in the working report.

## 2. Fatigue damage analysis framework to support fleet management

### 2.1 Nomenclature\*

<i>Parameter</i>	<i>Definition</i>
Flight task	A defined rule for performing a flight, usually indicated by syllabus code and mission type.
Flight syllabus	Set of flight tasks for a single aircraft with specific training objectives.
Flight syllabi	Set of flight tasks assigned to several aircraft with specific training objectives of the fleet.
$i = 1, \dots, N$	Flight task index.
$j = 1, \dots, L_i, \dots, \tilde{L}_i$	Index of occurrence of flight task $i$ . $L_i$ denotes the performed number of flight task $i$ in the fleet. $\tilde{L}_i$ denotes the number of flight task $i$ planned for the fleet.
$k = 1, \dots, K$	Index of structure unit or aircraft ('tailnumber')
$j = 1, \dots, L_i^k, \dots, \tilde{L}_i^k$	Index of the occurrence of flight task $i$ related to structure unit $k$ . $L_i^k$ denotes the past number of flight task $i$ , whereas $\tilde{L}_i^k$ denotes the planned number of flight task $i$ for structure unit $k$ .  We have the relationships $L_i = \sum_{k=1}^K L_i^k$ and $\tilde{L}_i = \sum_{k=1}^K \tilde{L}_i^k$ .
$m(\text{OLM})$	Measurement signal obtained from the OLM-system with strain gauges. The signal represents fatigue <i>load</i> of the monitored structure (Fig. 1a).
$m(g)$	Measurement signal obtained from the $g$ -measurement system with acceleration sensors. The signal represents fatigue <i>load</i> of the monitored structure (Fig. 1a).
$x$	Indirect measure $x = f(m(\text{OLM}))$ of fatigue <i>damage</i> . Function $f$ is an increasing monotonous function (Fig. 1a).
$y$	Indirect measure $y = h(m(g))$ of fatigue <i>damage</i> . Function $h$ is an increasing monotonous function (Fig. 1a).
$\mu_i$	Mean of fatigue damage related to flight task $i$ .
$\sigma_i$	Standard error of fatigue damage related to flight task $i$ .



$\eta_{iy}$	Bias of measurement value $y$ .
$\sigma_{iy}$	Standard error of measurement value $y$ .
$\sigma_{ix}$	Standard error of measurement value $x$ .
$\theta_{ij}^k$	Fatigue damage of structure unit $k$ induced by the $j^{\text{th}}$ flight of flight task $i$ .
$\psi^k$	Cumulative fatigue damage or fatigue life of structure unit $k$ , also denoted by fatigue index $FI^k$ (reference levels are defined in Fig. 1b).
$\tilde{\psi}^k$	Cumulative fatigue damage of structure unit $k$ after planned set of flight tasks, also denoted by $\tilde{FI}^k$ (reference levels are defined in Fig. 1b).
$\mathbf{H}$	Matrix of the number of different flight tasks performed involving all similar structures of the fleet (see text).
$\mathbf{H}^k$	Array of the number of different flight tasks performed involving structure unit $k$ (see text).
$\tilde{\mathbf{H}}$	Matrix of the number of different flight tasks planned for the fleet (see text).
$\tilde{\mathbf{H}}^k$	Matrix of the number of different flight tasks planned to involve structure unit $k$ (see text).
$\mathbf{Z} = (\mathbf{z}_1, \dots, \mathbf{z}_i, \dots, \mathbf{z}_N)$	Data matrix of fatigue damage measurement value pairs $\mathbf{z}_i = ((x_{i1}, y_{i1}), \dots, (x_{ij}, y_{ij}), \dots)$ from the whole fleet listed according to flight task $i$
$\mathbf{Z}^k = (\mathbf{z}_1^k, \dots, \mathbf{z}_N^k)$	Data matrix of fatigue damage measurement value pairs $\mathbf{z}_i^k = ((x_{i1}^k, y_{i1}^k), \dots, (x_{ij}^k, y_{ij}^k), \dots)$ of structure unit $k$ ( $\mathbf{Z}^k \subset \mathbf{Z}$ )

\*the most important quantities in the fatigue analysis framework are listed

Measures related to a flight task  $i$ :

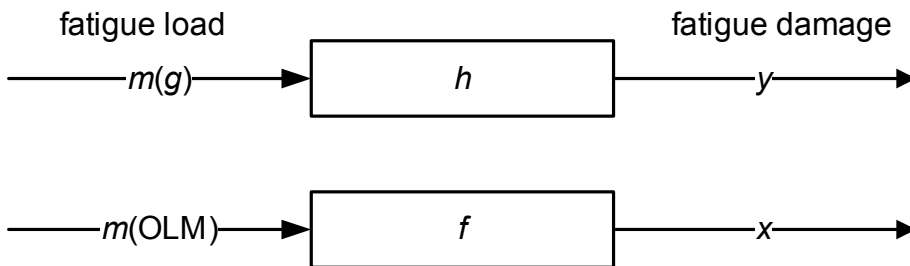


Figure 1a. Measures (data) related to the fatigue of an aircraft structure subject to flight tasks.

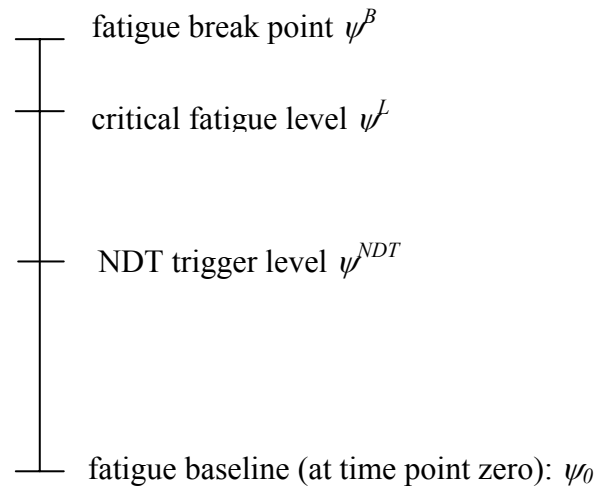


Figure 1b. Cumulative fatigue damage (fatigue life) reference levels used to define probabilistic decision criteria used in the assessment and evaluation of the condition of an aircraft structure.

## 2.2 Description of the fatigue analysis framework

### 2.2.1 Basic decision analyses for fleet management

In the following, we treat an aircraft structure subject to similar flight tasks as a statistical unit. A statistical population is the group of similar aircraft structures subject to similar flight tasks. Variations in the observed fatigue damage measurements are due to variations in plane&pilot combinations and human&machine interactions (pilot performance), as well as measurement errors. As there are no track record on the plane and pilot combinations, it is not possible by statistical means to compare the fatigue damage contributions of the pilots; if an extreme fatigue measurement value is observed, we need separate analyses to identify the causes. As a consequence, we hold all measurement values obtained from the pilot & plane – units statistically equally representative for the considered flight task.

Due to the probabilistic approach, decision-making is guided by defining probabilistic decision criteria (risk criteria) representing fleet management strategy. Risk criteria can be defined to support

- i) *data outlier indication* to attract attention to possible extreme fatigue damages or faults in the measurement system;
- ii) *fatigue life comparison* to balance the fatigue loading / damage of structures in the fleet,

- iii) *safety evaluation* to check whether risk limits are complied with or not;
- iv) *flight task performance assessment* to support specification of operational requirements related to the execution of a flight task;
- v) *fleet availability assessment* to check the availability of the required number of aircraft for a planned flight operation;
- vi) *maintenance cost assessment* to check if budget and Life Cycle Cost constraints are met.

The above assessments and evaluations may be performed after every OLM- and *g*-measurement. As the fatigue damage data history grows, refined estimations of the fatigue damages related to flight tasks, and predictions of the cumulative fatigue damage related to the planned flight syllabi, can be made. The assessments and evaluations may indicate a need for re-planning the flight syllabi. The iterative nature of fatigue load and damage measurement, fatigue damage estimation, fatigue life prediction, and syllabi planning, within the fatigue analysis framework, is shown in Fig. 2.

The analysis framework can be viewed as an empirical fatigue analysis framework supporting what-if analysis of optional flight syllabi and training programs. As the planning of the flight syllabi is a complex task of tailoring together sets of flight tasks for each aircraft, with several operational and training objectives to achieve together with safety and cost constraints, planning cannot, in general, be reduced to an optimisation problem yielding an optimal assignment of flight tasks and syllabi for pilot & plane - units until the Out of Service Date.

## Fatigue analysis framework to support fleet management

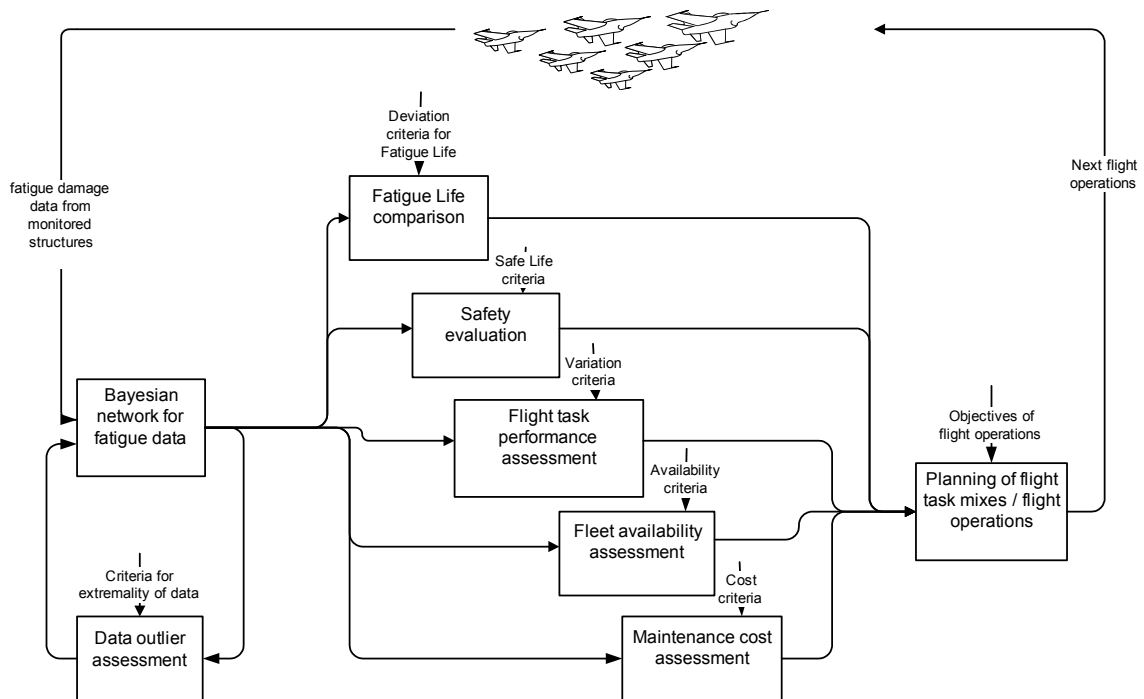


Figure 2. Analysis framework for guiding the planning of flight syllabi iteratively as observations on fatigue damage become available.

### 2.2.2 Main assumption for modelling fatigue damage

The main assumption for modelling fatigue damage related to a flight task, and the accumulation of fatigue damage as a function of flight tasks, is *order-invariance*, i.e. *similar flight tasks damage a structure statistically similarly irrespective of the order and distribution of the flight tasks in a flight syllabi*.

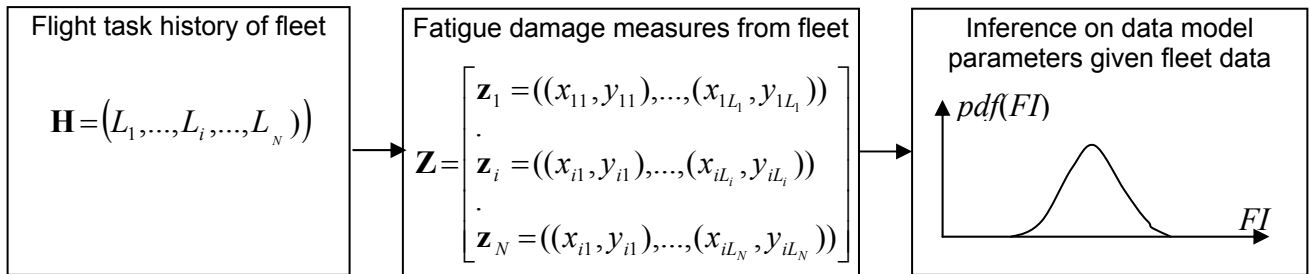
As a first corollary, the overall damage incurred by a flight syllabus (comprising of similar and different flight tasks) is statistically the same regardless of the order of the flight tasks in the syllabus. As a second corollary, variations in the external environment and the man-machine interactions (pilot performance) are assumed to be jointly statistically stable (with no trend in time).

As a consequence of the fatigue damage order-invariance assumption, we can model fatigue damage accumulation by an *additive fatigue damage function* (see section 2.3.1).

### 2.2.3 The inferential procedure for estimation and prediction

The inferential procedure is two-staged: Firstly, all fatigue measurement values obtained from all structures in the fleet, and related to similar structures and flight tasks, are used for statistical inference on parameters denoting fatigue damage. Secondly, the obtained probability functions on the fatigue damage parameters are used for estimation and prediction of the cumulative fatigue damage (=fatigue life) of a structure unit (aircraft level). The information flow between fatigue measurement, inference, estimation, prediction, and assessment, in the fatigue analysis framework, is outlined below:

#### Data and inference at fleet level (inference step 1):

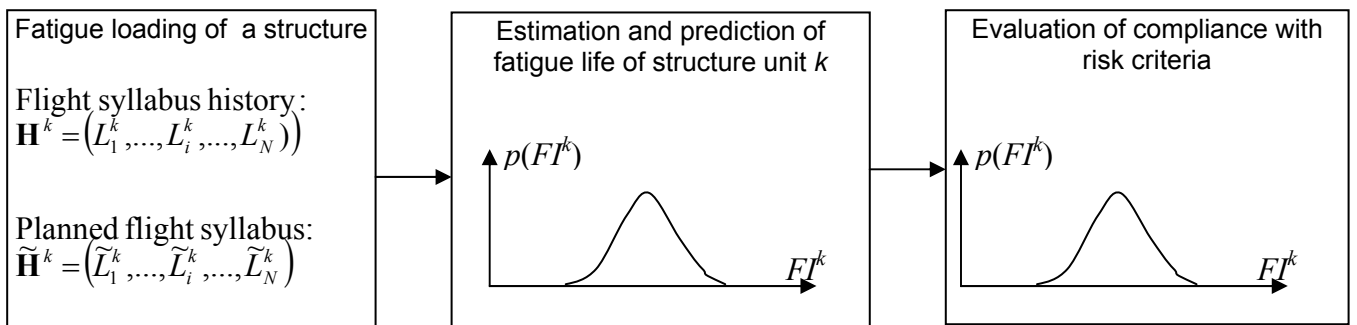


Vector denoting the flight task history of the fleet as the number of flights distributed over the flight tasks  $i$ , i.e. the total fatigue load in terms of flight task type related to an assemble of structures of some type.

Data matrix representing the fatigue damage data:  $(x, y) = (f(m(OLM)), g(m(g)))$  data pairs. Note that for most data entries  $x = N/A$  as OLM-measurements are only available from a few structures!

Set of probability functions related to the data model parameters. The primary parameters are the fatigue damage parameters related to the different flight tasks and the cumulative fatigue life  $FI$ .

#### Prediction and ‘what-if’ analysis at aircraft level (inference step 2):



Matrix denoting planned flight syllabus, i.e. the number of different flight tasks that structure unit  $k$  will be subject to.

Probability function of fatigue life of structure unit  $k$  subject to the fatigue loads of the performed (planned) flight tasks

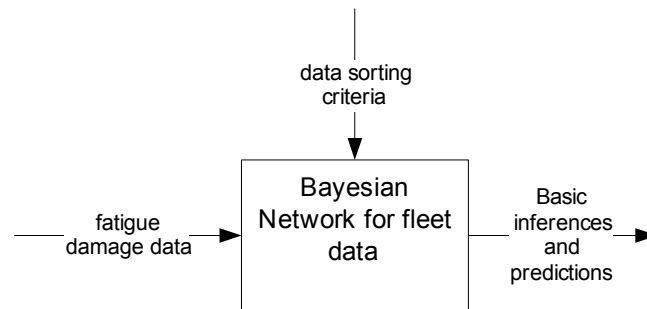
Are structure-specific risk criteria complied with or not? If yes, then no extra action. If no, perform required action.

In the the second step, estimations and predictions on the fatigue life of a structure unit will be computed and used in the evaluation of compliance with given risk criteria for decision-making regarding the feasibility and acceptability of the planned flight syllabus of each aircraft. The outcome of the evaluation guides the flight syllabus planner in a manner corresponding to ‘what-if’ analyses.

## 2.3 Bayesian Network model for fatigue damage data

### 2.3.1 Basic inferences and predictions

Reference to Fig. 2:



1. Each plane has a number of different structures (wing, hull, etc) which are monitored by a set of sensors. In the following, one structure type will be considered without any loss of generality of the approach.
2. Each structure is subject to fatigue loads determined by the flight tasks related to a flight syllabus. The different flight tasks  $i = 1, \dots, N$  occur in any order and number in different flight syllabi. The content of the flight syllabi is determined by the training objectives.
3. The order of fatigue loads in the fleet, related to flight task  $i$ , is indicated by  $j = 1, \dots, L_i$ , where  $L_i$  is the current accumulated number of performed flight task  $i$ . The order of fatigue loads from flight task  $i$ , experienced by the structure unit  $k$ , is indicated by  $j = 1, \dots, L_i^k$ .
4. Each performed flight task is coupled with a  $g$ -measurement value indicating fatigue load of a structure. Each performed flight task is also coupled with an OLM-measurement value indicating fatigue load of a structure that belongs to the subpopulation of structures (i.e. *fleet leaders*) that are OLM-monitored.
5. Each structure goes through a set of flight tasks before the next fatigue load and fatigue damage analysis.

6. Based on the basic assumptions stated in section 2.1., an *additive fatigue damage function* defining the *fatigue life* (cumulative fatigue damage) of a structure unit  $k$  can be defined as:

$$\psi^k = \sum_{i=1}^N \sum_{j=1}^{L_i^k} \theta_{ij}^k \quad (1)$$

where  $\psi^k$  denotes the fatigue life of a structure unit  $k$ ,  $\theta_{ij}^k$  denotes the underlying (hidden) fatigue damage related to the  $j^{\text{th}}$  flight of flight task  $i$ , and  $L_i^k$  is the number of flight tasks  $i$  performed involving structure unit  $k$ . The quantity  $\psi^k$  will also denote the *fatigue index*  $FI^k$ .

7. In *prediction* the above equation has to be extended to incorporate the planned set of future flight tasks (planned flight syllabus):

$$\tilde{\psi}^k = \sum_{i=1}^N \sum_{j=1}^{L_i^k + \tilde{L}_i^k} \theta_{ij}^k \quad (2)$$

where  $\tilde{\psi}^k$  denotes the *predictive* cumulative fatigue damage of a structure.  $\tilde{L}_i^k$  denotes the number of planned flights of flight tasks  $i$ . The quantity  $\tilde{\psi}^k$  also denotes the *predicted fatigue index*  $\tilde{FI}^k$ .

8. In the Bayesian statistical approach different types of evidence can be combined for the inference on the underlying fatigue damage parameters  $\theta_{ij}^k$ . We use the  $x = f(m(\text{OLM}))$  and the  $y = h(m(g))$  measures as *evidence on fatigue damage* and use them in the inference on the fatigue damage parameters  $\theta_{ij}^k$ ,  $\psi^k$  and  $\tilde{\psi}^k$ .
9. The dependence between the fatigue damage parameters in the data model, and the  $(x,y)$ -measurement values related to flight task  $i$ , are depicted by the Bayesian Network in Fig. 3. For each flight task  $i = 1, \dots, N$ , a similar Bayesian Network is specified. (For those structures that are not OLM-monitored we mark the  $x$ -data entry by N/A);
10. The Bayes Network in Fig. 3 includes, in the first inferential step, all evidence obtained from the whole population of the monitored and similar structures (wing, helm, etc.). In the second inferential step, the fatigue life of each structure unit  $k$  is estimated and/or predicted (see section 2.2.3).

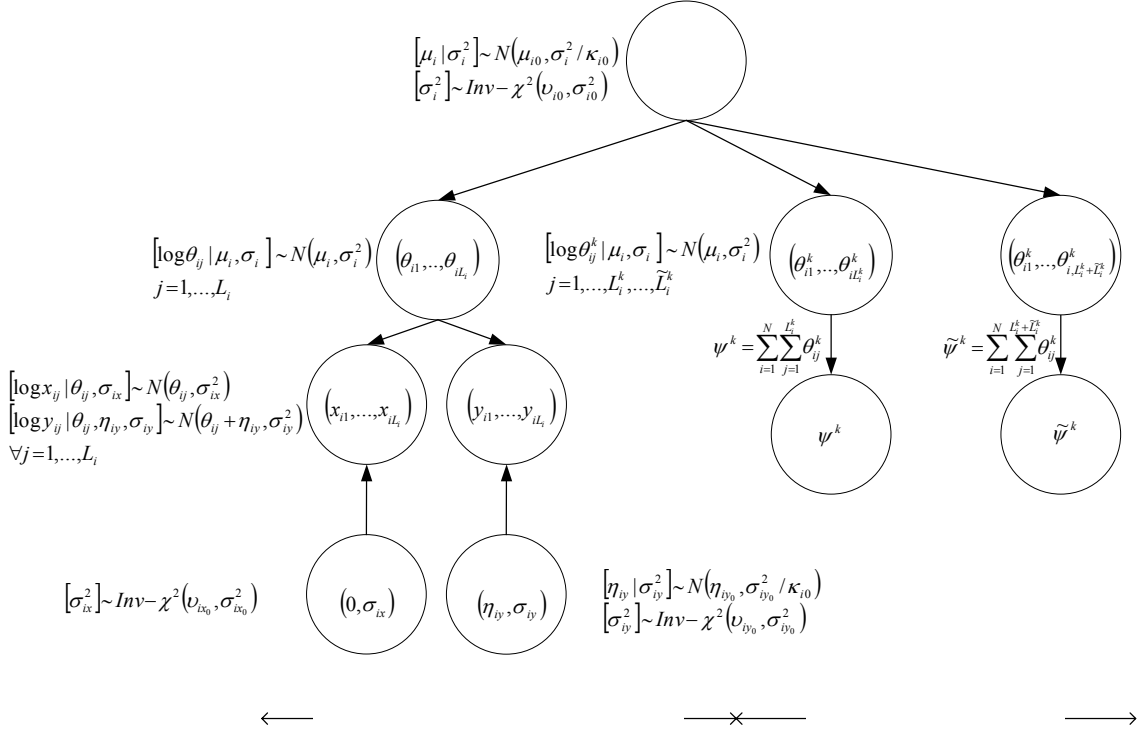


Figure 3. A Bayesian Network depicting the relationship between the fleet data  $\mathbf{z}_i = (x_j, y_j)_{j=1}^{L_i}$ , the hidden fatigue damage parameters  $\theta_{ij}$  and their distribution parameters  $(\mu_i, \sigma_i)$ , the measurement error distribution parameters  $(0, \sigma_{ix})$  and  $(\eta_{iy}, \sigma_{iy})$ , and the estimated and predicted fatigue life parameters  $\psi^k, \tilde{\psi}^k$  of structure unit  $k$ . Index  $i$  denotes flight task, index  $j$  the order of flight, and  $k$  the structure unit.

#### 11. Features of the ‘fatigue damage Bayes Network’:

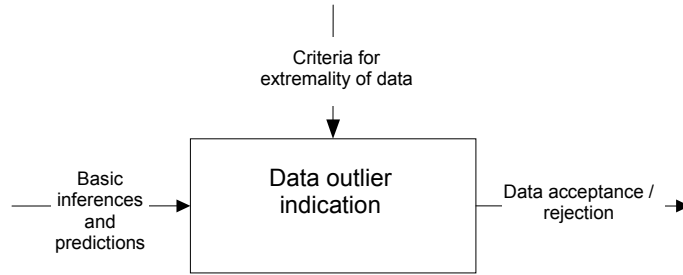
- The variation in the performance of the pilot, the combination of pilot and plane, the external conditions when performing a specified flight task is reflected in variations in the fatigue load and fatigue damage and is represented by the probability function parameters  $(\mu_i, \sigma_i)$ , specified by fleet fatigue damage matrix  $\mathbf{Z}$ .
- It is assumed that the measurement values  $x = f(m(\text{OLM}))$  are unbiased, whereas  $y = h(m(g))$  are biased, with the log-transformed values normally distributed given the parameters  $(0, \sigma_{ix})$ ,  $(\mu_{iy}, \sigma_{iy})$ . Thus the measurement error related to the OLM-measurement from fleet leaders has zero mean.



- The prior probability functions related to the parameter pairs  $(\mu_i, \sigma_i)$  and  $(0, \sigma_{ix})$ ,  $(\mu_{iy}, \sigma_{iy})$  may be defined according to Gelman et al. (2004). Other alternatives are possible.
- In the inference depicted by the Bayes Network we obtain the (predictive) posterior probability functions related to the fatigue damage parameters given the fatigue damage data of the fleet, i.e.  $\{p(\theta_{ij}^k | \mathbf{z}_i), j=1, \dots, L_i^k, \dots, L_i^k + \tilde{L}_i^k, i=1, \dots, N\}$ . These probability functions are then used to compute the fatigue life estimates and predictions used in the various assessments after the data-analysis, as shown in Fig. 2.
- The inference is in practice obtained numerically by Markov Chain Monte Carlo – methods (Gelman et al., 2004) if analytic solutions are not derivable.

### 2.3.2 Data outlier indication

Reference to Fig. 2:



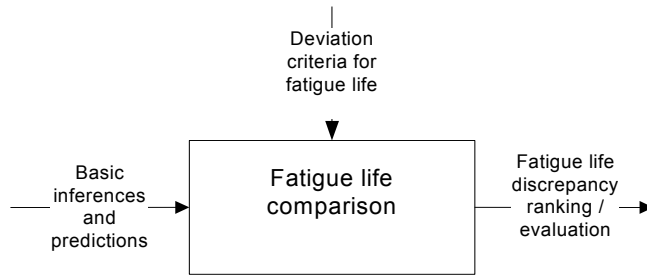
Data analysis / fleet management question: “What are the reasons for extreme fatigue damage measurement value(s)?”

The analysis procedure is as follows:

12. The extremality of an observation of fatigue damage;  $(x_{ij}, y_{ij})$  can be assessed in terms of its likelihood. Denote a  $p$ -fractile related to e.g.  $y$ -data by  $y_p$ . Let the probabilities  $p = p_l$  and  $p = p_u$  define fractiles related to extremely small and big fatigue damage observations, respectively. The probabilities of observing extremal  $y$  – measurement values related to the  $j^{\text{th}}$  realisation of flight task  $i$  are  $P(Y_{ij} > y_{p_u} | \theta_{ij}, \eta_{iy}, \sigma_{iy})$  and  $P(Y_{ij} < y_{p_l} | \theta_{ij}, \eta_{iy}, \sigma_{iy})$ . By specifying the probability limit values  $p_l$  and  $p_u$ , the corresponding  $p$ -fractiles  $y_{p_l}$  and  $y_{p_u}$  can be determined.
13. Extremal fatigue damage  $y$ -observations, low or high, related to flight task I, can now be alerted by the indicator functions  $i^l(y_{ij} < y_{p_l})$  and  $i^u(y_{ij} > y_{p_u})$ , respectively.

### 2.3.3 Fatigue Life comparison

Reference to Fig. 2:



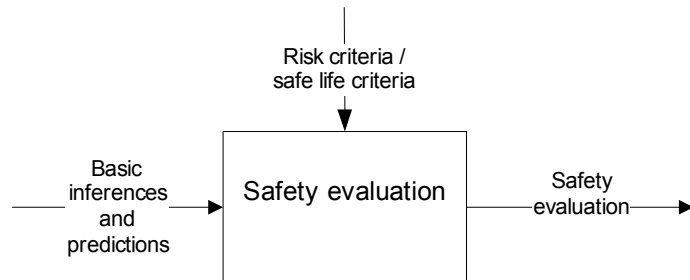
Fleet management question: “Is the population of similar structures in the fleet loaded evenly?”

The analysis procedure is as follows:

14. The estimated structure – specific fatigue lives  $\{\psi^k\}$  can be compared with each other in terms of a deviation measure, such as  $D^k = |\psi^k - \bar{\psi}|$  (where  $\bar{\psi}$  is the expected sum of the random variables  $\psi^k$ ,  $k = 1, \dots, K$ ), to check for balanced fatigue loading of the structures.
15. A probabilistic criterion indicating unbalanced fatigue damage can be defined as  $P(D^k \geq d^C | \mathbf{Z}^k)$ ,  $k=1, \dots, K$  that is the posterior probability of observing a deviation larger than some critical value  $d^C$ .
16. Critical differences in fatigue damages can be alerted by an indicator function  $i^D(P(D^k \geq d^C | \mathbf{Z}^k) \geq \alpha)$ ,  $k=1, \dots, K$ , where  $\alpha$  is a pre-defined risk level ( $\alpha \ll 1$ ).

### 2.3.4 Safety evaluation

Reference to Fig. 2:



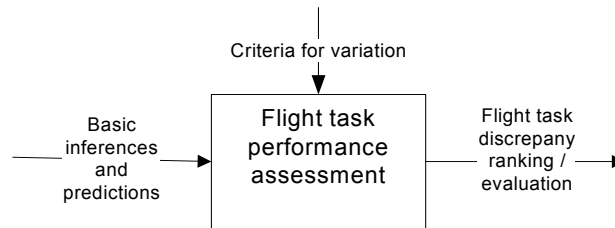
Fleet management question: “Are the next planned flight tasks safe enough?”

The analysis procedure is as follows:

17. As the cumulative fatigue damage  $\psi^k$  of structure unit  $k$  increases it approaches the critical fatigue level  $\psi^L$  specifying the "safe life" of the structure. (This limit should be selected such that the probability of finding a macro crack in the structure is miniscule when this limit is reached.)
18. The predictive posterior probability of structure unit  $k$  exceeding the critical fatigue limit  $\psi^L$ , before the end of the planned flight mixes  $(\tilde{L}_1^k, \dots, \tilde{L}_i^k, \dots, \tilde{L}_N^k)$ , is  $P(\tilde{\Psi}^k(\tilde{L}_1^k, \dots, \tilde{L}_i^k, \dots, \tilde{L}_N^k) \geq \psi^L | \mathbf{Z}^k)$ ,  $k=1, \dots, K$
19. A risk indicator function alerting of a cumulative fatigue damage prediction exceeding the critical fatigue limit  $\psi^L$  by a probability larger than a pre-defined risk level  $\alpha \ll 1$ , can be written as  $i^S(P(\tilde{\Psi}^k \geq \psi^L | \mathbf{Z}^k) \geq \alpha)$

### 2.3.5 Flight task performance assessment

Reference to Fig. 2:



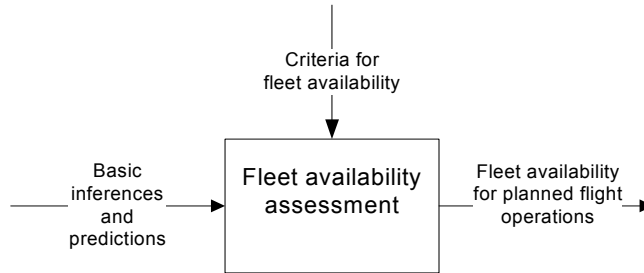
Fleet management questions: “Is the fatigue damage related to any flight task widely spread?”; “Are the technical specifications related to a flight task too broadly stated, leaving room for subjective interpretation about how to perform the flight task?”

The analysis procedure is as follows:

20. A measure that takes into account both the expected value and the standard deviation of the fatigue damage related to flight task  $i$  is the ‘posterior’ coefficient of variation:  $CV_i(\mathbf{z}_i) = E[\sigma_i | \mathbf{z}_i] / E[\mu_i | \mathbf{z}_i]$ .
21. Those flight tasks  $i = 1, \dots, N$ , which are associated with high  $CV$  - values, are more uncertain and may be prioritized for further fatigue damage analyses where the causes of the variations are identified, supporting better specification of the operational requirements related to the flight tasks.
22. Probabilistic criteria can be defined for alerting excessive variation in flight performance.

### 2.3.6 Fleet availability assessment

Reference to Fig. 2:



Fleet management question: “Are the fatigue damage levels of structures such that enough structures are available for the next planned flight program?”

The analysis procedure is as follows:

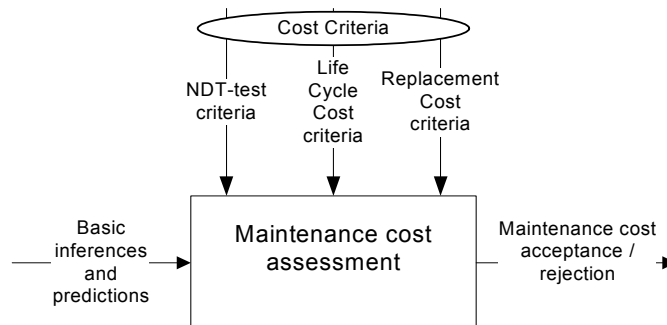
23. The predictive density of the cumulative fatigue damage for a structure depends on the planned flight task mix for the structure:  $p(\tilde{\Psi}^k(\tilde{L}_1^k, \dots, \tilde{L}_i^k, \dots, \tilde{L}_N^k) | \mathbf{Z}^k), k=1, \dots, K$

24. If  $K'$  structures (aircraft) are required to meet the operational objectives set for a flight program then unavailability of enough safe structures (at risk level  $\alpha$ ) can be indicated based on the safety indicator  $i^S$ :  $i^A \left( \sum_{k=1}^K i^S \left( P(\tilde{\Psi}^k \geq \psi^L | \mathbf{Z}^k) \geq \alpha \right) \geq K - K' \right)$

25. If unavailability is indicated, alternative flight task mixes / flight programs need to be designed or structures have to be renewed.

### 2.3.7 Maintenance cost assessment

Reference to Fig. 2:

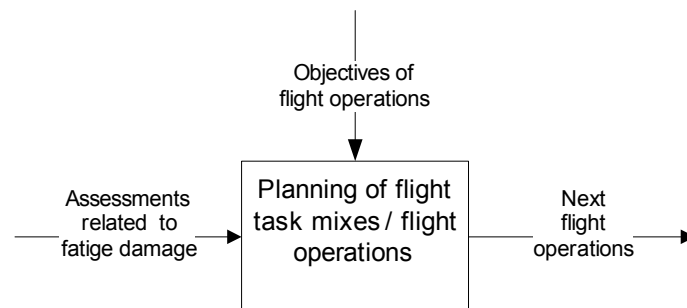


Fleet management question: “Do the next planned flight tasks meet maintenance cost criteria?”

The analysis procedure is as follows:

26. The utilised maintenance strategy is condition-based replacement, i.e. when a structure's fatigue life exceeds, for instance, the critical fatigue life  $\psi^L$ , the structure is considered used, and is replaced by a similar new structure, incurring a cost  $\rho_R$ .
27. The expected number of replacements of similar structures after the next planned fleet operation is  $\bar{n}_R = E \left[ \sum_{k=1}^K i^R_{\{\tilde{\psi}^k > \psi^L\}} \right] = \sum_{k=1}^K P(\tilde{\Psi}^k > \psi^L | \mathbf{Z}^k)$ , where  $i^R(\cdot)$  is an indicator function indicating that the critical fatigue damage limit (safe life) is exceeded.
28. The expected cost of replacements is now  $\bar{c}_R = \rho_R * \bar{n}_R$ .
29. Every planned flight program can be associated with an expected number of NDT-tests to be performed for the  $K$  structures of similar type. Now, the expected number of NDT-tests is  $\bar{n}_{NDT} = E \left[ \sum_{k=1}^K i^{NDT}_{\{\tilde{\psi}^k > \psi^{NDT}\}} \right] = \sum_{k=1}^K P(\tilde{\Psi}^k > \psi^{NDT} | \mathbf{Z}^k)$ , where the indicator function indicates that a NDT-test, with a cost  $\rho_{NDT}$ , is performed when the fatigue damage threshold value  $\psi^{NDT}$  is exceeded.
30. The expected NDT-cost associated with the flight program is  $\bar{c}_{NDT} = \rho_{NDT} * \bar{n}_{NDT}$
31. If cost criteria have been set for NDT- and replacement costs, then compliance / non-compliance can be checked for, steering the planning of flight programs.
32. The cost assessment can be extended to Life Cycle Cost assessment by assessing the above costs for flight programs extending to the planned Out of Service Date of the fleet.

### 2.3.8 Syllabi planning



33. The planning of flight syllabi has to meet several objectives related to training, safety and cost. Safety and cost objectives may be looked as constraints for the specification of flight syllabi. In particular, the planned lifetime of the fleet, i.e. the

Out Of Service date, has to be met. Utilising the fatigue analysis framework, planning is guided by what-if analyses of fatigue life predictions related to flight operations, and relies on the skills of the planner to specify flight syllabi that are feasible.

## **2.4 Refinements of the Bayesian Network model**

### **2.4.1 Refinement of the model by uncoupling planes and pilots**

Ideally, pilots and planes would be systematically mixed after each flight syllabus. A personal track record of the fatigue damage incurred by each pilot would serve as a basis for statistical analysis of deviating pilot performances for particular flight tasks under similar conditions. Such information could be used to monitor and learn from those pilots that achieve the objectives of the flight syllabus with minimal fatigue life expenditure.

### **2.4.2 Value of information**

The fatigue data model can, in principle, be used to assess the limit value of additional information by installing more OLM- measurement systems in the population of aircraft. By more accurate estimations and predictions it is possible to resolve uncertainty and therefore prevent premature deletion of structures based on the adopted risk criteria.

### **3. Software implementation of the fatigue analysis framework**

The implementation of the analysis framework outlined above for fatigue management of a fleet of aircraft requires the development of a software tool. The artefacts of the software development process should be:

- i) a data sorting interface for grouping fatigue load sets which are used as input for the basic inferences in the fatigue analysis framework;
- ii) an algorithm that computes the inferences needed for the different sub-assessments and evaluations;
- iii) a graphical interface for simultaneous display of sub-assessment results;
- iv) a verification report on the proper computer implementation of the Bayesian data model by comparing test data with analytically obtained results;
- v) a validation report based on real fatigue damage measurement values and observations of macro cracks in structures

The resources needed for the development of the framework is estimated to be 1,5–2 man-years.

## **4. Conclusions**

A fatigue analysis framework for estimating and predicting (cumulative) fatigue damage of aircraft structures, based on fatigue damage data from a fleet of aircraft, is described. The data model is defined by a Bayesian Network. Assessments related to safety, maintenance and life cycle cost, flight task performance and fatigue damage distribution between structures, are supported.

The fatigue analysis framework guides the flight planner in specifying flight syllabi that maximise operational and training effectiveness while simultaneously meeting safety and cost constraints.

The implementation of the fatigue analysis framework requires computerised functions for data sorting, algorithms related to the Bayesian statistical inferences of the fatigue damage data model, and a graphical display of multiple analysis results.



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